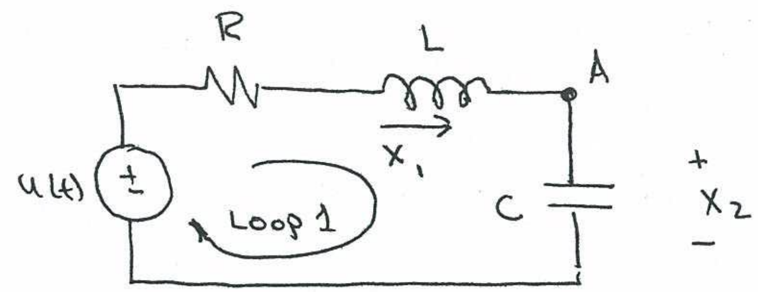
Exercises chapter 9.1

1.



States : x_1 = current through inductor

x_2 = voltage across capacitor

KCL: $x_1 = C\dfrac{dx_2}{dt} = C\dot{x}_2 \Rightarrow \dot{x}_2 = \dfrac{1}{C}x_1$

KVL: $u = L\dfrac{dx_1}{dt} + Rx_1 + x_2 \Rightarrow \dot{x}_1 = \dfrac{u}{L} - \dfrac{R}{L}x_1 - \dfrac{1}{L}x_2$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{R}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}}_{\underline{b}} u$$

2. $\{A, \underline{b}\}$ as in problem 1.

output equation: $y = x_2 \Rightarrow c = [0 \quad 1],\ d = 0$

3. $\{A, \underline{b}\}$ as in problem 1

output equation: $y = u - Rx_1 - x_2 \Rightarrow c = [-R \quad -1],$

$d = 1$

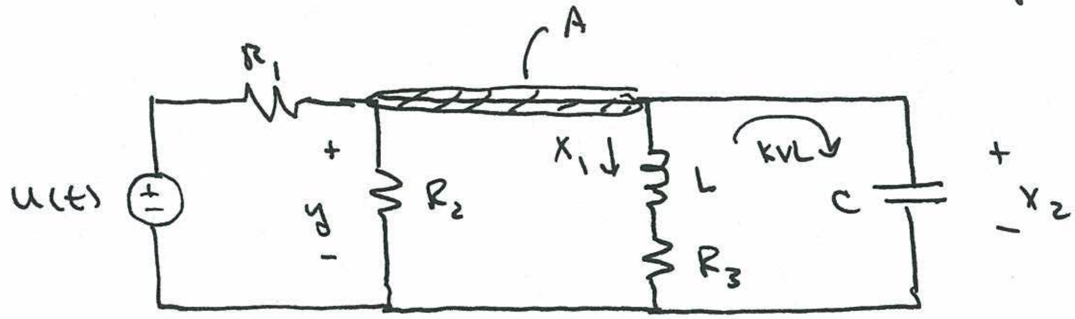
4. $\{A, \underline{b}\}$ as in problem 1

output equation: $y = Rx_1 \Rightarrow c = [R \quad 0],\ d = 0$

Exercises

Chapter 9.1

5.



KCL at node A:

$$\frac{u - x_2}{R_1} = \frac{x_2}{R_2} + x_1 + C\dot{x}_2 \Rightarrow \dot{x}_2 = -\frac{1}{C}x_1 - \frac{1}{C}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)x_2 + \frac{u}{R_1C}$$

KVL, right loop:

$$L\dot{x}_1 + R_3 x_1 = x_2 \Rightarrow \dot{x}_1 = -\frac{1}{R_3 L}x_1 + \frac{1}{L}x_2$$

Output equation: $y = x_2$

In matrix form, the model is:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{R_3 L} & \frac{1}{L} \\ -\frac{1}{C} & -\frac{1}{C}\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{R_1 C} \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0 \cdot u$$

