Exercises
Chapter 7.1

1. (a)

i. Equation is: $mc_p \frac{d(T_B - T_0)}{dt} + h(T_B - T_0) = 0$

as $t \to \infty$, temperatures become constant, and $\frac{d(T_B - T_0)}{dt} \to 0$. Therefore:

$$h(T_B - T_0) = 0 \quad \Rightarrow \quad \underline{\underline{T_B = T_0}}$$

Body temperature approaches ambient temperature as $t \to \infty$

ii. To find time constant, write equation in the form $\frac{dy}{dt} + \frac{1}{\tau} y = 0$:

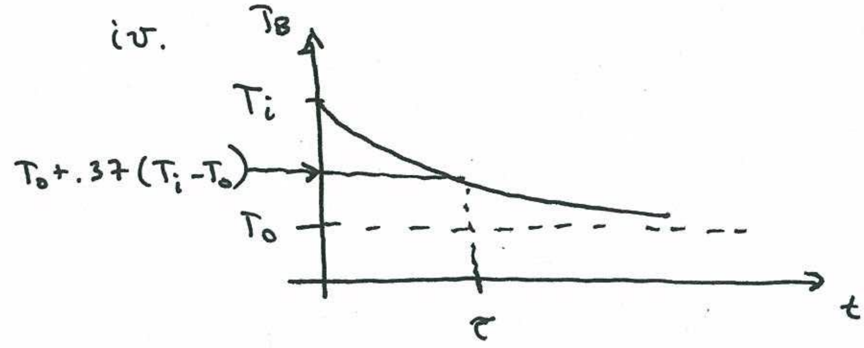
$$\frac{d(T_B - T_0)}{dt} + \frac{h}{mc_p}(T_B - T_0) = 0 \quad \Rightarrow \quad \underline{\underline{\tau = \frac{mc_p}{h}}}$$

iii. If mass doubles, the time constant doubles. Makes sense, since a larger mass should take longer to cool down.

Exercises
Chapter 7.1

1(a) continued

iv.



(b)

i) Governing equation: $m c_p \dfrac{d(T_B - T_o)}{dt} + h(T_B - T_o) = q_{in}$

The input is constant, so all temperatures become constant as $t \to \infty$ and $\dfrac{d(T_B - T_o)}{dt} \to 0$:

$$h(T_B - T_o) = q_{in} \Rightarrow \underline{\underline{T_B = \frac{q_{in}}{h} + T_o}} \text{ as } t \to \infty$$

ii. Doubling the heat input doubles the <u>increase</u> in body temperature (above T_o). Reasonable - more heat gives higher temp.

iii. Doubling mass does not affect the final temperature. Mass only affects how quickly the mass can heat up & cool down.

1(b) - continued

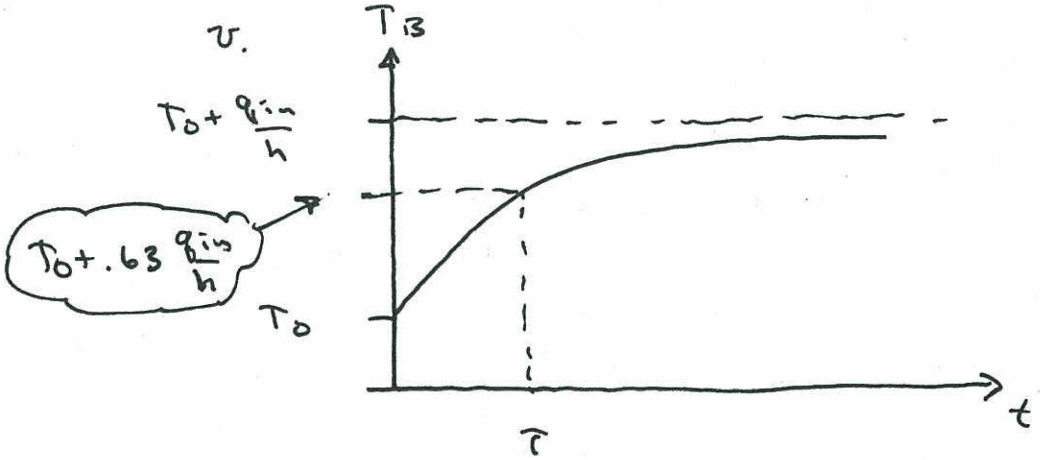
iv. Write equation in form $\frac{dy}{dt} + \frac{1}{\tau} y = f(t)$

$$\frac{d(T_B - T_0)}{dt} + \frac{h}{mC_p}(T_B - T_0) = \frac{q_{in}}{mC_p}$$

$$\underline{\underline{\tau = \frac{mC_p}{h}}}$$

doubling mass doubles time constant, as in part (a)

v.

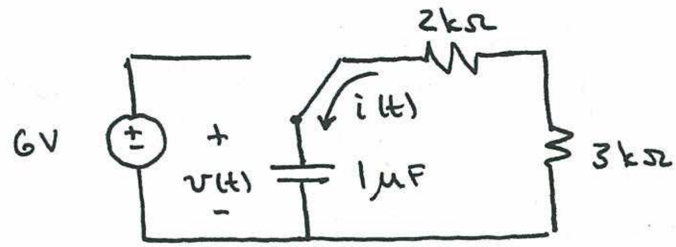


Exercises

Chaper 7.2

1.

(a) Circuit for $t > 0$:



KVL, right loop: $v(t) + (3k\Omega)\,i(t) + (2k\Omega)\,i(t) = 0$

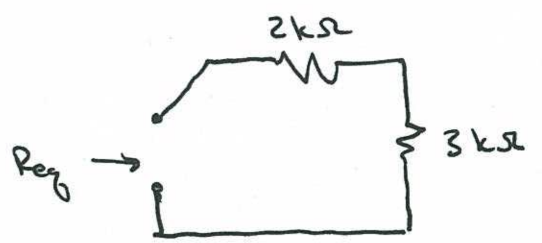
$$i(t) = C\frac{dv}{dt}$$

$$v(t) + 5k\Omega\left(1\mu F\,\frac{dv}{dt}\right) = 0$$

$$\underline{\underline{\frac{dv(t)}{dt} + 200\,v(t) = 0}}$$

(b) $\tau = \frac{1}{200}\text{ sec} = \underline{\underline{0.005\text{ sec}}}$

(c) Equivalent resistance:



$R_{eq} = 5k\Omega$

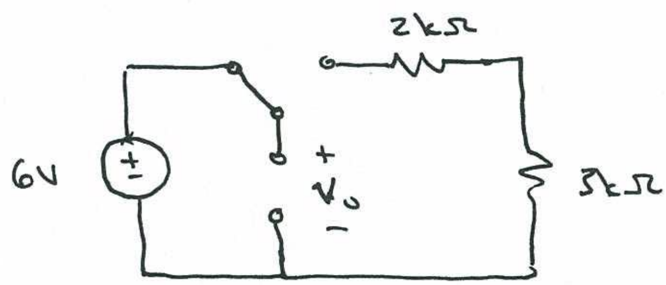
$\tau = R_{eq}C = (5k\Omega)(1\mu F)$

$\underline{\underline{\tau = 0.005\text{ sec}}}$

CHECKS!

1. (cont'd)

(d) For $t<0$, the switch is in position A and the capacitor looks like an open circuit (constant input, switch has been in position A for a long time) and the circuit looks like:



$\Leftarrow$ $\underline{\underline{V_o = 6V}}$

Voltages across capacitors can't change suddenly, so initial condition is:

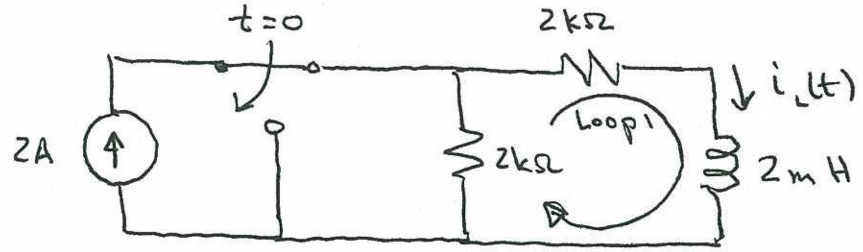
$$v(t=0^+) = v(t=0^-) = \underline{\underline{6V}}$$

(e) Form of solution is $v(t) = V_o e^{-t/\tau}$, so:

$$\underline{\underline{v(t) = 6e^{-200t}\ V, \quad t>0}}$$

Exercises
Chapter 7.3

1.



$+$
$L \frac{di_L}{dt}$
$-$

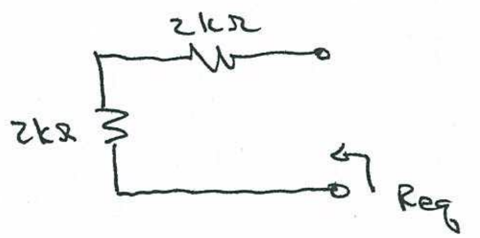
(a) KVL, loop 1, $t > 0$:

$$(2k\Omega)\, i_L + (2k\Omega)\, i_L + 2mH \frac{di_L}{dt} = 0$$

$$\underline{\underline{\frac{di_L}{dt} + 2 \times 10^6\, i_L = 0}}$$

(b) From (a), $\tau = \frac{1}{2} \mu sec$

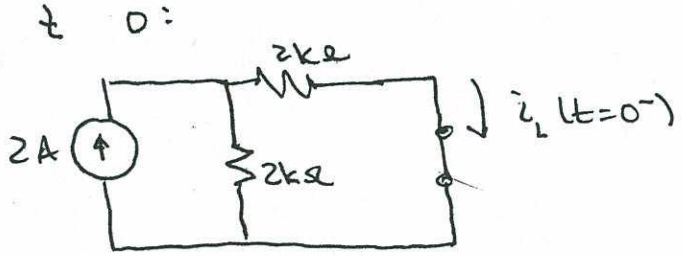
(c)



$\Rightarrow$ $R_{eq} = 2k\Omega + 2k\Omega = 4k\Omega$

$$\tau = \frac{L}{R_{eq}} = \frac{2mH}{4k\Omega} = \underline{\underline{\frac{1}{2} \mu sec}}$$

(d) t 0:



} inductor acts as short circuit at $t=0^-$

$i_L(t=0^-) = 1A \Leftarrow$ circuit is a current divider

$i_L(t=0^+) = i_L(t=0^-) = \underline{\underline{1A}} \Leftarrow$ inductor current cannot change instantly.

Exercises
Chapter 7.3

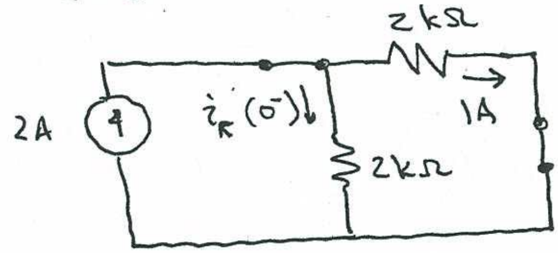
(e) From (a), $\frac{di_L}{dt} + 2\times10^6\, i_L = 0 \Rightarrow \tau = 0.5$

From (b), $i_L(t=0^+) = 1A = I_0$

Form of solution: $i_L(t) = I_0\, e^{-t/\tau}$

$$\underline{\underline{i_L(t) = 1\, e^{-2\times10^6 t}, \quad t>0}}$$

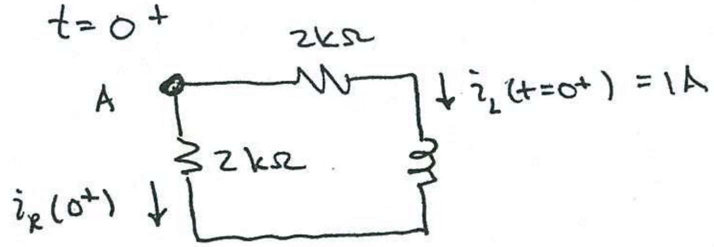
(f) $t=0^-$:



$i_R(t=0^-) = 1A$ (current divider)

$i_L(t=0^-) = 1A$ (current divider)

$t=0+$



At $t=0^+$, $i_L(t=0^+) = 1A$
(inductor current can't change suddenly)

$\Downarrow$

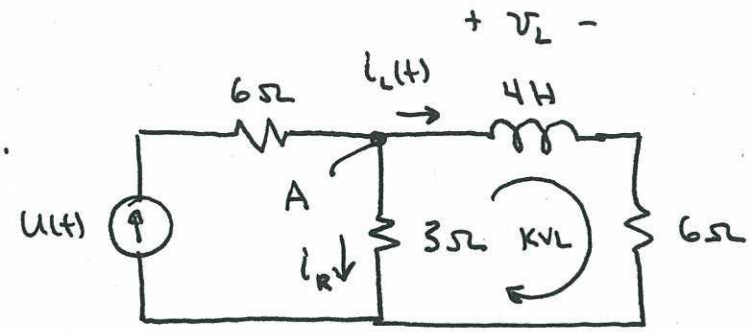
KCL at A: $i_R(t=0^+) = -$

Resistor current is <u>not</u> <u>continuous with</u> <u>time</u>. (It changes suddenly to compensate for the change in applied current.) The <u>other</u> resistor's current is continuous with time.

Exercises

Chapter 7.4

1.



KCL at node A: $i_R = u(t) - i_L(t)$

KVL around right loop:

$$3\Omega\,(i_R) = v_L + 6\Omega\,(i_L) \quad ; \quad v_L = 4\frac{di_L}{dt}$$

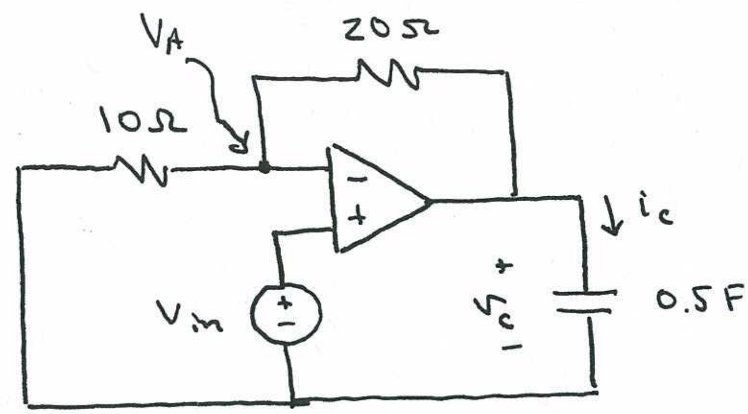
$$3\Omega\,(u - i_L) = 4\frac{di_L}{dt} + 6\Omega\,(i_L)$$

$$\underline{\underline{3u = 4\frac{di_L}{dt} + 9i_L}}$$

Exercises

Chapter 7.4

2.



Op-amp rules : $V_A = V_{in}$

KCL at inverting input: $\frac{V_A - 0V}{10\Omega} + \frac{V_A - V_c}{20\Omega} = 0$

$V_c = 3V_A = 3V_{in}$

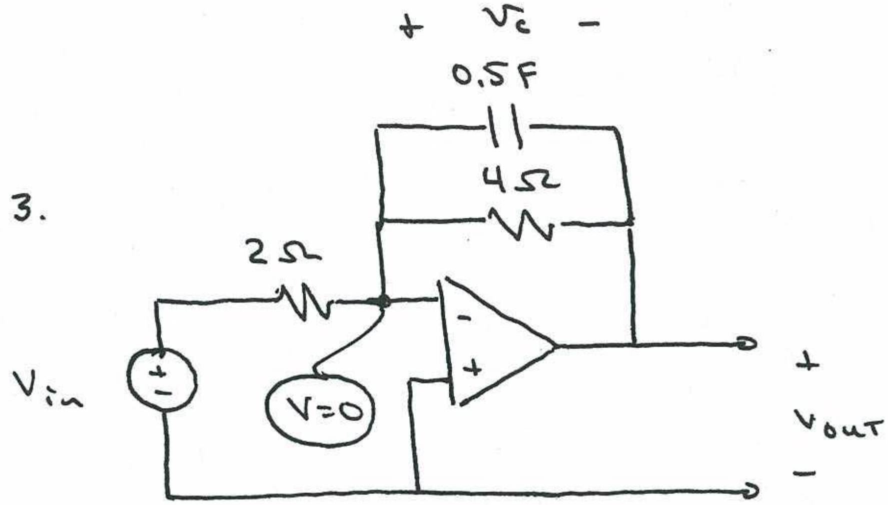
$i_c = (0.5F)\frac{dV_c}{dt} = 0.5\left[\frac{d}{dt}(3V_{in})\right]$

$$\underline{\underline{i_c(t) = \frac{3}{2}\frac{dV_{in}}{dt}}}$$

Exercises

Chapter 7.4

3.



KCL at inverting input:

$$\frac{V_{in} - 0V}{2\Omega} = (0.5F)\frac{dV_c}{dt} + \frac{0 - V_{OUT}}{4\Omega}$$

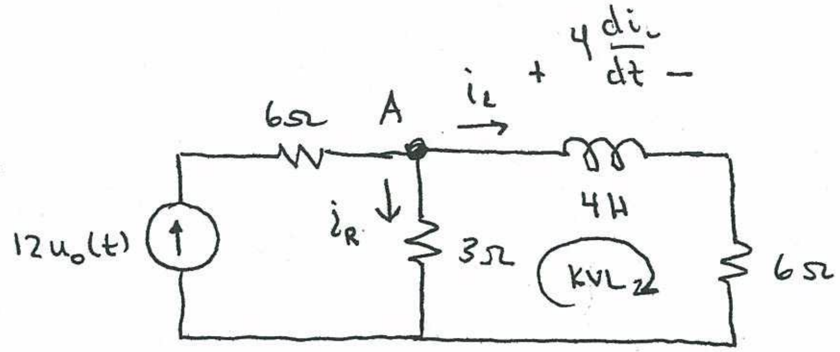
$V_c = -V_{OUT}$

$$\frac{V_{in}}{2} = -0.5\frac{dV_{OUT}}{dt} - \frac{V_{OUT}}{4}$$

$$2\frac{dV_{OUT}}{dt} + V_{OUT} = -2V_{in}$$

Exercises
Chapter 7.5

1.



(a) KCL at node A gives: $i_R = 12A - i_L$

KVL around right loop then results in:

$$3\Omega\,(12A - i_L) = 4\frac{di_L}{dt} + 6\Omega\,(i_L)$$

$$36 = 4\frac{di_L}{dt} + 6 i_L$$

$$\underline{\underline{\frac{di_L}{dt} + \frac{9}{4} i_L = 9}}$$

(b) From (a), $\frac{9}{4} = \frac{1}{\tau} \Rightarrow \underline{\underline{\tau = \frac{4}{9}\text{ sec}}}$

(c) $i_L(t) = K_1 + K_2 e^{-t/\tau}$

(d)

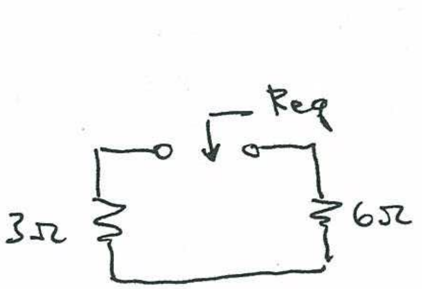
$t = 0^+ \Rightarrow i_L(0^+) = i_L(0^-) = 0A$

$t \to \infty \Rightarrow i_L(t \to \infty) = 4A$

$$\Rightarrow \underline{\underline{i_L(t) = 4\left[1 - e^{-4t/9}\right]}}$$

$\underline{\underline{(t > 0)}}$

(e)



$\Rightarrow R_{eq} = 3\Omega + 6\Omega = 9\Omega$

$\tau = \frac{L}{R} = \frac{4H}{9\Omega} \Rightarrow \underline{\underline{\tau = \frac{4}{9}\text{ sec}}}$ ✓

Exercises
chapter 7.5

2. $$2\frac{di(t)}{dt} + 3i(t) = 5v(t)$$

DC gain: $\frac{di}{dt} \to 0 \Rightarrow 3i(t\to\infty) = 5\,v(t\to\infty)$

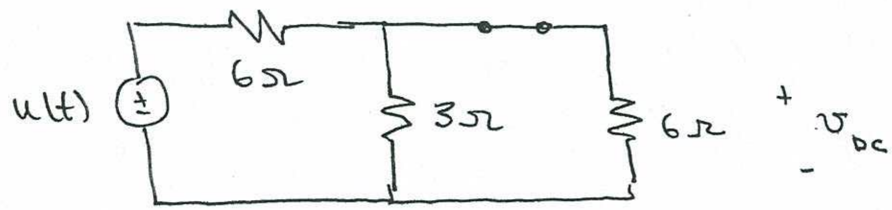
$$\left.\frac{i(t)}{v(t)}\right|_{t\to\infty} = \underline{\underline{\frac{5}{3}}}$$

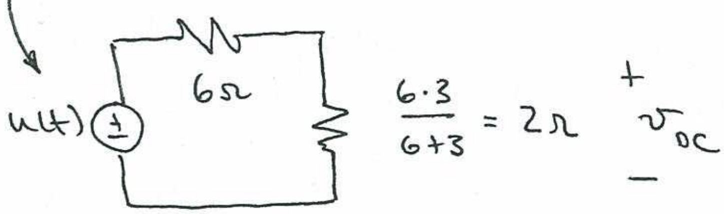
Units are $\underline{\underline{A/V}}$.

Time constant: $\frac{di(t)}{dt} + \underbrace{\frac{3}{2}}_{\frac{1}{\tau}} i(t) = \frac{5}{2} v(t)$

$$\underline{\underline{\tau = \frac{2}{3}\text{ sec}}}$$

3. In steady-state, with $u(t) = $ constant, the inductors becomes a short circuit:





Voltage divider: $v_{DC} = \frac{2}{8}u \Rightarrow$ DC gain $= \underline{\underline{\frac{1}{4}}}$

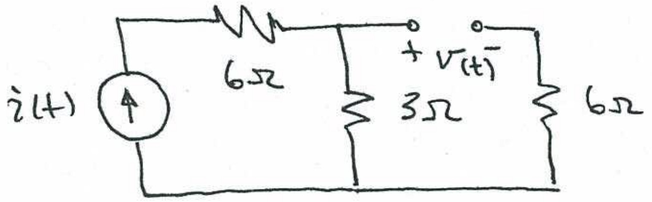
units are V/V
(unitless)

Exercises
Chapter 7.5

4. From differential equation, with $\frac{dv}{dt} = 0$:

$\frac{1}{9} v(t \to \infty) = i(t \to \infty) \Rightarrow$ DC gain = 9 (from Diff. eqn.)

From circuit, with capacitor open-circuited:



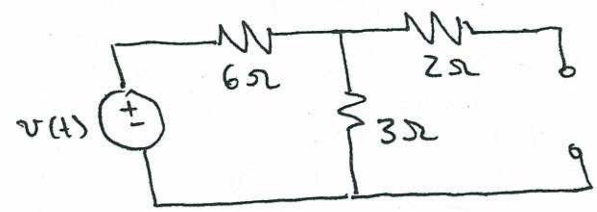
$v(t) = 3\, i(t)$

$\Rightarrow$ D gain = 3

DC gain from circuit different than DC gain from differential equation $\Rightarrow$ <u>D.E. not accurate</u>

5. $3 \frac{dv_c}{dt} + v_c(t) = v(t) \Rightarrow$ DC gain = 1

circuit:



+
$v_c(t)$
−
$\Rightarrow$ DC gain = $\frac{1}{2}$

<u>Inconsistent</u>

$\Rightarrow$ Diff. equation probably inaccurate.