9.3.2: Second Order Circuit Response

Overview:

This lab will emphasize the use of the state variable approach for modeling electrical circuits.

In this assignment, we will create a state variable model of a second order electrical circuit and simulate the response of the circuit using MATLAB and/or Octave. We will build and test the circuit, and measure both of the circuit states. The experimentally-determined responses and state trajectory will be compared with the simulated responses.

Before beginning this lab, you should be able to:

- Identify appropriate state variables for electrical systems
- Determine an appropriate state variable model for an electrical system from a schematic of the circuit.
- Use MATLAB or Octave to simulate the response of a state variable model to an arbitrary input

After completing this lab, you should be able to:

- Use state variable methods to model electrical systems
- Experimentally measure the response of a system's state variables to an arbitrary input
- Plot state trajectories for a system from experimental data and simulation results

This lab exercise requires:

- Analog Discovery module
- Digilent Analog Parts Kit
- Digital multimeter (optional)

Symbol Key:

DEMO — Demonstrate circuit operation to teaching assistant; teaching assistant should initial lab notebook and grade sheet, indicating that circuit operation is acceptable.

ANALYSIS — Analysis; include principle results of analysis in laboratory report.

SIM — Numerical simulation (using PSPICE or MATLAB as indicated); include results of MATLAB numerical analysis and/or simulation in laboratory report.

DATA — Record data in your lab notebook.

General Discussion:

In this lab assignment, we will create a state variable model of the circuit shown in Figure 1. We will simulate the response of the circuit to a step input and compare the simulated step response to the measured step response for the circuit.

The circuit of Figure 1 has two independent energy storage elements, and thus has two states. We will define states as the voltages across the capacitor and the current through the inductor. Our lab equipment does not readily allow for measurement of time-varying currents; therefore we will use the measured voltage across the resistor and Ohm's law to infer the current through the inductor.

We will be measuring both system states in order to plot a *state trajectory* for the system. The state variables can be thought of as the axes of a coordinate system; the values of the state variables at any point in time uniquely define a point in that coordinate system. The coordinate system is called a *state space*, it corresponds to the space of all possible combinations of the state variables. Typically, when an input is applied to a system, the state variables will respond by changing from one value to another. The path in the state space through which the state variables pass is called the *trajectory*.

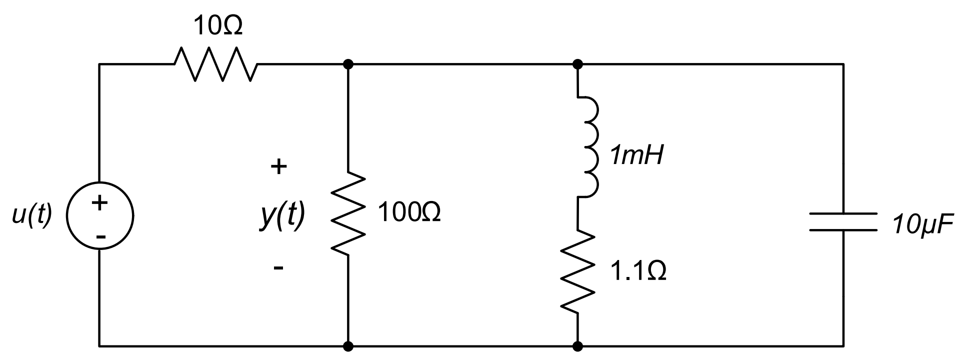

Figure 1. Circuit schematic.

Pre-lab:

ANALYSIS Determine a state variable model for the circuit shown in Figure 1. Use the voltage across the capacitor and the current through the inductor as your states. Simulate the response of your system using MATLAB or Octave. Plot the responses of both system states vs. time and the state trajectory. Your model should output both system states. The input voltage $V_{in}(t) = u_0(t)$ Volts, where $u_0(t)$ is the unit step function.