Exercises
Chapter 9.2

1. State variable model for this circuit was created in exercise 1 of chapter 9.1.

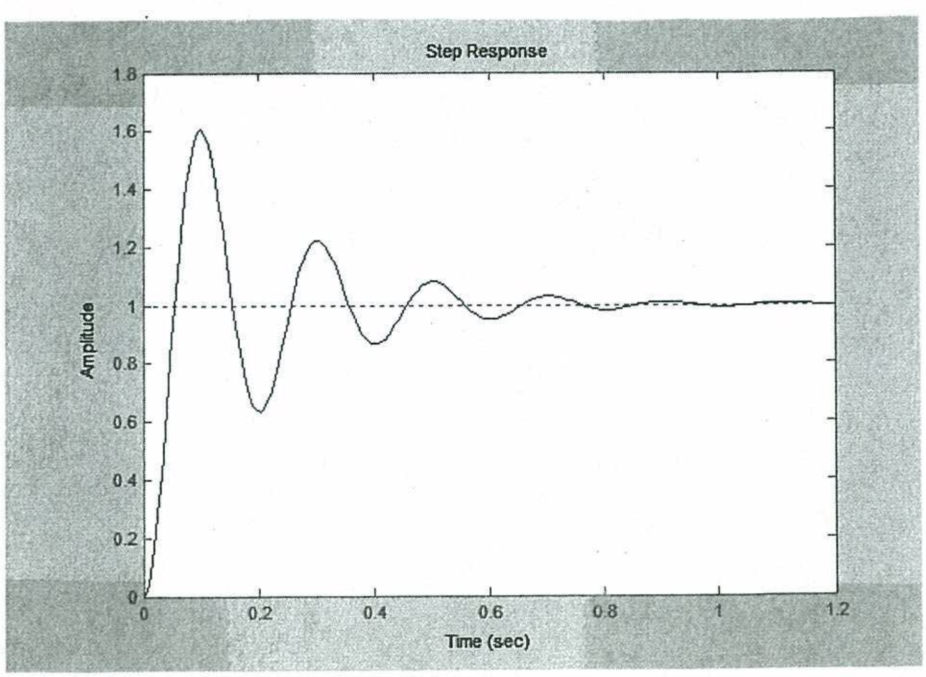
We just need to plug in numbers and use MATLAB's "step" command. The commands I used are below:

```
R = 10;
L = 1;
C = 1e-3;

A = [-R/L  -1/L; 1/C 0];
b = [1/L; 0];
c = [0  1];
d = 0;

sys = ss(A,b,c,d);
step(sys)
```

Resulting Figure:



Exercises
Chapter 9.2

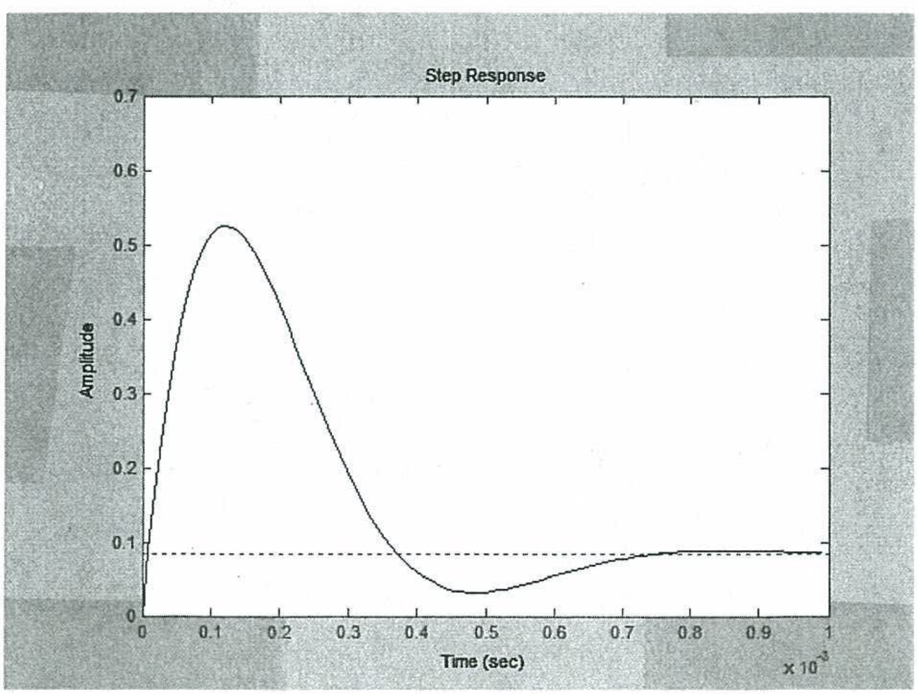
2. The state variable model for this circuit was created in exercise 2 of Chapter 9.1. We just need to plug in numbers and use MATLAB's "step" command, as shown below:

```
R1 = 10;
R2 = 100;
R3 = 1.1;
L = 1e-3;
C = 10e-6;

A = [-1/R3/L   1/L; -1/C   -1/C*(1/R1+1/R2)];
b = [0; 1/(R1*C)];
c = [0  1];
d = 0;

sys = ss(A,b,c,d);
step(sys)
```

