Exercises
Chapter 8.1

1. $$\frac{d^2y(t)}{dt^2} + 32\frac{dy(t)}{dt} + 64y(t) = 0$$

(a) $\omega_n = \sqrt{64} = \underline{\underline{8 \text{ rad/sec}}}$

(b) $2\xi\omega_n = 32 \Rightarrow 2\xi(8) = 32 \Rightarrow \underline{\underline{\xi = 2}}$

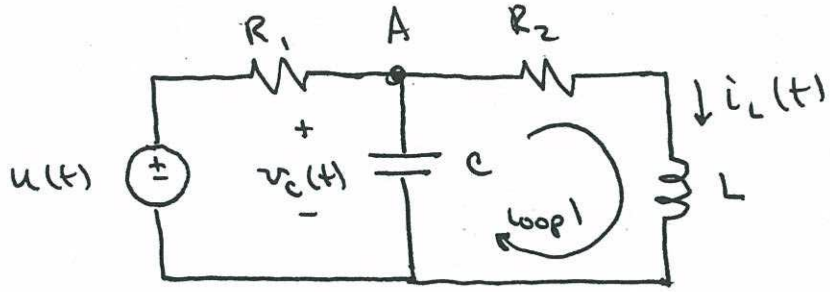
2. $$\frac{d^2y(t)}{dt^2} + 8\frac{dy(t)}{dt} + 64y(t) = 0$$

(a) $\omega_n = \sqrt{64} = 8 \text{ rad/sec}$

(b) $2\xi\omega_n = 8 \Rightarrow 2\xi(8) = 8 \Rightarrow \underline{\underline{\xi = 0.5}}$

Exercises
Chapter 8.1

3.



KCL at A:
$$\frac{u - v_c}{R_1} = C\frac{dv_c}{dt} + i_L \qquad (1)$$

KVL, Loop 1:
$$v_c = R_2 i_L + L\frac{di_L}{dt} \qquad (2)$$

From (1),
$$i_L = \frac{u}{R_1} - \frac{v_c}{R_1} - C\frac{dv_c}{dt} \qquad (3)$$

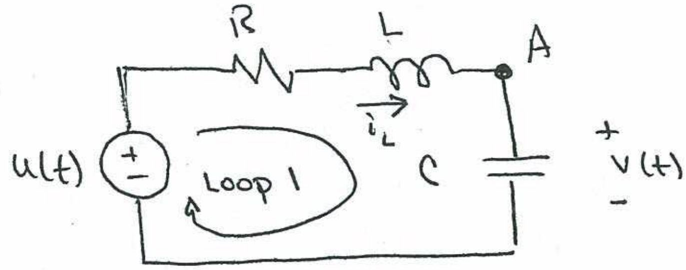
$$\frac{di_L}{dt} = \frac{1}{R_1}\frac{du}{dt} - \frac{1}{R_1}\frac{dv_c}{dt} - C\frac{d^2v_c}{dt} \qquad (4)$$

Substitute (3) & (4) into (2)

$$v_c = R_2\left[\frac{u}{R_1} - \frac{v_c}{R_1} - C\frac{dv_c}{dt}\right] + L\left[\frac{1}{R_1}\frac{du}{dt} - \frac{1}{R_1}\frac{dv_c}{dt} - C\frac{d^2v_c}{dt^2}\right]$$

This can be simplified, but that's not required by the problem statement...

4.



Exercises
chapter 8.1

KCL at A: $\underline{i_L = C\frac{dv}{dt}}$

KVL: $\underline{u(t) = R i_L + L\frac{di_L}{dt} + v}$

$$i_L = C\frac{dv}{dt} \Rightarrow \frac{di_L}{dt} = C\frac{d^2v}{dt^2}$$

$$u(t) = R i_L + L\frac{di_L}{dt} + v$$

$$\underline{\underline{u(t) = RC\frac{dv}{dt} + LC\frac{d^2v}{dt^2} + v}}$$

Exercises

Chapter 8.3

1. $\cos(\pi t - 30°)$

2. $v(t) = \text{Real}\left\{ e^{j100t} \right\}$

3. $V = 3 - j3 = \sqrt{3^2 + 3^2} \angle \tan^{-1}\left(\frac{-3}{3}\right) = 3\sqrt{2} \angle -45°$

or equivalently: $3\sqrt{2}\, e^{-j45°}$

4. $e^{j\pi} = 1 e^{j180°} = -1$

5. $e^{-j\pi/2} = -j$

Exercises

Chapter 8.4

1. $$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 144y = 0$$

($6 \uparrow 2\xi\omega_n$, $144 \uparrow \omega_n^2$)

$\omega_n^2 = 144 \Rightarrow \omega_n = 12$ rad/sec

$2\xi\omega_n = 6 \Rightarrow 2\xi(12) = 6 \Rightarrow \xi = \frac{1}{4}$

circuit is underdamped ($\xi < 1$)

2. $$\frac{d^2y}{dt^2} + 32\frac{dy}{dt} + 64y = 0$$

(a) $\omega_n = \sqrt{64} \Rightarrow \omega = 8$ rad/sec

(b) $2\xi\omega_n = 32 \Rightarrow 2\xi(8) = 32 \Rightarrow \xi = 2$

(c) $$-\frac{1}{\tau_{1,2}} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$$

$$\tau_1 = +\left[\frac{1}{\xi\omega_n + \omega_n\sqrt{\xi^2-1}}\right] = \frac{1}{2(8)+8} = \frac{1}{24} \text{ sec}$$

$$\tau_2 = +\left[\frac{1}{\xi\omega_n - \omega_n\sqrt{\xi^2-1}}\right] = \frac{1}{2(8)-8} = \frac{1}{8} \text{ sec}$$

Exercises

Chapter 8.4

3. $\frac{d^2y}{dt^2} + 8\frac{dy}{dt} + 64y = 0$

(a) $\omega_n = \sqrt{64} = \underline{8 \text{ rad/s}}$

(b) $2\zeta\omega_n = 8 \Rightarrow 2\zeta(8) = 8 \Rightarrow \underline{\zeta = \frac{1}{2}}$

(c) $\omega_d = \omega_n\sqrt{1-\zeta^2} = 8\sqrt{1-.25}$

$$\underline{\omega_d = 6.93 \text{ rad/sec}}$$

Exercises

Chapter 8.5

1. $\frac{d^2v}{dt^2} + 12\frac{dv}{dt} + 400v = 400\,u(t)$

$u(t) = 2u_0(t)$

(a) $\omega_n^2 = 400 \Rightarrow \omega_n = 20$ rad/s

(b) $2\xi\omega_n = 12 \Rightarrow 2\xi(20) = 12 \Rightarrow \xi = 0.3$

(c) $M_p \approx 1 - \frac{\xi}{0.6} = 0.5$

$t_r = \frac{1.8}{\omega_n} = \frac{1.8}{20} \Rightarrow t_r = 0.09$ sec

$400\,v_{ss} = 400\,[2u_0(t \to \infty)] \leftarrow \frac{dv}{dt}, \frac{d^2v}{dt} = 0$

$v_{ss} = 2$ V

(d) $v_{max} = v_{ss}[1 + M_p] \Rightarrow v_{max} = 2[1 + 0.5] = 3$ V