Lab Procedures:

DATA Construct the circuit shown in Figure 1. Record actual values for all resistors and the capacitor. (If your DMM does not have the capability of measuring capacitance, you may omit the capacitance measurement.)

a. Use a square wave input from your function generator to emulate a 1V step input to the system. Record the amplitude of the square wave you are applying to the system; you will need that in order to compare your data with your analysis from the pre-lab. Note: be sure that the frequency of the square wave is low enough to allow the system to reach a steady-state response. DATA

b. Demonstrate operation of your circuit to the TA and have them initial your lab notebook and the lab worksheet. DEMO

c. Set up the oscilloscope to measure the voltage across the capacitor and the current through the inductor. To do this, we will refer to the circuit schematic shown in Figure 2 below. The capacitor voltage measurement is simple – simply use one of the oscilloscope channels (say, channel 1) to measure $v_c(t)$ indicated on Figure 2. DATA

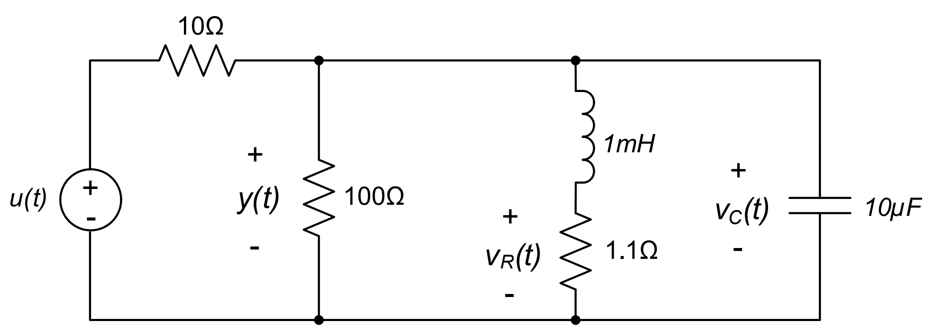

Figure 2. Schematic indicating measured voltages.

Oscilloscopes do not provide the capability to measure time-varying currents, so we will infer the current through the inductor from the voltage across the nominal 1.1Ω resistor Figure 1. If we measure the voltage $v_R(t)$, we can calculate the inductor current as $v_R(t) / 1.1\Omega$. To set up your oscilloscope to display the current through the inductor, do the following:

i. Use, for example, channel 2 (CH2) to measure the input voltage $v_R(t)$.

ii. Click on the Add Channel button on the oscilloscope toolbar, select **Add Mathematic Channel** and choose **Custom**. A custom math channel control panel should appear.

iii. Click on the button on the bottom of this window to set up the math expression; a math editor window should open. In the text box in this window, type the expression which determines current from the measured voltage on channel 2: C2/1.1. (Assuming that the resistance value of the resistor is 1.1Ω; you should actually divide $v_R(t)$ by whatever the actual resistance value is that you used.)

iv. Click OK to create the math channel.

DATA

d. Record an image of the oscilloscope window, showing the responses of both states. Also record the responses of the system states to a file[1].

DEMO

e. Demonstrate operation of your circuit to the TA and have them initial your lab notebook and the lab worksheet.

f. Plot the responses measured in part (d) using your favorite plotting software (Matlab, Octave, Excel,…)

g. Plot the state trajectory as measured in part (d).

ANALYSIS

h. Compare the step responses and the state trajectory you measured to the expected response from the pre-lab. Discuss the comparison in your lab report.

Post-lab Exercises:

ANALYSIS

a. Overlay the simulated output, y(t), vs. time from your pre-lab simulations and the measured output response vs. time. Comment on the differences between the two.

SIM

b. Re-do your simulations from the pre-lab using the actual component values. Also, include the resistance of the inductor in your simulation. (This resistance can be added directly to the 1.1Ω resistance, since the inductor and resistor are in series. No fundamental changes to your model are necessary.) Overlay plots of the simulation results (state responses vs. time <u>and</u> state trajectories) with the measured state responses and state trajectories. Comment on the differences between the simulation and the measured data.

[1] Hint: the Export button on the scope instrument can be used to export the data to either a .csv or .txt file. Most application software (e.g. MATLAB, Excel) allow you to import data in either of these formats.