

8.5.1: Series RLC Circuit Step Response

Overview:

This lab will emphasize modeling and testing of a series RLC second order circuit. This lab assignment will consist of two parts:

- In Part I of this assignment, the step response of a given circuit is analyzed and tested. The measured response of the circuit is compared with expectations based on the damping ratio and natural frequency of the circuit.
- Part II of this assignment consists of a simple design problem: the circuit of Part I is to be re-designed to make it critically damped, without changing either the natural frequency or the DC gain. Again, the circuit step response is measured and compared to expectations.

Before beginning this lab, you should be able to:

- Model practical inductors as an ideal inductor in series with a resistance (Lab 6.4.2)
- Determine natural frequency and damping ratio for second order circuits
- State the relationship between damping ratio and maximum overshoot for underdamped systems
- State the relationship between natural frequency and rise time for underdamped systems
- Use the Analog Discovery to apply and measure time-varying waveforms

After completing this lab, you should be able to:

- Measure the step response of a second order electrical circuit
- Estimate the damping ratio and natural frequency of an underdamped second order circuit from measured step response data

This lab exercise requires:

- Analog Discovery module
- Digilent Analog Parts Kit

Symbol Key:

DEMO

Demonstrate circuit operation to teaching assistant; teaching assistant should initial lab notebook and grade sheet, indicating that circuit operation is acceptable.

ANALYSIS

Analysis; include principle results of analysis in laboratory report.

SIM

Numerical simulation (using PSPICE or MATLAB as indicated); include results of MATLAB numerical analysis and/or simulation in laboratory report.

DATA

Record data in your lab notebook.

Second Order Series RLC Circuit:

The general differential equation governing a second order system is:

$$\frac{d^2y(t)}{dt^2} + 2\zeta\omega_n \frac{dy(t)}{dt} + \omega_n^2 y(t) = f(t) \quad (1)$$

where $y(t)$ is any system parameter of interest (for example, a voltage or current in an electrical circuit), ω_n and ζ are the *undamped natural frequency* and the *damping ratio* of the system, respectively, and $f(t)$ is a forcing function applied to the system. Overdamped systems have a damping ratio greater than one, underdamped systems have a damping ratio less than one, and critically damped systems have a damping ratio of exactly one.

Pre-lab:

- ANALYSIS** a. Write the differential equation relating V_{out} and V_{in} for the system shown in Figure 1. Your solution will be a function of the circuit parameters R, L, and C.
- ANALYSIS** b. From your differential equation, estimate the damping ratio, natural frequency, damped natural frequency, and DC gain of the circuit. These parameters will be functions of R, L, and C.
- ANALYSIS** c. Estimate the rise time (t_r), overshoot (M_p), and frequency of any oscillations you would expect to see in the step response of the circuit. (Also as functions of R, L, and C.)

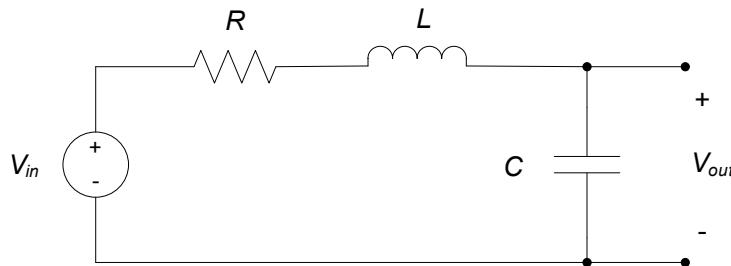


Figure 1. Series RLC circuit.

Lab Procedures:

DATA

Construct the circuit shown in Figure 1, using $C = 100\text{nF}$, $L = 1\text{mH}$, and $R = 1.1\Omega$. (As always, measure the actual resistance values and the capacitance value if you have a DMM which measures capacitance; you may assume that the nominal inductance value is correct. Also, use your DMM to measure the inductor resistance; the non-ideal inductor effects can have a strong effect on your results in this lab assignment.)

- a. Use your arbitrary waveform generator to apply a 2V step input to the circuit, at a low enough frequency so that the circuit can reach steady-state between pulses¹. Use your oscilloscope channels to measure both the input and output voltages (V_{out} and V_{in}) for the circuit². Record an image of the oscilloscope window, showing the signals $V_{in}(t)$ and $V_{out}(t)$.
- b. Estimate the rise time, overshoot, damping ratio, natural frequency, DC gain, and the period of any oscillations in the response and compare them to the values calculated in the pre-lab. Comment on any differences in your lab notebook. (Hint: the capacitor and inductor you use are non-ideal; the inductor, in particular, may have a non-negligible resistance. See Chapter 6.4 for a practical inductor model, including an inductor resistance.)
- c. Demonstrate operation of your circuit to the TA and have them initial your lab notebook and the lab checklist.

DATA

ANALYSIS

DEMO

Critically Damped Series RLC Circuit:

We will now modify the circuit of Part I in order to make the circuit critically damped, without changing the natural frequency or DC gain of the circuit.

Pre-lab:

ANALYSIS

For the circuit of Figure 1, determine values of R, L, and C necessary to make the circuit critically damped, with the same natural frequency of the circuit in Part I. The circuit should have a DC gain of one, as in Part I of this assignment.

Lab Procedures:

DATA

DEMO

- a. Construct the circuit you designed in the pre-lab and experimentally determine the step response of your circuit. Record an image of the oscilloscope window, showing the signals $V_{in}(t)$ and $V_{out}(t)$. Comment on your results vs. your expectations for a critically damped circuit.
- b. Demonstrate operation of your circuit to the TA and have them initial your lab notebook and the lab checklist.

¹ Suggestion: A square wave with 1V amplitude and 1V offset can provide an input that goes from 0V to 2V. As long as the period of the square wave period is more than about 10msec, the system should have more than enough time to reach steady-state.

² The input voltage V_{in} provides a good source for a trigger. If you trigger off the rising edge of this signal, your zero time should correspond to the time at which the step input is applied.