Resulting Figure:



Exercises
Chapter 9.3

1. The state variable model for this circuit was created in exercise 1 of chapter 9.1. We can just plug in numbers and do the simulation with Octave:

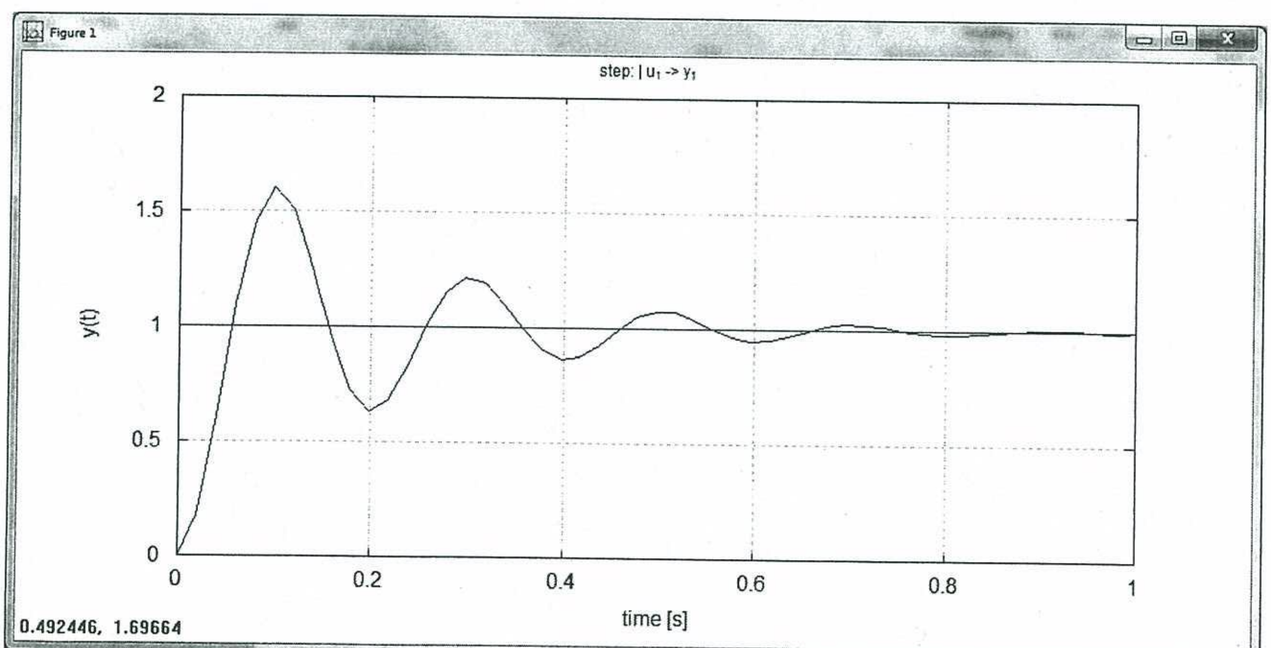
Octave commands:

```
R = 10;
L = 1;
C = 1e-3;

A = [-R/L  -1/L; 1/C 0];
b = [1/L; 0];
c = [0  1];
d = 0;

sys = ss(A,b,c,d);
step(sys)
```

Resulting Figure:



Exercises
chapter 9.3

2. The state variable model for this circuit was created in exercise 2 of chapter 9.1. We just need to plug in numbers and use Octave to simulate the response:

Octave commands:

```
R1 = 10;
R2 = 100;
R3 = 1.1;
L = 1e-3;
C = 10e-6;

A = [-1/R3/L   1/L; -1/C   -1/C*(1/R1+1/R2)];
b = [0; 1/(R1*C)];
c = [0  1];
d = 0;

sys = ss(A,b,c,d);
step(sys)
```

Resulting figure: