Real Analog Chapter 7: First Order Circuits

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7 Introduction and Chapter Objectives

First order systems are, by definition, systems whose input-output relationship is a *first order* differential equation. A first order differential equation contains a *first order derivative* but no derivative higher than first order – the order of a differential equation is the order of the highest order derivative present in the equation.

First order systems contain a *single energy storage element*. In general, the order of the input-output differential equation will be the same as the number of <u>independent</u> energy storage elements in the system. Independent energy storage cannot be combined with other energy storage elements to form a single equivalent energy storage element. For example, we previously learned that two capacitors in parallel can be modeled as a single equivalent capacitor – therefore, a parallel combination of two capacitors forms a single independent energy storage element.

First order systems are an extremely important class of systems. Many practical systems are first order; for example, the mass-damper system and the mass heating examples from section 6.1 are both first order systems. Higher order systems can often be approximated as first order systems to a reasonable degree of accuracy if they have a *dominant first order mode*. (System modes will be discussed later in this text.) Understanding first order systems and their responses is an important aspect to design and analysis of systems in general.

First order electrical circuits are a special class of first order systems – they are first order systems which are composed of electrical components. Since the electrical components which store energy are capacitors and inductors, first order circuits will contain either one (equivalent) capacitor or one (equivalent) inductor.

In this textbook, we are really interested only in the analysis of electrical circuits, so the bulk of this chapter, of course, deals with analysis of first order electrical circuits. However, at this stage of your studies, you probably do not have an intuitive grasp of the mechanisms of energy storage in electrical circuits and the response of electrical circuits with energy storage elements. Therefore, this chapter begins in Section 7.1 with a general discussion of the response of first order systems, using a sliding mass as an example within which to frame the basic concepts. This provides a relatively gentle introduction to the nomenclature and mathematics which will be used throughout this chapter, in the context of an example for which the student should have some physical insight. Section 7.1 can be omitted, however, without loss of clarity of the remaining sections. Sections 7.2 and 7.3 present the natural response of RC and RL circuits, respectively (RC circuits have a capacitor as an energy storage element, while RL circuits contain an inductor). The natural response of a system corresponds to the system response to some initial condition, with no forcing function provided to the system. In section 7.4, we present the force response of first order circuits, and in section 7.5 we examine the response of first order circuits to a specific forcing function – a step input.

After completing this chapter, you should be able to:

- Write the general form of the differential equation governing a first order system
- State, in physical terms, the significance of a differential equation's homogeneous and particular solutions
- Define, from memory, the relationships between a system's unforced response, zero-input response, natural response, and the homogeneous solution to the differential equation governing the system
- Define, from memory, the relationships between a system's forced response, zero-state response, and the particular solution to the differential equation governing the system
- Determine the time constant of a first order system from the differential equation governing the system
- Write mathematical expressions from memory, giving the form of the natural and step responses of a first order system
- Sketch the natural response of a first order system from the differential equation governing the system and the system's initial condition
- Sketch the step response of a first order system from the differential equation governing the system and the amplitude of the input step function
- Write the differential equation governing RC and RL circuits
- Determine the time constant of RC and RL circuits from their governing differential equations
- Determine the time constant of RC and RL circuits directly from the circuits themselves
- Determine initial conditions on arbitrary RC and RL circuits
- Write from memory the form of the natural responses of RC and RL circuits
- Determine the natural response of RC and RL circuits, given the governing differential equation and initial conditions
- Write the form of the differential equations governing forced first order electrical circuits
- Determine the time constant of a forced electrical circuit from the governing differential equation
- Write differential equations governing passive and active first order circuits
- Determine the differential equation governing the step response of a first order electrical circuit
- Write the form of the particular solution of a first order differential equation, to a step input
- Write the form of the step response of a first order electrical circuit
- Determine the final conditions (steady-state response) of a first order electrical circuit, to a step input
- Define DC gain for a circuit and relate it to the steady-state response to a step input
- Determine the step response of a first order electrical circuit from the governing differential equation, the initial conditions, and the final conditions

7.1 Introduction to First Order Systems

In this section, we introduce some basic nomenclature relative to first order system responses and illustrate these terms in the context of an example for which the reader may have an intuitive understanding: a mass sliding on a surface. This example, though not directly relevant to the study of electrical circuits, is intended to allow the reader to develop some physical insight into the terminology and concepts relative to the solution of first order differential equations. The concepts and results obtained with this example are then generalized to apply to any arbitrary first order system. These results are used in later sections to provide insight in the analysis of electrical circuits, for which the reader may not yet have an intuitive understanding.

Before discussing first-order electrical systems specifically, we will introduce the response of general first order systems. A general first order system is governed by a differential equation of the form:

$$a_1 \frac{dy(t)}{dt} + a_0 y(t) = ft,\ t > 0 \qquad \text{Eq. 7.1}$$

Where *f(t)* is the (known) input to the system and *y(t)* is the response of the system. a_1 and a_0 are constants specific to the system being analyzed. We assume in equation (7.1) that the input function is applied only for times $t>t_0$. Thus, from equation (7.1), we can only determine the response of the system for times $t>t_0$.

In order to find the solution to equation (7.1), we require knowledge of the system's *initial condition*:

$$y(t = t_0) = y_0 \qquad \text{Eq. 7.2}$$

The initial condition, y_0, defines the state of the system at time $t=t_0$. Since equation (7.1) describes a system which stores energy, the effect of the initial condition is to provide information as to the amount of energy stored in the system at time $t=t_0$.

The system described by equations (7.1) and (7.2) can be illustrated in block diagram form as shown in Fig. 7.1. The output of the system depends upon the initial condition, y_0, and the input function *f(t)*. The initial condition provides information relative to the energy stored in the system prior to application of the input function. The input function provides information relative to the energy being applied to the system from external sources. The input-output equation describes how the system transfers the energy initially present in the system and the energy added to the system to the system output.

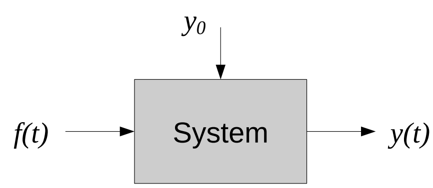


Figure 7.1. System block diagram.

The solution to equation (7.1) consists of two parts – the *homogeneous solution*, $y_h(t)$, and the particular solution, $y_p(t)$, as shown below:

$$y(t) = y_h(t) + y_p(t) \qquad \text{Eq. 7.3}$$

The homogeneous solution is due to the properties of the system and the initial conditions applied to the system; it describes the response of the system if no input is applied to the system, so *f(t)=0*. The homogeneous solution is sometimes called the systems *natural response*, the *unforced response*, or the *zero input response*. Since all physical systems dissipate energy (according to the second law of Thermodynamics) the homogeneous solution must die out with time; thus, $y_h(t) \rightarrow 0$ as $t \rightarrow \infty$.

The particular solution describes the systems response to the particular forcing function applied to the system; the form of the particular solution is dictated by the form of the forcing function applied to the system. The particular solution is also called the *forced response* or the zero *state response*.

Since we are concerned only with linear systems, superposition principles are applicable, and the overall system response is the sum of the homogeneous and particular solutions. Thus, equation (7.3) provides the system's overall response to both initial conditions and the particular forcing function being applied to the system.

The previous concepts are rather abstract, so we provide below an example of the application of the above concepts to a system for whose response the students should have some intuitive expectations. This example is intended to provide some physical insight into the concepts presented above prior to applying these concepts to electrical systems.

7.1.1 Mass-damper System Example

As an example of a system which includes energy storage elements we revisit the mass-damper system of section 6.1. The system under consideration is shown in Fig. 7.2. The applied force *F(t)* pushes the mass to the right. The mass's velocity is *v(t)*. The mass slides on a surface with sliding coefficient of friction *b*, which induces a force which opposes the mass's motion. The mass will have some initial velocity:

$$v(t=0) = v_0 \qquad \text{Eq. 7.4}$$

Consistent with section 6.1, we consider the applied force to be the input to our system and the mass's velocity to be the output. Figure 7.3 illustrates the system input-output relationship and initial conditions in block diagram form.

The governing equation for the system was determined in section 4.1 to be the first order differential equation:

$$m\frac{dv(t)}{dt} + bv(t) = F(t) \qquad \text{Eq. 7.5}$$

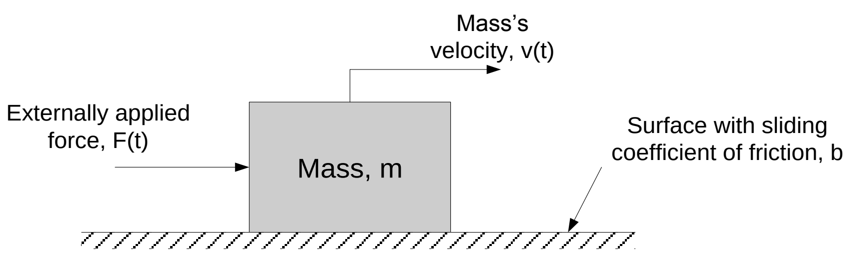


Figure 7.2. Sliding mass on surface with friction coefficient, b.

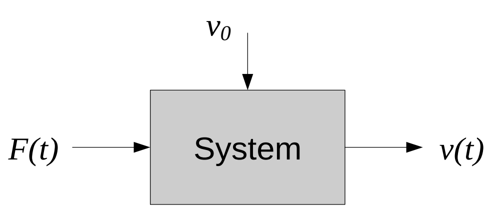


Figure 7.3. Block diagram of system shown in Figure 1.

We consider two cases of specific forcing functions in the following cases. In the first case, the forcing function is zero, and we determine the system's natural response or the homogeneous solution to equation (7.5) above. In the second case, the forcing function is a constant nonzero force applied to the mass with zero initial velocity.

Case i: Natural (Homogeneous) Response

Let us consider first the case in which the mass has some initial velocity but no external force is applied to the mass. Intuitively, we expect that the velocity of the mass will decrease until the mass comes to rest. In this example, we will determine the solution of the differential equation (7.5) and compare this solution with our expectations.

With no applied forcing function, the differential equation governing the system is:

$$m\frac{dv(t)}{dt} + bv(t) = 0 \qquad \text{Eq. 7.6}$$

The initial condition is given by equation (7.4) above, repeated here for convenience:

$$v(t=0) = v_0$$

Equation (7.6) is a homogeneous differential equation, since there is no forcing function applied to the system. Thus, the particular solution in this case is $y_p(t)=0$ and our overall solution is simply the homogeneous solution, $y(t)=y_h(t)$.

To solve the above differential equation, we rearrange equation (7.6) to give:

$$\frac{m}{b}\cdot\frac{dv(t)}{dt} = -v(t) \qquad \text{Eq. 7.7}$$

Separating variables in equation (7.7) results in:

$$\frac{dv(t)}{v} = -\frac{b}{m}dt \qquad \text{Eq. 7.8}$$

Incorporating dummy variables of integration and integrating both sides of (7.8) gives:

$$\int_{v_0}^{v(t)} \frac{d\xi(t)}{\xi} = -\frac{b}{m}\int_0^t d\varsigma$$

Which evaluates to:

$$ln(v)\Big|_{v_0}^{v(t)} = -\frac{b}{m}t \Rightarrow \ln[v(t)] - \ln[v_0] = -\frac{b}{m}t \Rightarrow ln\left[\frac{v(t)}{v_0}\right] = -\frac{b}{m}t$$

Taking the exponent of both sides of the above provides our final result:

$$v(t) = v_0 e^{\frac{-bt}{m}} \qquad \text{Eq. 7.9}$$

A plot of the response given in equation (7.9) is shown in Fig. 7.4. This plot matches our previous expectations, the velocity of the mass at time $t=0$ is v_0 and the velocity decreases exponentially until the mass is (essentially) at rest. Referring to section 6.2, we see that the response of equation (7.9) can be written in terms of a time constant as:

$$v(t) = v_0 e^{\frac{-t}{\tau}}$$

Where the time constant $\tau = \frac{m}{b}$. This result also agrees with our intuition: as the friction coefficient decreases, the time constant increases and the mass comes to rest more slowly. Likewise, increasing the mass causes the time constant to increase – a larger mass will tend to "coast" for a longer time.

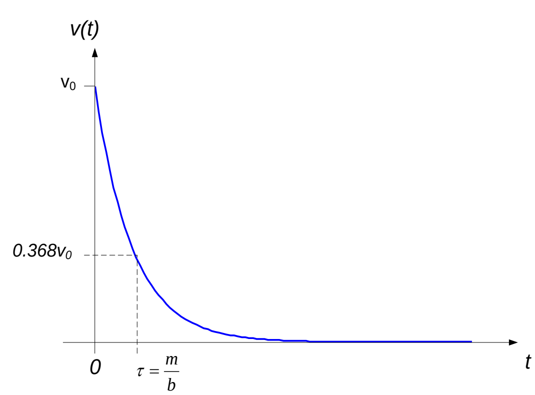


Figure 7.4. Homogeneous response of mass-damper system.

Note: The velocity of the mass tells us how much kinetic energy is being stored by the system. The initial condition provides the energy initially stored in the system. The calculated response describes how this energy is dissipated through the sliding friction. No energy is added to the system in this case, since the external applied force is zero.

Case ii: Response to Step Input

We will now consider the case in which the mass is initially at rest, and a constant force is applied to the mass at time *t=0*. Intuitively, we expect the velocity of the mass to increase to some final value; the final velocity of the mass corresponds to the condition in which the frictional force is equal and opposite to the applied force (recall that in our model, the frictional force is proportional to velocity – as the velocity increases, the frictional force opposing the motion also increases). We now solve the governing differential equation for this system and compare the results to our expectations.

The differential equation governing the system, valid for *t>0*, and initial condition, providing the energy in the system at *t=0*, are:

$$m\frac{dv(t)}{dt} + bv(t) = F$$

$$v(t = 0) = 0 \qquad \text{Eq. 7.10}$$

Where F is the magnitude of the (constant) applied force. Note that since F is constant, and only applied for times *t>0*, we have a step input with magnitude F. We want to solve the above differential equation for *t>0*; since the input forcing function can be represented as a step function, this resulting solution is called the step response of the system.

For this case, we have both a nonzero forcing function and an initial condition to consider. Thus, we must determine both the homogeneous and particular solutions and superimpose the result per equation (7.3) above.

The homogeneous solution is determined from:

$$m\frac{dv_h(t)}{dt} + bv_h(t) = 0 \qquad \text{Eq. 7.11}$$

Where vh(t) is the homogeneous solution. This equation has been solved as case *i*; the <u>form</u> of the solution is:

$$v_h(t) = K_1 e^{\frac{-bt}{m}} \qquad \text{Eq. 7.12}$$

Where K_1 is (in this case) an unknown constant which will be determined from our initial conditions.

The particular solution is determined from:

$$m\frac{dv_p(t)}{dt} + bv_p(t) = F \qquad \text{Eq. 7.13}$$

Where $v_p(t)$ is the particular solution to the differential equation in equation (7.10). Since the right-hand side of equation (7.13) is constant for *t>0*, the left-hand side of the equation must also be constant for *t>0* and $v_p(t)$ must be constant for *t>0*. If $v_p(t)$ is constant, $\frac{dv_p(t)}{dt}$ is zero and equation (7.13) simplifies to:

$$bv_p(t) = F$$

So that:

$$v_p(t) = \frac{F}{b} \qquad \text{Eq. 7.14}$$

Superimposing equations (7.12) and (7.14), per the principle expressed in equation (7.3) results in:

$$v(t) = v_k(t) + v_p(t) = K_1 e^{\frac{-bt}{m}} + \frac{F}{b} \qquad \text{Eq. 7.15}$$

We can now use our initial condition, *v(t=0)=0*, to determine the constant K_1. Evaluating equation (7.15) at *t=0* and applying the initial condition results in:

$$v(t=0) = 0 = K_1 e^{\frac{-b\cdot 0}{m}} + \frac{F}{b} \qquad \text{Eq. 7.16}$$

Since $e^{-b\cdot 0}m = 1$, equation (7.16) results in:

$$K_1 = -\frac{F}{b} \qquad \text{Eq. 7.17}$$

Substituting equation (7.17) into equation (7.15) results in the overall solution:

$$v(t) = -\frac{F}{b} e^{\frac{-bt}{m}} + \frac{F}{b} = \frac{F}{b}\left(1 - e^{\frac{-bt}{m}}\right) \qquad \text{Eq. 7.18}$$

If, as in case *i*, we define the time constant $\tau = \frac{m}{b}$, equation (7.13) can be expressed as:

$$v(t) = \frac{F}{b}\left(1 - e^{\frac{-t}{\tau}}\right) \qquad \text{Eq. 7.19}$$

A plot of the system response is shown in Fig. 7.5. This plot matches our intuitive expectations: the initial velocity is zero; the applied force causes the mass to move. When the frictional and applied forces balance, the velocity of the mass becomes constant. The time constant is determined by the mass and the frictional coefficient; a larger mass results in a longer time constant – it takes longer to get a large mass to its final velocity than a small mass. The frictional coefficient also affects the system time constant; a smaller friction coefficient results in a longer time constant. This result seems counter-intuitive at first, since a smaller frictional coefficient should allow us to accelerate the mass more rapidly. However, the smaller frictional coefficient also results in a higher final velocity – since the time constant is defined by the time required to reach approximately 63.2% of the final velocity, the higher final velocity causes a longer time constant even though the mass is accelerating more rapidly. (If the damping coefficient is zero, the time constant goes to infinity. However, the final velocity also goes to infinity – it takes an infinite amount of time to get to 63.2% of an infinite velocity!)

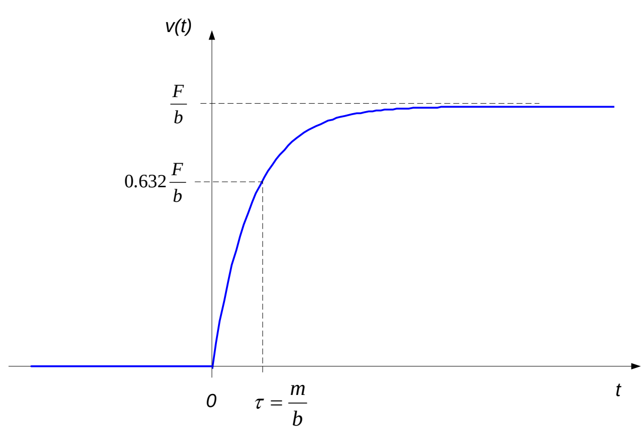


Figure 7.5. Step response of mass-damper system.

Note: The velocity of the mass again describes the energy stored by the system; in this case, the initial velocity is zero and the system has no energy before the force is applied. The applied force adds energy to the system by causing the mass to move. When the rate of energy addition by the applied force and energy dissipation by the friction balance, the velocity of the mass becomes constant and the energy stored in the system becomes constant.

Summary

We use the results of the above examples to re-state some primary results in more general terms. It is seen above that the natural and step responses of first order systems are strongly influenced by the system time constant, τ. The original, general, differential equation – equation (7.1) above – can be re-written directly in terms of the system time constant. We do this by dividing equation (7.1) by the coefficient a_1. This results in:

$$\frac{dy(t)}{dt} + \frac{a_0}{a_1} y(t) = \frac{1}{a_1} f(t), \quad t > t_0 \qquad \text{Eq. 7.20}$$

Defining $\tau = \frac{a_1}{a_0}$, equation (7.20) becomes:

$$\frac{dy(t)}{dt} + \frac{1}{\tau} y(t) = \frac{1}{a_1} f(t) \qquad \text{Eq. 7.21}$$

The initial condition on equation (7.21) is as before:

$$y(t = t_0) = y_0 \qquad \text{Eq. 7.22}$$

The cases of the system homogeneous response (or *natural* or *unforced* response) and step response are now stated more generally, for the system described by equations (7.21) and (7.22).

1. **Homogeneous response**

   For the homogenous response $f(t) = 0$, and the system response is

   $$y(t) = y_0 e^{\frac{-t}{\tau}} \, for \ t \geq 0 \qquad \text{Eq. 7.23}$$

   The response is shown graphically in Fig. 7.6.

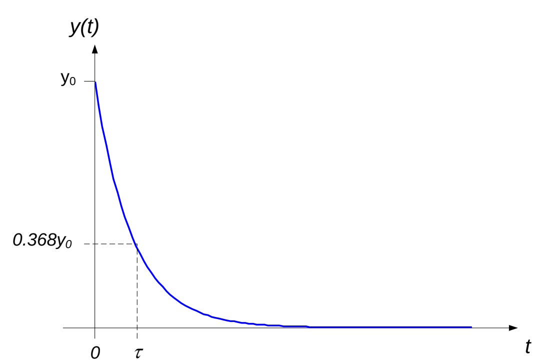


Figure 7.6. First-order system homogeneous response.

2. **Step Response**

   For a step input of amplitude A, $f(t) = Au_0(t)$ where $u_0(t)$ is the unit step function defined in section 6.1. Substituting this input function into equation (7.21):

$$\frac{dy(t)}{dt} + \frac{1}{\tau}y(t) = \frac{A}{a_1}, \quad t > 0 \qquad \text{Eq. 7.24}$$

Using the approach of case 2. of our previous mass-damper system example, we determine the system response to be:

$$y(t) = \frac{A}{a_0}\left[1 - e^{\frac{-1}{\tau}}\right] \qquad \text{Eq. 7.25}$$

This response is shown graphically in Fig. 7.7.

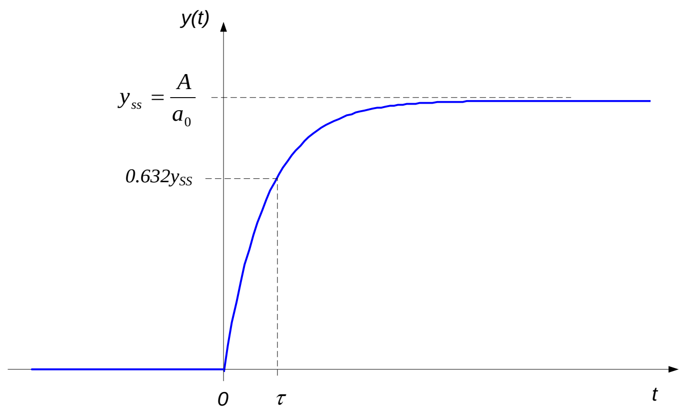


Figure 7.7. First order system response to step input with amplitude A.

Section Summary

- A first order system is described by a first order differential equation. The order of the differential equation describing a system is the same as the number of independent energy storage elements in the system – a first order system has one independent energy storage element. (The number of energy dissipation elements is arbitrary, however.)
- The differential equation governing a first order system is of the form:

$$a_1\frac{dy(t)}{dt} + a_0y(t) = f(t)$$

  Where *y(t)* is the system output, *f(t)* is the applied input to the system, and a_0 and a_1 are constants.

- The differential equation governing a first order system can also be written in the form:

$$\frac{dy(t)}{dt} + \frac{1}{\tau}y(t) = \frac{f(t)}{a_1}$$

  Where *y(t)* is the system output, *f(t)* is the applied input to the system, and τ is the system time constant.

- The system time constant is a primary parameter used to describe the response of first order systems.
- In this chapter, we considered two types of forcing functions: a zero-input case, in which $f(t)=0$ and $y(t=0)=y_0$, and a step input case, in which $f(t)=Au_0(t)$ and y(t=0)=0. For the zero-input case, the response is:

$$y(t) = y_0e^{\frac{-t}{\tau}}, \qquad t > 0$$

  For the step input case, the so-called *step response* is:

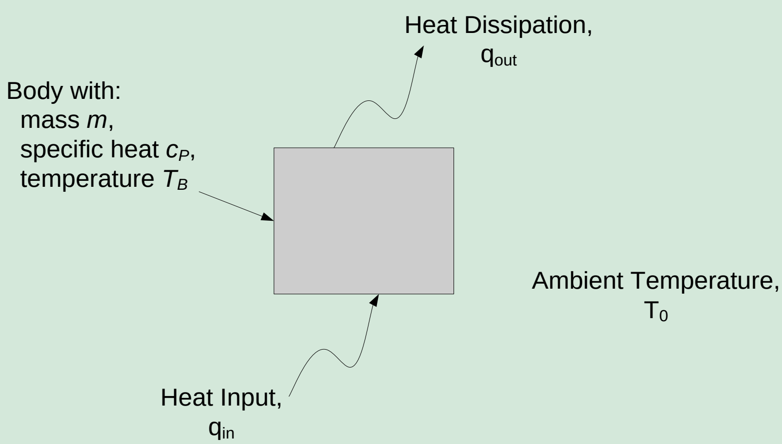
$$y(t) = \frac{A}{a_0}[1 - e^{-t}\tau], \;\; t > 0$$

- The system response consists of two parts: a *homogeneous solution* and a *particular solution*. The response can also be considered to consist of a *transient response* and a *steady-state response*. The homogeneous solution and the transient response die out with time; they are due to a combination of the system characteristics and the initial conditions. The particular solution and the steady state response have the same form as the forcing function; they persist as $t \to \infty$. It can be seen from the above that, for the zero-input case, the steady state response is zero (since the forcing function is zero). The steady state step response is $\frac{A}{a_0}$; it is a constant value and is proportional to the magnitude of the input forcing function.

7.1 Exercises

1. In Example 6.2, we examined a body which was subjected to external heating. The system is shown in the figure below. The mass of the body is m, the body material has a specific heat, c_P, and is at some temperature T_B. The surroundings are at an ambient temperature T_0. A heat input q_{in} is applied to the body, the heat dissipation between the body and its surroundings is q_{out}. It is common to assume that the heat dissipation q_{out} is proportional to the difference in temperature between the mass and its surrounds, so that $q_{out}=h(T_B-T_0)$. Incorporating this assumption into the governing equation for the system provided in Example 6.2, results in the following differential equation relating q_{in} and T_B:

$$mc_p\frac{d(T_B - T_0)}{dt} + h(T_B - T_0) = q_{in}$$



   a. If the body has some initial temperature T_i, and no heat is applied to the body (e.g. $q_{in} = 0$),
      i. What is the final temperature of the body?
      ii. What is the time constant (in terms of m, c_p, and h)?
      iii. If the body mass is doubled, what is the effect on the time constant? Does this agree with your expectations based on your intuition?
      iv. Sketch the response of the body temperature (T_B) vs. time. Label the initial condition and time constant on the sketch.
   b. If the body is initiallt at a temperature T_0 (the same as the ambient surroundings) and a constant heat input q_{in} is applied starting at t = 0,
      i. What is the final temperature of the body (in terms of m, c_p, q_{in}, and h)
      ii. What is the effect of doubling the heat input q_{in} on the final temperature? Does this agree with your intuition?
      iii. What is the effect of doubling the mass on the final temperature. Does this agree with your intuition?

iv. What is the effect of doubling the mass on the time constant? Does this agree with your intuition?
v. Sketch the response T_B vs. time. Label the initial temperature, final temperature, and time constant on the sketch.

7.2 Natural Response of RC Circuits

In this section, we consider source-free circuits containing only resistors and a single capacitor – commonly referred to as RC circuits. Since these circuits contain only a single energy storage element – the capacitor – the governing equations for the circuits will be first order differential equations. Since the circuits are source-free, no input is applied to the system and the governing differential equation will be homogeneous. Thus, in this section we will be examining the *natural response of RC circuits*.

To begin our investigation of the natural response of RC circuits, consider the simple resistor-capacitor combination shown in Figure 7.8. We assume that the capacitor is initially charged to some voltage, V_0, at time $t=0$ (so that $v(0)=V_0$). We want to determine the capacitor voltage, $v(t)$, for $t>0$.

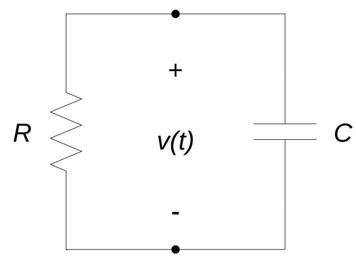


Figure 7.8. RC circuit; $v(t=0) = V_0$.

Applying Kirchhoff's current law at the positive terminal of the capacitor, as shown in Fig. 7.9, results in:

$$i_C(t) + i_R(t) = 0 \qquad \text{Eq. 7.26}$$

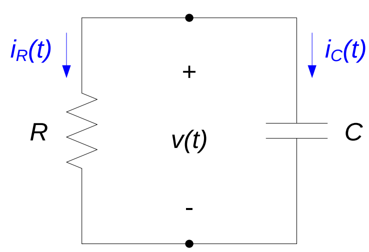


Figure 7.9. Currents in RC circuit.

Since $i_R(t) = \frac{v(t)}{R}$ and $i_C(t) = C\frac{dv(t)}{dt}$, equation (7.26) can be written in terms of the capacitor voltage as:

$$C\frac{dv(t)}{dt} + \frac{v(t)}{R} = 0$$

Separation of variables results in:

$$\frac{dv}{v} = -\frac{1}{RC}dt$$

The integral of the above is:

$$\int_{V_0}^{v(t)} \frac{dv}{v} = -\frac{1}{RC}\int_0^t dt$$

Which evaluates to:

$$\ln[v(t)] - lm[V_0] = -\frac{t}{RC}$$

Or

$$ln\left[\frac{v(t)}{V_0}\right] = -\frac{t}{RC}$$

Since $e^{\ln(x)} = x$, the above becomes:

$$v(t) = V_0 e^{\frac{-t}{RC}}$$

Alternate Approach to Solving the Above Differential Equation:

Since $\frac{dv(t)}{dt} = -\frac{1}{RC}v(t)$ we see that the form of the voltage signal must not change as a result of differentiation. Thus, assume that the voltage signal is of the form:

$$v(t) = Ke^{-st}$$

Where K and s are unknown constants. If we substitute this into the original differential equation, we obtain:

$$-Kse^{-st} = -\frac{K}{RC}e^{-st}$$

This is satisfied if we choose $s = -\frac{1}{RC}$. Employing our initial condition, $v(0) = V_0$, gives:

$$Ke^{\frac{-0}{RC}} = K = V_0$$

Results in:

$$v(t) = V_0 e^{\frac{-t}{RC}}$$

As before.

The capacitor voltage response is shown graphically in Fig. 7.10. The voltage response is a decaying exponential with a time constant:

$$\tau = RC \qquad \text{Eq. 7.27}$$

Thus, if we increase the resistance without changing capacitance, the circuit's time constant will increase. Likewise, increasing capacitance while maintaining the resistance constant will also increase the system's time constant. It is important to note that neither the resistance nor the capacitance alone specify the time constant, it is determined by the product of the two – if we simultaneously double the resistance and halve the capacitance, the system's time constant is unchanged.

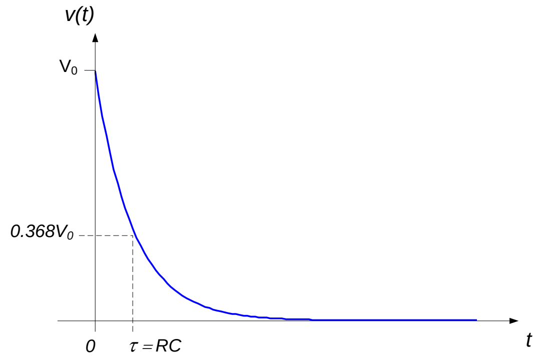


Figure 7.10. RC circuit natural response.

We can also obtain the above result by writing the governing differential equation directly in terms of the time constant. Previously, applying KCL to the circuit of Fig. 7.9, we obtained:

$$C\frac{dv(t)}{dt}+\frac{v(t)}{R}=0$$

Which can be re-written (by dividing the equation by the capacitance, *C*, as:

$$\frac{dv(t)}{dt}+\frac{v(t)}{RC}=0$$

Since the time constant for this circuit is $\tau=RC$, the above can be re-written as:

$$\frac{dv(t)}{dt}+\frac{1}{\tau}v(t)=0 \qquad \text{Eq. 7.28}$$

With initial condition:

$$v(0)=V_0 \qquad \text{Eq. 7.29}$$

The solution to equation (7.28), subject to the initial condition of equation (7.29) is (per our results above):

$$v(t)=V_0e^{-t}\tau \;\text{ for } t\geq 0 \qquad \text{Eq. 7.30}$$

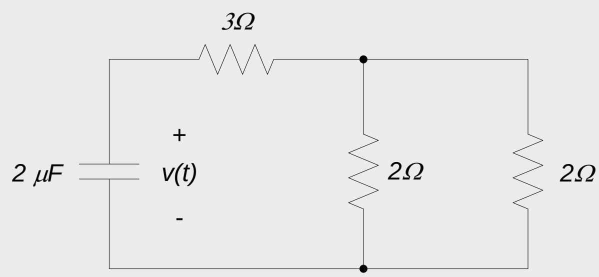
Where the time constant is defined per equation (7.27). The above approach matches our previous result. We will use the problem description provided by equations (7.28) and (7.29) and the solution of the form (7.30) most commonly in the subsequent examples. It should be emphasized, however, that the results are not dependent upon the solution approach – either of the other two approaches presented above yield the same conclusions.

7.2.1 Generalization to Multiple Resistors

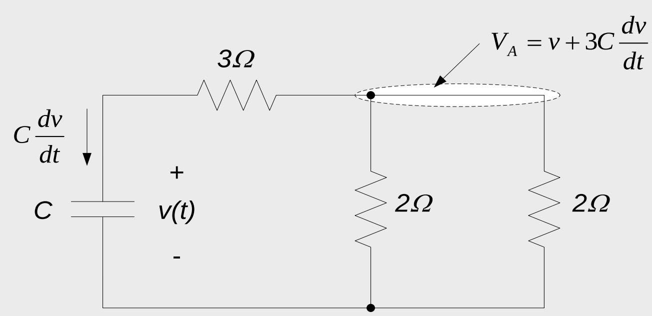
The resistance in the time constant of equation (7.27) can be more generally defined as the equivalent overall resistance of the circuit as seen by the capacitor. Thus, if we remove the capacitor from the circuit and create a Thevenin equivalent resistance as seen by the capacitor, the time constant will be the product of the capacitance and this equivalent resistance. We illustrate this point with an example.

Example 7.1

Determine the voltage *v(t)* for the circuit below if *v(0)=5V*.



We will first solve the problem by writing the first order differential equation governing the system. To aid this process we re-draw the circuit as shown below, labeling the current through the capacitor, defining node "A", and labeling the voltage at node "A". For simplicity, we label the capacitor as having capacitance "C" in the figure below.



The voltage at node A $V_A = v + (3\Omega)C\frac{dv}{dt}$ is obtained by applying KVL around the outer loop of the circuit.

Applying KCL at node A results in:

$$C\frac{dv}{dt} + \frac{V_A}{2\Omega} + \frac{V_A}{2\Omega} = C\frac{dv}{dt} + V_A = 0$$

Which, when substituting $V_A = v + 3C\frac{dv}{dt}$ results in:

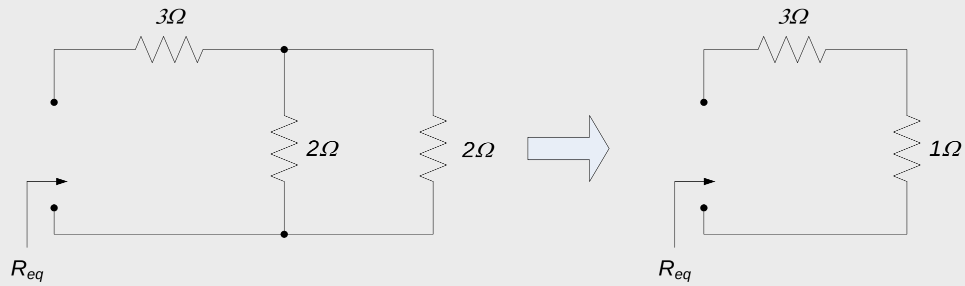
$$4C\frac{dv}{dt} + V_A = 0$$

This can be placed in the "standard form", $\frac{dv(t)}{dt} + \frac{1}{\tau v(t)} = 0$, by dividing through by 4C:

$$\frac{dv}{dt} + \frac{1}{4C}V_A = 0$$

Thus, the time constant $\tau = 4C = 8 \times 10^{-6}$ seconds.

Removing the capacitor and using circuit reduction to determine an equivalent resistance results in the circuit shown to the left below. The parallel combination of two, 2Ω resistors results in an equivalent 1Ω resistor, as shown in the figure to the right below. The resulting series combinations simplifies to a single 4Ω resistance, so R_{eq}=4Ω.



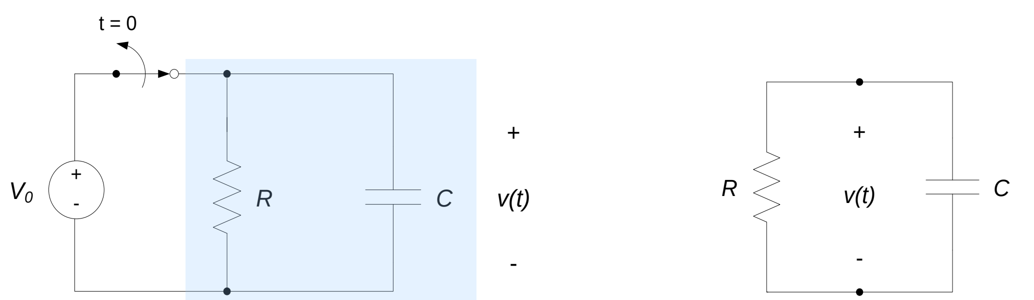
The circuit time constant can now be determined from R_{eq} and C. Thus:

$$\tau = R_{eq}C = (4\Omega)(2 \times 10^{-6}F) = 8 \times 10^{-6}\text{seconds}$$

Since the initial capacitor voltage is given to be 5V, $v(t) = 5e^{\frac{-t}{8\times10^{-6}}} = 5e^{1.25\times10^{5}t}V$.

7.2.2 Determining Initial Conditions

The circuits we have considered so far in this chapter contain no sources, the circuits' initial conditions are given. In general, we will need to determine the initial conditions from a given source and/or switching operation. For example, a conceptual circuit showing how the initial condition for the circuit of Fig. 7.8 can be created is shown in Fig. 7.11(a). The switch in Fig. 7.11(a) has been closed for a long time; thus, just before the switch opens, the voltage across the capacitor is V_0. Opening the switch removes the source from the circuit of interest. Since the voltage across a capacitor cannot change suddenly, the capacitor still has voltage V_0 immediately after the switch opens. (Mathematically, we say that $v(t=0)=v(t=0^+)=V_0$. Where time $t=0$ is an infinitesimal time before the switch opens, and the time $t=0^+$ is an infinitesimal time after the circuit opens.) Thus, for times $t>0$, the shaded portion of the circuit of Fig. 7.11(a) is identical to the circuit of Fig.7.11(b) from the viewpoint of the capacitor voltage. An example of the analysis of an RC circuit with an included source is provided below; note that the circuit being analyzed is still <u>unforced</u> - the source only provides an initial condition.

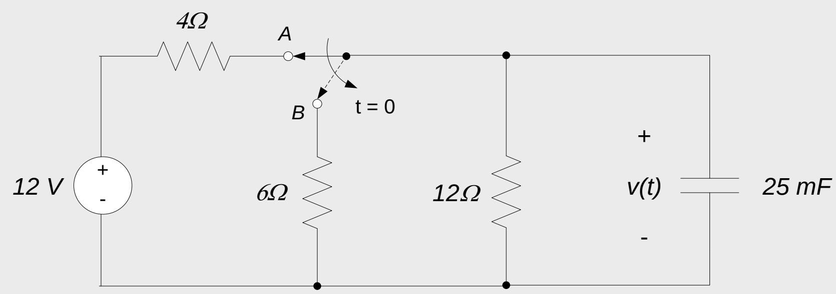


(a) RC circuit with source *(b) RC circuit with initial voltage*

Figure 7.11. Capacitor energy storage.

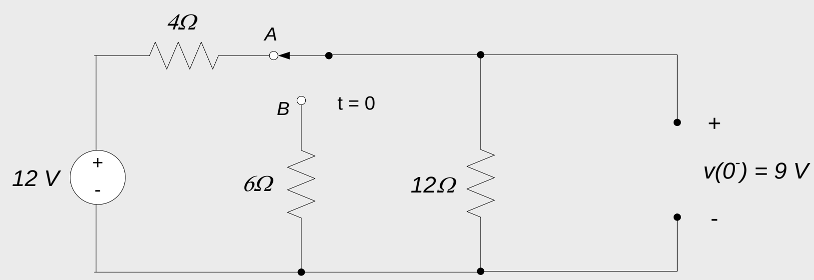
Example 7.2: Switched Circuit Natural Response

Consider the circuit shown below. The switch is originally at position A; at time $t=0$ seconds, the switch moves to position B in the circuit. We wish to determine the capacitor voltage, $v(t)$ for $t>0$.



Before time *t=0*, we assume that the switch has been at position A for a long time – all transients have died off, and any voltages and currents in the circuit have become constant. Since the capacitor voltage-current relationship is $i = C\frac{dv}{dt}$, if all parameters are constant the capacitor current is zero and the capacitor looks like an open circuit. Replacing the capacitor with an open circuit, as shown in the figure below, allows us to determine the voltage across the capacitor before the switch moved to position B. It is fairly easy to see that the capacitor voltage is the same as the voltage across the 12Ω resistor. Since no current flows through the 6Ω resistor in the circuit, the voltage across the 12Ω resistor can be determined by a voltage division between the 12Ω resistor and the 4Ω resistor.

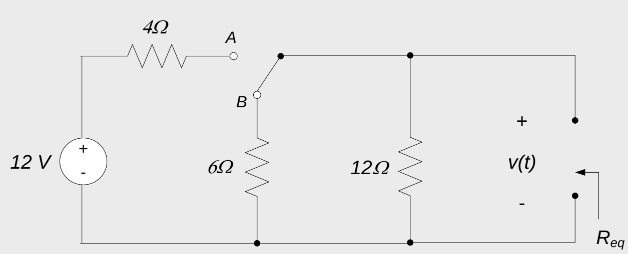
$$v(0^-) = 12V\left[\frac{12\Omega}{12\Omega + 4\Omega}\right] = 9V$$



Since a capacitor cannot change its voltage instantaneously, the capacitor voltage just after the switch moves to position B is $v(0^+) = v(0^-) = 9V$, which gives us our initial condition on the capacitor voltage.

The system time constant can be determined from the capacitance and the equivalent resistance seen by the capacitor. The equivalent resistance can be determined by looking into the capacitor terminals with the switch at position B (recall that we are solving the differential equation for *t>0*). The appropriate circuit is shown below. The equivalent resistance consists of a parallel combination of the 12Ω and 6Ω resistors (note that the 4Ω resistor and the voltage source are no longer relevant to the problem – they are isolated from the capacitor after the switch changes to position B). The equivalent resistance is thus:

$$R_{eq} = \frac{(6\Omega)(12\Omega)}{6\Omega + 12\Omega} = 4\Omega$$



The system time constant is, therefore, $\tau = R_{eq}C = (4\Omega)(25 \times 10^{-3}F) = 100 \times 10^{-3}$ seconds or 0.1 seconds. Appropriate substitution of the initial condition and time constant into equation (7.30) gives:

$$v(t) = 9e^{\frac{-t}{0.1}} = 9e^{-10t}V$$

Section Summary

- The natural response of an RC circuit describes the capacitor voltage in a circuit consisting only of resistors and a single equivalent capacitance. The circuit is source-free; the response is entirely due to energy initially stored in the capacitor.
- The differential equation for an unforced RC circuit is of the form:

$$\frac{dv(t)}{dt} + \frac{1}{R_{eq}C}v(t) = 0$$

  Where R_{eq} is the equivalent resistance "seen" by the capacitor.

- The RC circuit natural response is of the form:

$$v(t) = V_0e^{\frac{-t}{\tau}} \text{ for } t \geq 0$$

  Where V_0 is the initial voltage across the capacitor and τ is the circuit time constant.

- The time constant for <u>any</u> first order system can be determined from the differential equation governing the system. If the governing differential equation is written in the form:

$$\frac{dv(t)}{dt} + \frac{1}{\tau}v(t) = 0$$

  The time constant τ can be determined by inspection. Thus, by comparison with the above differential equation for RC circuits, $\tau=R_{eq}C$.

- Alternatively, for the special case of an RC circuit, the time constant can also be determined by:

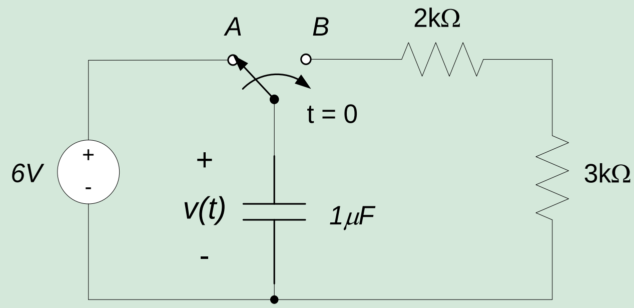
$$\tau = R_{eq}C$$

  Where R^{eq} is the equivalent resistance "seen" by the capacitor and C is the capacitance in the circuit. (Notice, that to use this relation, we must accurately identify the circuit as a first order RC circuit before proceeding.)

- The capacitor properties can be useful in determining initial conditions for an RC circuit:
  - Capacitors behave like open-circuits when all circuit parameters are constant, and
  - Capacitor voltages cannot change instantaneously

7.2 Exercises

1. The switch in the circuit below moves from position A to position B at time *t = 0*.
   a. Write the differential equation governing *v(t), t > 0*.
   b. Determine the time constant of the circuit from the differential equation of part a.
   c. Use the capacitance and the equivalent resistance seen by the capacitor to check your answer to part b.
   d. Determine the initial condition on the capacitor voltage, $v(t=0^+)$
   e. Determine *v(t), t >0*.



7.3 Natural Response of RL Circuits

In this section, we consider source-free circuits containing only resistors and a single inductor – commonly referred to as RL circuits. Like RC circuits, these circuits contain only a single energy storage element – the inductor – and the governing equations for the circuits will be first order differential equations. Since the circuits are source-free, no input is applied to the system and the governing differential equation will be homogeneous; the response of the circuit is due entirely to any energy initially stored in the inductor. We will thus be concerned with the *natural response* of RL circuits.

We will base our discussion of RL circuit natural responses on the series resistor-inductor circuit shown in Figure 7.12. We assume that the inductor has some initial current, I_0, flowing through it at time $t=0$ (so that $i(0)=I_0$). We will determine the inductor current, $i(t)$, for $t>0$.

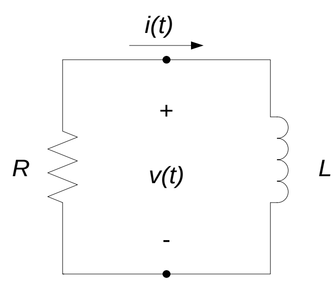


Figure 7.12. RL circuit with initial condition $i(t=0) = I_0$.

Since the inductor's voltage drop and current are related by $v(t) = L\frac{di(t)}{dt}$, application of Kirchhoff's voltage law around the single loop in the circuit results in:

$$L\frac{di(t)}{dt} + Ri(t) = 0 \qquad \text{Eq. 7.31}$$

In this chapter, we will solve this differential equation using the "alternate approach" presented in section 7.2 for capacitors. This approach consists of assuming a form of the solution, based on the form of the differential equation being solved. The assumed solution will contain unknown constants; these constants will be determined by plugging the assumed solution into the original differential and forcing the solution to satisfy the original differential equation and initial conditions. Since the differential equation is linear and has constant coefficients, the solution to the differential equation is unique – thus, if we can find <u>any</u> solution, we have found the <u>only</u> solution. This approach is an extremely common differential equation solution method, we will use it regularly in subsequent chapters.

The form of equation (7.31) indicates that i(t) must be a function which does not change form when it is differentiated ($L\frac{di(t)}{dt}$ must cancel out *Ri(t)*). The only function with this property is an exponential function. Thus, we assume that the current is of the form:

$$i(t) = Ke^{-st} \quad \text{Eq. 7.32}$$

Where K and s are unknown constants. Substituting equation (7.32) into equation (7.31) results in:

$$L(-Kse^{-st}) + R(Ke^{-st}) = 0$$

The above simplifies to:

$$(R - Ls)Ke^{-st} = 0$$

Which is satisfied if $s = \frac{R}{L}$ or $Ke^{-st} = 0$. Choosing $Ke^{-st} = 0$ results in the trivial solution *i(t)=0*, which will not, in general, satisfy the initial condition on the circuit. By the process of elimination, we choose $s = \frac{R}{L}$ and the form of our solution becomes:

$$i(t) = Ke^{\frac{-tR}{L}} \quad \text{Eq. 7.33}$$

The unknown constant K is determined by applying the initial condition, $i(0)=I_0$. Evaluating equation (7.33) at time *t=0*, and equating the result to the given initial condition, we obtain:

$$i(0) = Ke^{\frac{-0 \cdot R}{L}} = K = I_0$$

Thus, $K = I_0$, and the solution to the differential equation is:

$$i(t) = I_0 e^{\frac{-tR}{L}} = I_0 e^{\frac{-t}{\tau}} \quad \text{Eq. 7.34}$$

And the circuit time constant is:

$$\tau = \frac{L}{R} \quad \text{Eq. 7.35}$$

Equation (7.35) indicates that increasing L or decreasing R causes the time constant to increase. Conversely, decreasing L or increasing R decreases the time constant. A plot of the response of equation (7.34) is shown in Fig. 7.13.

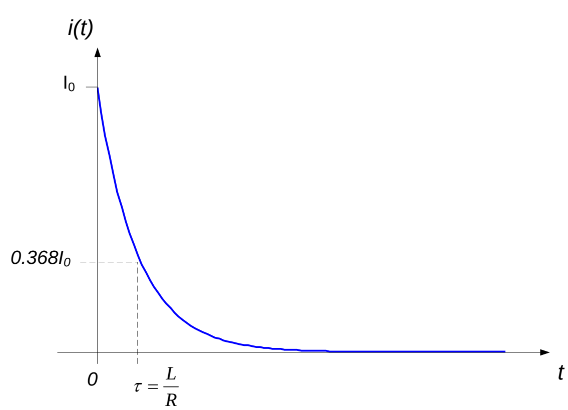


Figure 7.13. RL circuit natural response.

7.3.1 Generalization to Multiple Resistors

As with the RC circuit, the resistance in the time constant of equation (7.35) can be more generally defined as the equivalent overall resistance of the circuit as seen by the inductor. Thus, if we remove the inductor from the circuit and create a Thevenin equivalent resistance as seen by the inductor, the time constant will be the quotient of the inductance and the equivalent resistance.

7.3.2 Determining Initial Conditions

Though the initial condition is given in the above example, in general we will need to determine the initial condition from the application of some source and/or switching condition. For example, the circuit of Fig. 7.14 can be used to generate the initial condition of the example circuit above. In the circuit of Fig. 7.14, we assume that the switch has been closed for a long time and all circuit voltages and currents have become constant. When all circuit operating conditions are constant, the inductor behaves like a short circuit and all the current applied by the current source goes through the inductor and the inductor current is I_0. Since the current through an inductor cannot change suddenly, the inductor still has current I_0 immediately after the switch opens. Mathematically, $i(t=0^-)=i(t=0^+)=I_0$. where time $t=0^-$ is an infinitesimal time before the switch opens, and the time $t=0^+$ is an infinitesimal time after the circuit opens. Thus, for times $t>0$, the shaded portion of the circuit of Fig. 7.14 is identical to the circuit of Fig. 7.12 from the viewpoint of the inductor current.

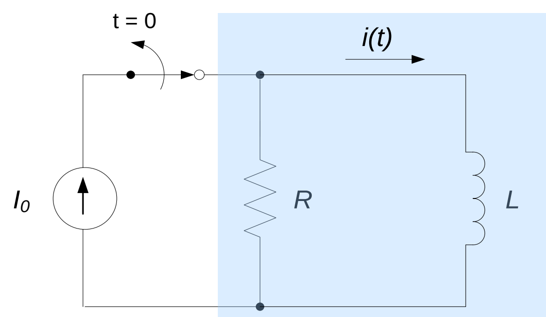
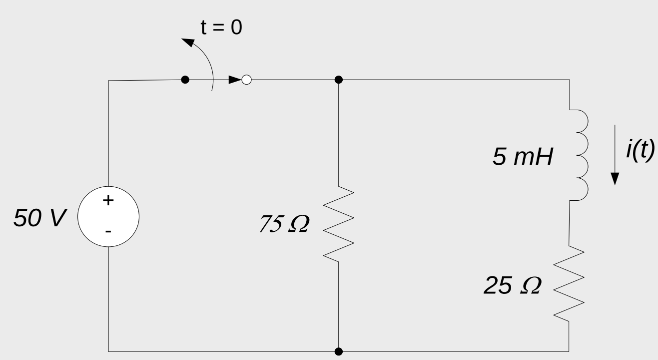


Figure 7.14. Circuit to realize the initial condition of the circuit of Figure 7.12.

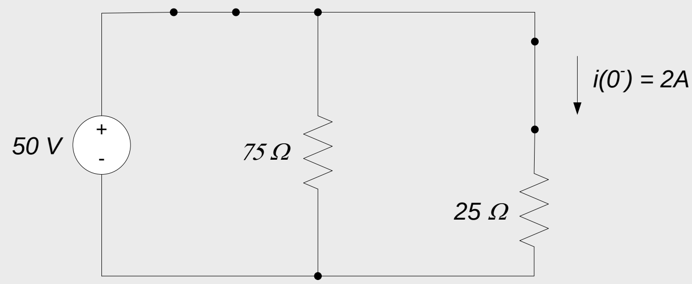
An example is provided below to illustrate the points made in this chapter.

Example 7.3: Switched RL Circuit Natural Response

Consider the circuit shown below. The switch has been closed for a long time; at time $t=0$ seconds, the switch suddenly opens. Determine the inductor current, $i(t)$ for $t>0$.

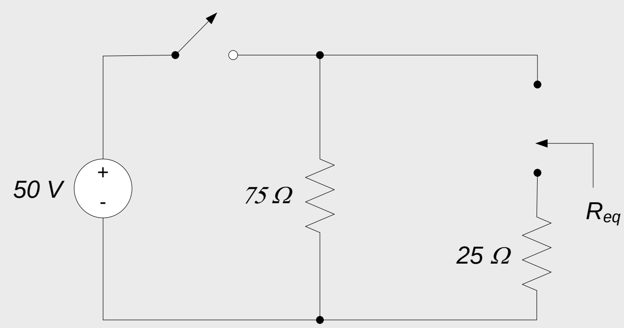


Since we are told that the switch has been closed for a long time, we assume that all voltages and currents in the circuit are constant before we open the switch. Therefore, before the switch is open the inductor behaves like a short circuit and the inductor current at time *t=0*⁻ can be determined by analyzing the circuit below:



From the above circuit, the inductor current just before the switch is opened is, from Ohm's law, $i(0^-) = \frac{50V}{25\Omega} = 2A$. Since an inductor cannot change its current suddenly, the current immediately after the circuit opens is $i(0^+) = i(0^-) = 2A$. This provides our initial condition on the inductor current.

The time constant of the circuit is determined from the inductance and the equivalent resistance of the circuit seen by the inductor, after the switch opens. The equivalent resistance can be determined by analyzing the circuit shown below.



From the circuit above it can be seen that, to the inductor, the 25Ω and 75Ω resistors are in series. Thus, $R_{eq} = 25\Omega + 75\Omega = 100\Omega$. The system time constant is, therefore:

$$\tau = \frac{L}{R_{eq}} = \frac{5 \times 10^{-3} H}{100\Omega} = 5 \times 10^{-5}\ seconds$$

Substitution of the initial condition and time constant into equation (4) gives:

$$i(t) = 2e^{\frac{-t}{5\times10^{-5}}} = 2e^{-20{,}000t} A$$

Section Summary

- The natural response of an RL circuit describes the inductor current in a circuit consisting only of resistors and a single equivalent inductance. The circuit is source-free; the response is entirely due to energy initially stored in the inductor.
- The differential equation for an unforced RL circuit is of the form:

$$\frac{di(t)}{dt} + \frac{R_{eq}}{L} i(t) = 0$$

Where R_{eq} is the equivalent resistance "seen" by the inductor.

- The RL circuit natural response is of the form

$$i(t) = I_0 e^{\frac{-t}{\tau}}, \text{for } t \geq 0$$

Where I_0 is the initial voltage across the capacitor and τ is the circuit time constant.

- The time constant for a first order system can be determined from the differential equation governing the system. If the governing differential equation is written in the form:

$$\frac{di(t)}{dt} + \frac{1}{\tau} i(t) = 0$$

The time constant τ can be determined by inspection. Thus, by comparison with the above differential equation for RL circuits, $\tau = \frac{L}{R_{eq}}$.

- Alternatively, for the special case of an RL circuit, the time constant can also be determined by:

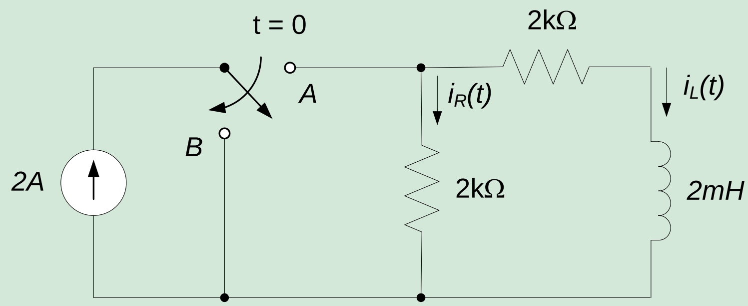
$$\tau = \frac{L}{R_{eq}}$$

Where R_{eq} is the equivalent resistance "seen" by the inductor and L is the inductance in the circuit. (Notice, that to use this relation, we must accurately identify the circuit as a first order RL circuit before proceeding.)

- The inductor properties can be useful in determining initial conditions for an RL circuit:
    - Inductors behave like short-circuits when all circuit parameters are constant, and
    - Inductors currents cannot change instantaneously

7.3 Exercises

1. The switch in the circuit below moves from position A to position B at time $t = 0$.
    a. Write the differential equation governing $i_L(t)$, $t > 0$.
    b. Determine the time constant of the circuit from the differential equation of part a.
    c. Use the inductance and the equivalent resistance seen by the inductor to check your answer to part b.
    d. Determine the initial condition on the inductor current, $i_L(t=0^+)$
    e. Determine $i_L(t)$, $t > 0$.
    f. Determine the resistor current $i_R(t)$ just before and just after the switch moves. (e.g. determine $i_R(t=0^-)$ and $i_R(t=0^+)$.) Is the resistor current continuous with time? Is the current through the other resistor continuous with time?

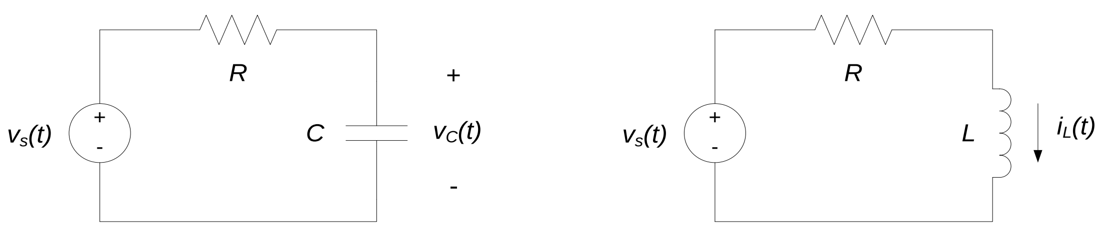


7.4 Forces Response of First Order Circuits

In sections 7.2 and 7.3, we were concerned with the natural response of electrical circuits containing a single energy storage element. For the natural response, any sources in the circuit were isolated from the circuit prior to determining the circuit response, so that the circuit being analyzed contained no sources. Thus, the circuit response of interest was entirely due to the energy initially stored in the circuit's capacitors or inductors. In these cases, all voltages and currents in the circuit die out with time.

In this section, we consider the case in which voltage or current sources are present in the first order circuit being analyzed. In this case, we must concern ourselves not only with the initial conditions in the circuit, but also with any *driving* or *forcing* functions applied to the circuit. The response of a system in the presence of an external input such as a voltage or current source is commonly called the *forced response* of the system. A primary difference between the natural response and the forced response of a system is that, although the natural response of a system always decays to zero, the forced response has no such restriction. In fact, the forced response of the system will take the same form as the forcing function, as time goes to infinity.

The differential equations governing the forced response of first order circuits are still, as implied, first order – thus, the circuits presented here will contain only a single energy storage element. Figure 7.15 shows two examples of forced first order circuits; Fig. 7.15(a) is a forced RC circuit and Fig. 7.15(b) is a forced RL circuit. The voltage sources $v_s(t)$ in Fig. 7.15 provide an arbitrary input voltage which can vary as a function of time. Without loss of generality, we will concern ourselves only with determining the forced response of the voltages across capacitors or the currents through inductors. In spite of the simplicity of the circuits shown in Fig. 7.15, their analysis provides a framework within which to place the analysis of an arbitrary first order electrical circuit. These two circuits are, therefore, analyzed below in order to present the forced solution of an arbitrary first order differential equation.



(a) Forced RC circuit *(b) Forced RL circuit*

Figure 7.15. Forced first order circuit examples.

Tip: In all circuits we analyze, we will define as unknown variables the voltages across capacitors and the currents through inductors. We will then solve for these variables and determine any other necessary circuit parameters from these variables.

In any electrical circuit, we can determine *any* circuit parameter from the voltages across capacitors and the currents through inductors. The reason for this is that capacitor voltages describe the capacitor energy storage and inductor currents describe the inductor energy storage. If we know the energy stored in all energy storage elements, we have completely characterized the circuit's operating parameters – mathematically, we say that we have defined the *state* of the system. We will formalize this concept later when we discuss *state variable models*.

Examples of this concept, in the context of Fig. 7.15 include:

- The voltage drop across the resistor in Fig. 7.15(a) is: $v_R(t) = v_s(t) - v_c(t)$
- The current through the capacitor in Fig. 7.15(a) is: $i_R(t) = \frac{v_S(t) - v_C(t)}{R}$
- The voltage drop across the resistor in Fig. 7.15(b) is: $V_R(t) = Ri_L(t)$

KCL at the positive terminal of the capacitor of the circuit shown in Fig. 7.15(a) provides:

$$\frac{v_S(t)-v_C(t)}{R} = C\frac{dv_C(t)}{dt} \qquad \text{Eq. 7.36}$$

Dividing through by the capacitance, C, and grouping terms results in the governing differential equation for the circuit of Fig. 7.15(a):

$$\frac{dv_C(t)}{dt} + \frac{1}{RC}v_c(t) = \frac{1}{RC}v_s(t) \qquad \text{Eq. 7.37}$$

KVL around the loop of the circuit of Fig. 7.15(b) results in:

$$-v_s(t) + Ri_L(t) + L\frac{di_L(t)}{dt} = 0 \qquad \text{Eq. 7.38}$$

Dividing through by the inductance, L, and rearranging results in the governing differential equation for the circuit of Fig. 7.15(b):

$$\frac{di_L(t)}{dt} + \frac{R}{L}i_L(t) = \frac{1}{L}v_s(t) \qquad \text{Eq. 7.39}$$

We notice now that the $\frac{1}{RC}$ term in equation (7.37) corresponds to $\frac{1}{\tau}$, where τ is the time constant of an RC circuit. Likewise, we note that the RL term in equation (7.39) corresponds to $\frac{1}{\tau}$, where τ is the time constant of an RL circuit. Thus, both equation (7.37) and equation (7.39) are of the form:

$$\frac{dy(t)}{dt} + \frac{1}{\tau}y(t) = u(t) \qquad \text{Eq. 7.40}$$

Where τ is the time constant of the system, *u(t)* is the input to the system, and *y(t)* is the desired system parameter (a voltage across a capacitor or a current through an inductor, for example). Equation (7.40) can be solved, given knowledge of the initial conditions on *y(t)*, $y(0)=y_0$. A block diagram of the system described by equation (7.40) is shown in Fig. 7.16.

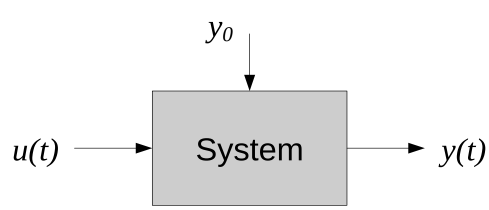


Figure 7.16. Block diagram of general forced first order system.

The solution to any forced differential equation can be considered to be formed of two parts: the *homogeneous solution* or *natural response* (which characterizes the portion of the response due to the system's time constant and initial conditions) and the *particular solution* or *forced response* (which characterizes the system's response to the forcing function *u(t)* after the natural response has died out). Thus, the system response *y(t)* in equation (7.40) and Fig. 7.16 can be expressed as:

$$y(t) = y_h(t) + y_p(t) \qquad \text{Eq. 7.41}$$

Where $y_h(t)$ is the homogeneous solution and $y_p(t)$ is the particular solution.

We will not attempt to analytically determine the solution of equation (7.40) for the general case of an arbitrary forcing function *u(t)*; instead, we will focus on specific types of inputs. Inputs of primary interest to us will consist of:

- Constant (step) input functions
- Sinusoidal input functions

The study of circuit responses to step functions is provided in Section 7.5. Sinusoidal input functions are discussed in later chapters.

7.4.1 Generalization to Multiple Resistors

As in the first order circuit natural response, the resistance in the time constant of equation (7.37) can be more generally defined as the equivalent overall resistance of the circuit as seen by the energy storage element. This conclusion follows directly from Thévenin's theorem. The circuits of Fig. 7.15 consist of energy storage elements (a capacitor and an inductor) which are connected to a circuit which can be considered to be the Thévenin equivalent circuit of a more complex circuit. Thus, the resistance R in the circuits of Fig. 7.15 can be the equivalent (Thévenin) resistance of an arbitrary linear circuit to which an energy storage element is connected. Thus, the resistances in the governing differential equations (7.37) and (7.39) can be considered to be Thévenin equivalent resistances. These equations thus can be written as:

$$\frac{dv_C(t)}{dt} + \frac{1}{R_{eq}C} v_c(t) = \frac{1}{R_{eq}C} v_s(t) \qquad \text{Eq. 7.42}$$

For an RC circuit, and :

$$\frac{di_L(t)}{dt} + \frac{R_{eq}}{L} i_L(t) = \frac{1}{L} v_s(t) \qquad \text{Eq. 7.43}$$

For an RL circuit, where R_{eq} is the equivalent (Thévenin) resistance seen by the energy storage element. This leads to the conclusion that the time constants for first order forced circuits can be written in terms of the Thevenin resistance seen by the energy storage element. The appropriate relationships are:

$$\tau = R_{eq}C \qquad \text{Eq. 7.44}$$

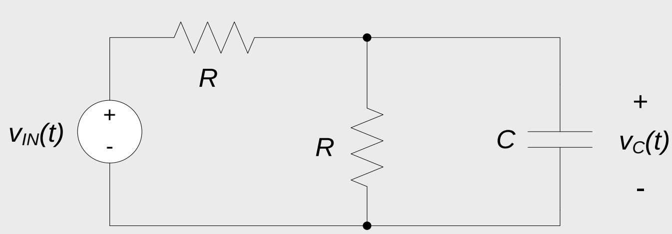
For RC circuits, and:

$$\tau = \frac{L}{R_{eq}} \qquad \text{Eq. 7.45}$$

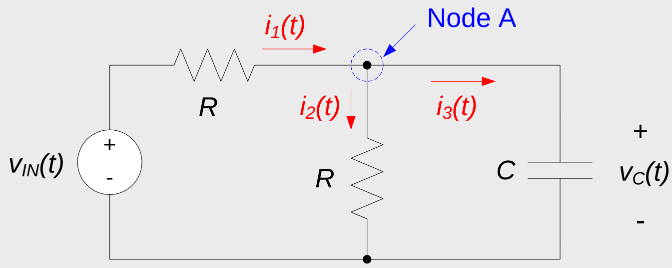
For RL circuits. Note that this conclusion is consistent with our previous results for unforced RC and RL circuits. We conclude this section with several examples in which we determine the differential equations governing first order electrical circuits. Note that we make no attempt to solve these differential equations – in fact, we cannot solve the differential equations, since we have not specified what the forcing functions are in the circuits below.

Example 7.4

Determine the differential equation relating $v_{in}(t)$ and $v_c(t)$ in the circuit below.



We will apply KCL at node "A", as indicated in the figure below, to begin our analysis.



Thus,

$$i_1(t) = i_2(t) + i_3(t)$$

Using the voltage-current relations to write these currents in terms of voltages results in:

$$\frac{v_{in}(t) - v_c(t)}{R} = \frac{v_c(t)}{R} + C\frac{dv_c(t)}{dt}$$

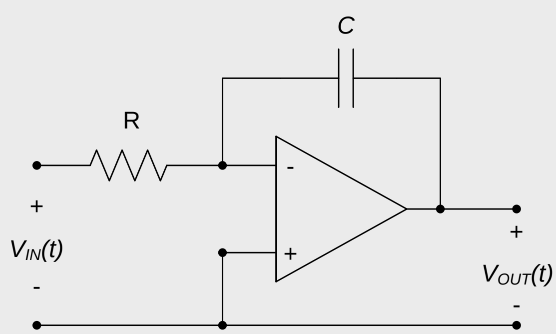
After a little algebra, the above results can be written in our standard first order circuit form as:

$$\frac{dv_c(t)}{dt} + \frac{2}{RC}v_c(t) = \frac{1}{RC}v_{in}(t)$$

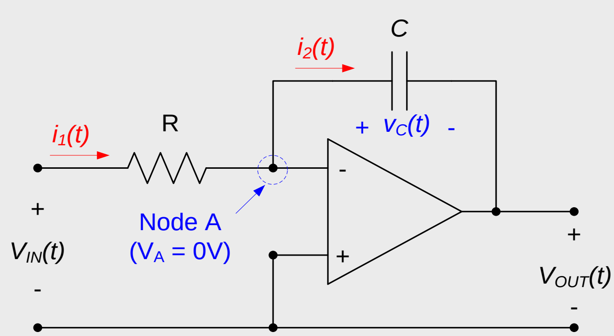
And the time constant of the circuit is $\tau = \frac{RC}{2}$.

Example 7.5

Determine the differential equation relating $V_{in}(t)$ and $V_{out}(t)$ in the circuit below.



Consistent with our approach of defining variables as voltages across capacitors and currents through inductors, we define the capacitor voltage as $v_c(t)$, as shown in the figure below. Also in the figure below, node A is defined and the rules governing ideal op-amps are used to identify the node voltage V_A=0V.



Applying KCL at node A in the circuit above gives:

$$i_1(t) = i_2(t)$$

The currents can be written in terms of the voltages in the circuit to provide:

$$\frac{V_{in}(t) - 0}{R} = C\frac{dv_c(t)}{dt}$$

The capacitor voltage can be written in terms of V_{OUT} (using KVL) as:

$$V_{OUT}(t) = -v_c(t)$$

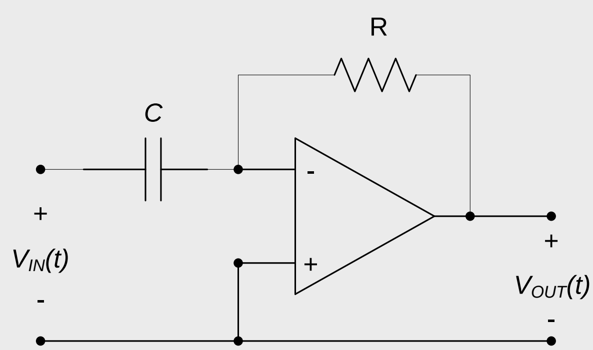
Thus, the governing differential equation for this circuit can be written as:

$$V_{OUT} = -\frac{1}{RC}\int V_{in}(t)dt$$

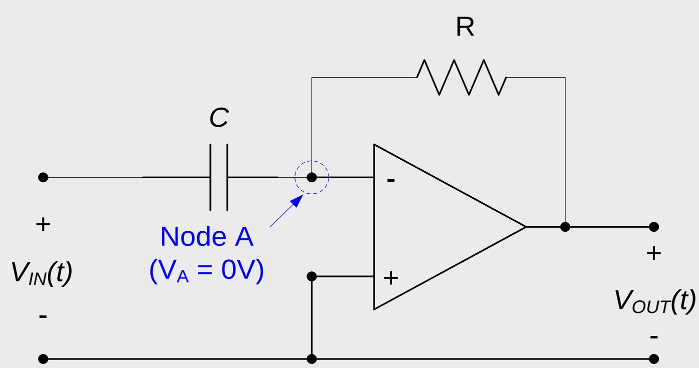
Note that this circuit is performing an integration.

Example 7.6

Determine the differential equation relating $V_{in}(t)$ and $V_{out}(t)$ in the circuit below.



As in example 7.5, we define node A is defined and use the rules governing ideal op-amps to identify the node voltage V_A=$0V$, as shown in the figure below.



Writing KCL at node A directly in terms of the node voltages results in:

$$C\frac{dV_{in}(t)}{dt} = -\frac{V_{OUT}}{R}$$

So that:

$$V_{OUT} = -RC\frac{dV_{in}(t)}{dt}$$

And the output voltage is proportional to the derivative of the input voltage.

7.4.2 Cross-checking Results

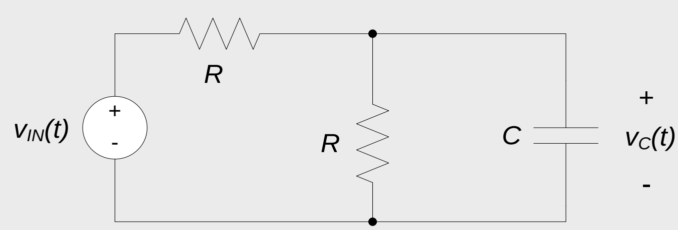
The above examples revolve entirely around determining the governing differential equation for the circuit. The actual circuit response depends upon the governing differential equation, the initial conditions, and the specific forcing function being applied to the circuit. In the above examples, the circuit time constants were inferred from the differential equation coefficients governing the forced response just as they were when we determined the natural response.

It is always desirable to check one's results in as many ways as possible. With this in mind, we would like to check to see if the differential equation we have written for a given electrical circuit makes sense before solving the equation for a specific forcing function. For first order circuits, at least, we can do this by determining a time constant directly from the circuit itself and comparing this time constant with the time constant inferred from the governing differential equation. The time constant of any first order forced circuit can be obtained by calculating the Thévenin resistance seen by the energy storage element and using equations (7.44) and (7.45) to provide the time constant. We will now revisit example 7.4 using this approach to validate the differential equation we previously determined for this circuit.

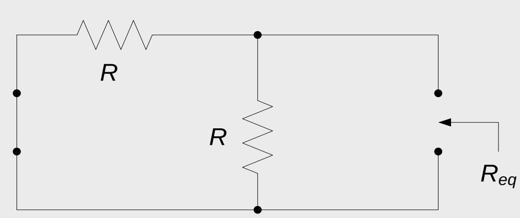
Example 7.7

Check the time constant for the circuit of Example 7.4 by calculating the equivalent resistance seen by the capacitor.

The circuit of example 7.4 is shown below for reference.



We can determine the equivalent resistance seen by the capacitor by replacing the voltage source with a short circuit and looking at the resistance seen across the capacitors terminals, as shown below:



The resistors are now in parallel, so that the equivalent resistance is R/2. The time constant is then $\tau = R_{eq}C = \frac{RC}{2}$, which agrees with the result of example 7.4

Section Summary

- The forced response of a first order circuit describes the response of the circuit in the presence of (in general) both a non-zero initial condition and an arbitrary time-varying input function.
- The differential equation describing the forced response of a first order circuit is of the form:

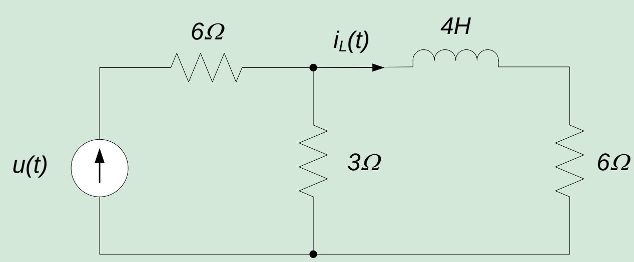
$$\frac{dy(t)}{dt} + \frac{1}{\tau} y(t) = f(t)$$

  Where *f(t)* is the forcing function applied to the circuit. The time constant, τ, of the circuit is readily obtained from the differential equation when it is written in the above form.

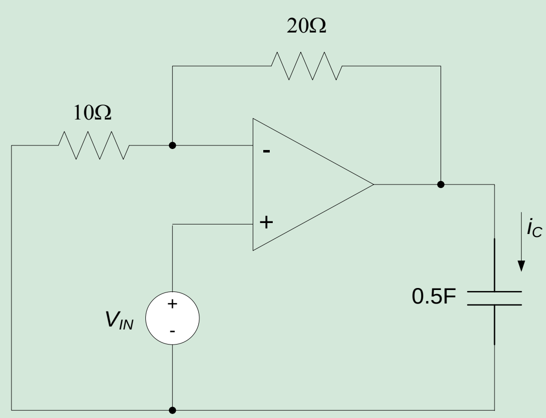
- The time constant for a first order forced system can also be determined directly from the circuit itself. The process is to determine the Thevenin resistance, R_{eq}, seen by the energy storage element and use that resistance in the appropriate time constant formula as introduced in sections 7.2 and 7.3. For an RL circuit, the time constant is $\tau = \frac{L}{R_{eq}}$, while for an RC circuit the time constant is $\tau = R_{eq}C$.
- Although the time constant can be determined from either the governing differential equation or the circuit itself, it is strongly recommended that both approaches be used and the results compared with one another to provide a cross-check of your analysis.

7.4 Exercises

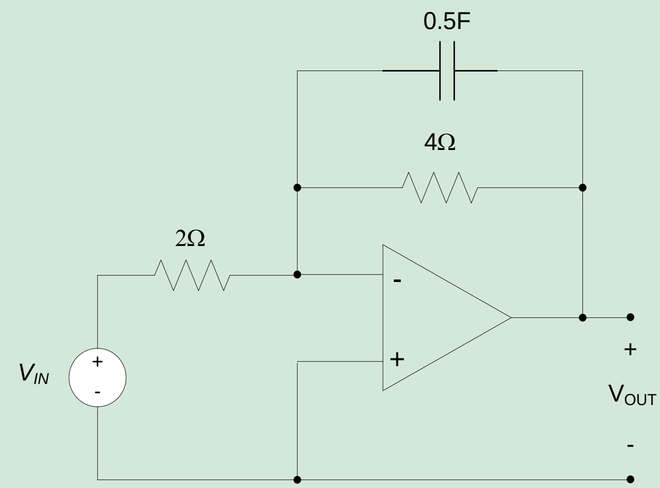
1. For the circuit below, write the differential equation governing $i_L(t)$. The input is the current source, u(t).



2. Determine the differential equation governing $i_C(t)$ in the circuit below:



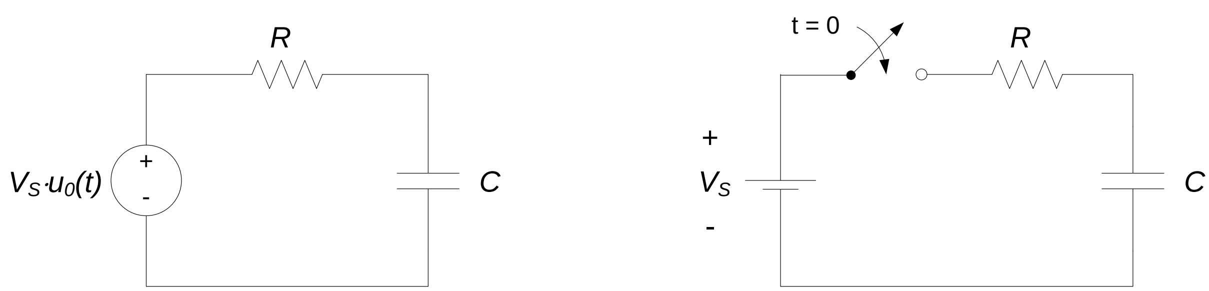
3. Determine the differential equation governing $V_{out}(t)$ in the circuit below.



7.5 Step Response of First Order Circuits

In section 7.4 we introduced the concept of the response of a first order circuit to an arbitrary forcing function. We will not attempt to solve this problem for an arbitrary forcing function; we will instead restrict our attention to specific forcing functions. In this section, we address the case in which the input consists of the sudden application of a constant voltage or current to a circuit; this type of input is typically modeled as a step function. The response of a system to this type of input, in the absence of any initial conditions, is called the *step response* of the system.

Figure 7.17 shows a conceptual circuit which applies a step input to an RC circuit, and an actual switched circuit which may be used to implement this forcing function.



(a) Time-varying voltage source

(b) Switched, constant voltage source

Figure 7.17. Circuits to provide step input to RC circuit.

In section 7.3, we saw that the differential equation governing the forced response of a first order circuit is of the form:

$$\frac{dy(t)}{dt} + \frac{1}{\tau} y(t) = u(t) \quad \text{Eq. 7.46}$$

where τ is the time constant of the system, *u(t)* is the input to the system, and *y(t)* is the desired system parameter (for example, the voltage across a capacitor or the current through an inductor). In the case of a step input to the system, the input *u(t)* to the system is a constant,

$$u(t) = Au_0(t) \quad \text{Eq. 7.47}$$

Where $u_0(t)$ is the unit step function.

We also saw, in section 7.3, that the solution of equation (7.46) can be written as the superposition of a homogeneous solution and the particular solution,

$$y(t) = y_h(t) + y_p(t) \qquad \text{Eq. 7.48}$$

The *homogeneous solution*, $y_h(t)$, characterizes the portion of the response due to the system's time constant and initial conditions while the *particular solution*, $y_p(t)$, characterizes the system's response to the forcing function $u(t)$ after the natural response has died out.

The homogeneous response is the solution to the homogeneous differential equation:

$$\frac{dy(t)}{dt} + \frac{1}{\tau}y(t) = 0 \qquad \text{Eq. 7.49}$$

In Section 7.1, we showed that the form of the solution of the homogeneous response is:

$$y_h(t) = K_1 e^{\frac{-t}{\tau}} \qquad \text{Eq. 7.50}$$

However, now we cannot determine K_1 directly at this point. Any conditions we can generate from the circuit with which to determine unknown coefficients apply to the entire forced solution, not the homogeneous or particular solution individually.

Our next step, therefore, is to determine the particular solution by substituting the input of equation (7.47) into the differential equation (7.46) and solving for $y_p(t)$. The appropriate differential equation is, therefore:

$$\frac{dy_p(t)}{dt} + \frac{1}{\tau}y_p(t) = Au_0(t) \qquad \text{Eq. 7.51}$$

The particular solution is appropriate after the homogeneous solution dies out. Thus, we evaluate equation (7.51) as $t \to \infty$. The right-hand side of equation (7.51) is a constant value as $t \to \infty$, since:

$$Au_0(t) = \begin{cases} 0, t < 0 \\ A, t > 0 \end{cases}$$

If the right-hand side of equation (7.51) is a constant, then the left-hand side of equation (6) must be a constant and the individual terms in the left-hand side of equation (7.51) must be constants. It follows, then, that $y_p(t)$ is a constant and that $\frac{dy_p}{dt} = 0$. Therefore, as $t \to \infty$, equation (7.51) becomes:

$$\frac{1}{\tau}y_p(t) = A \Rightarrow y_p(t) = K_2 \qquad \text{Eq. 7.52}$$

The overall solution, then, from equations (7.48), (7.50), and (7.52) is:

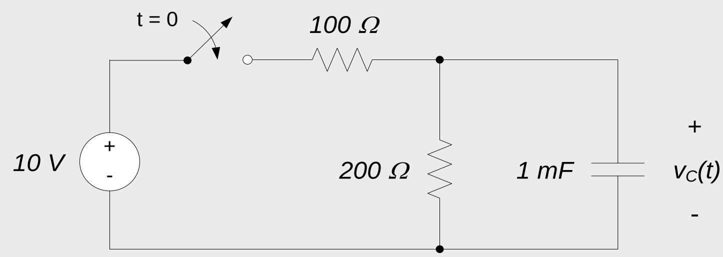
$$y(t) = K_1 e^{\frac{-t}{\tau}} + K_2 \qquad \text{Eq. 7.53}$$

The unknown constants K_1 and K_2 are typically determined from evaluating the circuit's behavior for $t=0$ and $t \to \infty$.

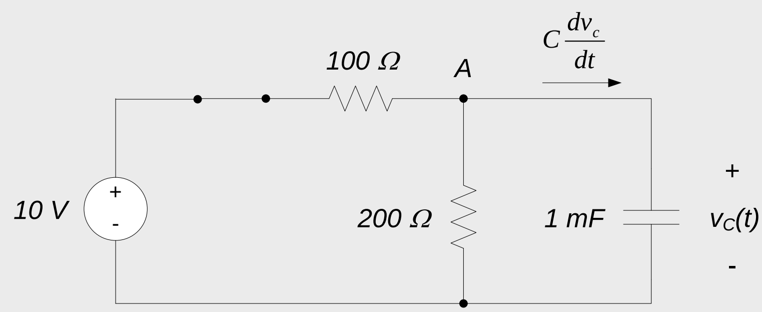
We illustrate the overall solution process with three examples below. The first two examples are of passive first order circuit responses; the third example is of an active first order circuit.

Example 7.8

Determine the voltage across the capacitor, $v_C(t)$ for $t>0$ in the circuit below. The switch has been open for a long time, and the initial voltage across the capacitor is zero.



When the switch is closed, the circuit is as shown below. The capacitor current has been labeled for later convenience. Note that the direction of the capacitor current is set to agree with the polarity of the capacitor voltage, according to the passive sign convention.



KCL at node "A" of the circuit shown above results in:

$$\frac{10 - v_c}{100\Omega} = \frac{v_c}{200\Omega} + 1 \times 10^{-3}\frac{dv_c}{dt}$$

Placing this in the form of equation (7.46) results in:

$$\frac{dv_C}{dt} + 15v_C = 100$$

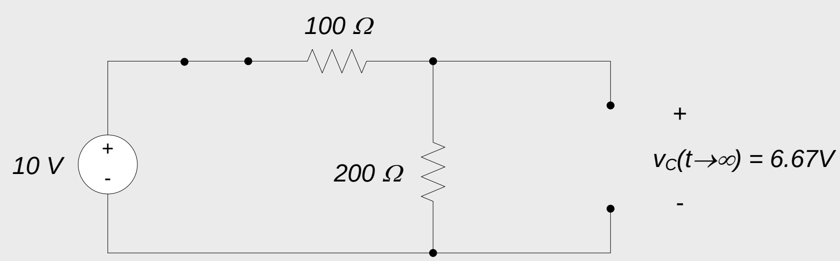
Thus, the time constant $\tau = \frac{1}{15}$ seconds. From equation (7.53) above, the form of the solution is:

$$v_C(t) = K_1 e^{\frac{-t}{\tau}} + K_2$$

We now apply the given initial condition, *vC(0)=0* to get:

$$0 = K_1 e^{\frac{-t}{\tau}} + K_2 \Rightarrow K_1 + K_2 = 0 \qquad (*)$$

Another condition for determining the unknown constants is the capacitor voltage as $t \to \infty$. As $t \to \infty$, for a constant input, the capacitor becomes an open circuit. Thus, the circuit above becomes:



And the capacitor voltage can be determined from voltage division to be:

$$v_C(t \to \infty) = 10V \frac{200\Omega}{100\Omega} + 200\Omega = 6.67V$$

Substituting this result into the expression for the capacitor voltage results in:

$$6.67V = K_1 e^{\frac{-\infty}{\tau}} + K_2 \Rightarrow K_2 = 6.67V \qquad (**)$$

Equations (*) and (* *) provide two equations in two unknowns. Solving these results in:

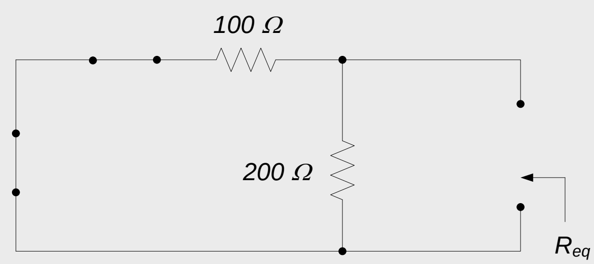
$$K_1 = -6.67V$$

$$K_2 = 6.67V$$

And the capacitor voltage becomes:

$$v_C(t) = 6.67(1 - e^{-15t})$$

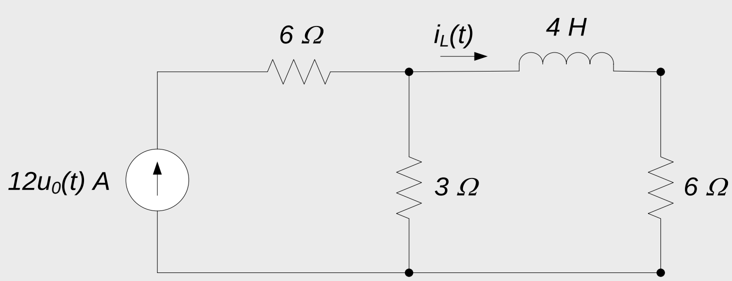
The value for τ can be checked by determining the equivalent resistance seen by the capacitor. To do this, we kill the sources and look into the capacitor terminals. The appropriate circuit is shown below.



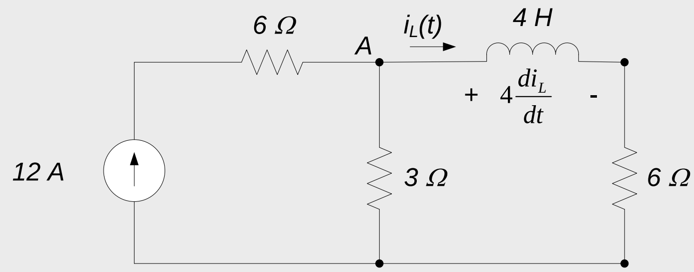
The equivalent resistance is $R_{eq} = \frac{(100\Omega)(200\Omega)}{100\Omega+200\Omega} = 66.67\Omega$ and $\tau = R_{eq}C = (66.67\Omega)(1mH) = \frac{1}{15}$ seconds, which checks our previous result.

Example 7.9

Determine the current through the inductor, $i_L(t)$, in the circuit below. No energy is stored in the circuit prior to $t=0$ seconds. The applied current input consists of a $12A$ step input applied at $t=0$.



The circuit is shown below for $t>0$, with the inductor current labeled for ease of reference.



KVL around the rightmost loop in the circuit results in the following voltage across the 3Ω resistor:

$$v_{3\Omega} = 4\frac{di_L}{dt} + 6i_L$$

Employing this result and applying KCL at node A results in:

$$-12 = i_L + \frac{1}{3\Omega}\left(4\frac{di_L}{dt} + 6i_L\right)$$

Placing the above equation in the form of equation (7.46) results in:

$$\frac{di_L}{dt} + \frac{1}{4}i_L = 9$$

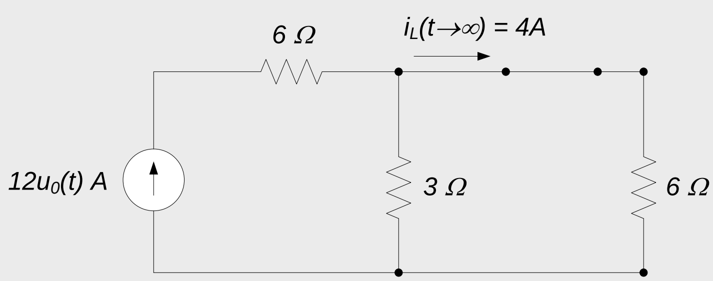
From this, we see that the circuit time constant is *τ=49* and the form of $i_L(t)$ is, from equation (7.53):

$$i_L(t) = K_1 e^{\frac{-4t}{9}} + K_2$$

From the given initial condition:

$$i_L(0) = K_1 + K_2 = 0$$

As $t \to \infty$, the inductor becomes a short circuit and the above circuit becomes:



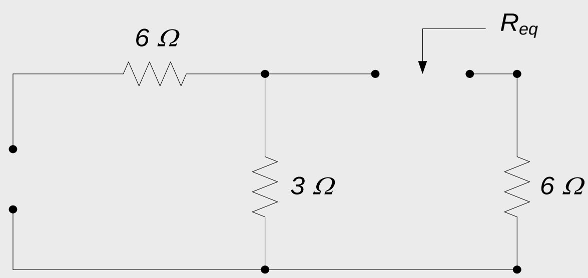
Current division allows us to determine that $i_L(t \to \infty) = 12A\frac{3\Omega}{3\Omega+6\Omega} = 4A$. Substituting this into equation governing $i_L(t)$, we obtain:

$$i_L(t \to \infty) = 4A = K_1 e^{-\infty} + K_2 \Rightarrow K_2 = 4A$$

Thus, the current through the inductor is:

$$i_L(t) = 4\left(1 - e^{\frac{-4t}{9}}\right)$$

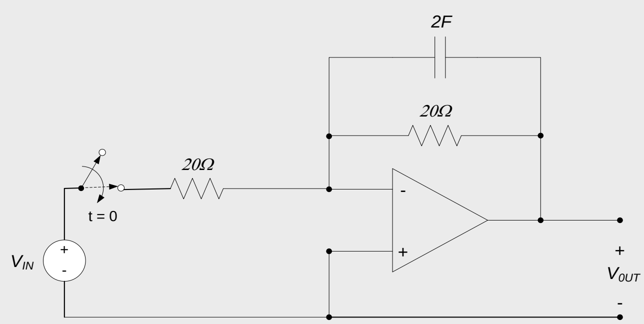
Note that, once again, we can check our value for the time constant by killing any sources and determining the equivalent resistance seen by the inductor. The appropriate circuit is shown below:



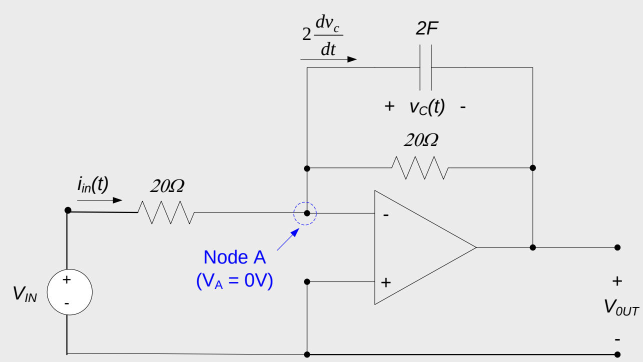
From the above circuit, the equivalent resistance is simply a series combination of the 3Ω and 6Ω resistors. Thus, $R_{eq}=9\Omega$. For an RL circuit, the time constant $\tau = \frac{L}{R} = \frac{4}{9}$ seconds. This agrees with the previous result we obtained from examining the form of the governing differential equation.

Example 7.10

The switch in the circuit below closes at time $t=0$. Find $V_{OUT}(t)$, for $t>0$. The capacitor has no energy stored in it prior to $t=0$.



For time $t>0$, the switch is closed and the circuit is as shown on the figure below. Labeling node A on this circuit as shown, we can determine from the ideal operational amplifier rules that $V_A=0V$.



Applying KCL at node A results in:

$$\frac{V_{in} - 0}{20\Omega} = \frac{v_C(t)}{20\Omega} + 2\frac{dv_C(t)}{dt}$$

Since $v_{C(t)} = 0 - V_{OUT}(t)$, the above equation can be written as:

$$\frac{V_{IN}}{20\Omega} = \frac{0 - V_{OUT}(t)}{20\Omega} - 2\frac{dV_{OUT}(t)}{dt}$$

Placing this in the form of equation (7.46) results in:

$$\frac{dV_{OUT}(t)}{dt} + 40V_{OUT}(t) = -V_{IN}$$

Thus, the time constant *τ=40* sec. From equation (7.53) above, the form of the solution is:

$$V_{OUT}(t) = K_1 e^{\frac{-t}{\tau}} + K_2$$

We now apply the given initial condition, $v_C(0)=0$ to get:

$$0 = K_1 e^{\frac{-0}{\tau}} + K_2 \Rightarrow K_1 + K_2 = 0 \qquad (*)$$

The output voltage as $t \to \infty$ can be determined from by open-circuiting the capacitor and analyzing the resulting circuit. The circuit with the capacitor open-circuited is simply an inverting voltage amplifier with a gain of one, so:

$$V_{OUT}(t \to \infty) = -V_{IN}$$

Substituting this result into the expression for the output voltage results in:

$$-V_{IN} = K_1 e^{\frac{-\infty}{\tau}} + K_2 \Rightarrow K_2 = -V_{IN} \qquad (**)$$

Equations (*) and (* *) provide two equations in two unknowns. Solving these results in:

$$K_1 = V_{IN}$$

$$K_2 = -V_{IN}$$

And the output voltage is:

$$V_{OUT}(t) = -V_{IN}\left(1 - e^{\frac{-t}{40}}\right)$$

7.5.1 Cross-checking Results

As in section 7.4, the examples above emphasize writing the governing differential equation for the circuit and determining the time constant from this differential equation. We also noted in section 7.4 that cross-checking results is crucial to producing reliable analyses, and that the time constant can also be determined directly from the circuit by calculating an equivalent resistance seen by the energy storage element. This cross-check was performed in exercises 7.8 and 7.9 above, as it should be.

For the case of a constant (or step) forcing function, we can use the final value of the solution as an additional cross-check of our results. In the above examples, we determined the final value of the response, $y(t \to \infty)$, based on the circuit behavior with capacitors replaced by open circuits and inductors replaced by short circuits. We can also determine the final value directly from the differential equation itself by examining the response of the differential equation as $r \to \infty$. This value is called the *steady-state* response of the circuit.

The general differential equation governing the step response of a first order circuit is given by equation (7.51) above. If we examine the solution to this differential equation as $t \to \infty$, we obtain only the particular solution, $y_p(t)$. (The homogeneous solution must go to zero as $t \to \infty$, leaving only the particular solution.) Thus, as $t \to \infty$ equation (7.51) becomes:

$$\frac{dy_p(t)}{dt} + \frac{1}{\tau} y_p(t) = A \qquad \text{Eq. 7.54}$$

Since the particular solution has the same form as the forcing function, and the forcing function is a constant, the derivative of the particular solution with respect to time is zero, and equation (7.54) becomes:

$$\frac{1}{\tau} y_p = A$$

And the steady state response, y_{ss}, is $y_{ss}=A \cdot \tau$. This value must agree with the final value obtained by short- circuiting inductors, open circuiting capacitors, and determining the final value from the circuit itself.

We apply this cross-check to the circuits of examples 7.8 and 7.9 below.

Example 7.11

In example 7.8, we determined (directly from the circuit behavior) that the final value of the capacitor voltage was:

$$v_C(t \to \infty) = 6.67V$$

And that the governing differential equation for the circuit was:

$$\frac{dv_C}{dt} + 15v_C = 100$$

We wish to check our final value of capacitor voltage relative to the differential equation behavior.

We can determine the final value of capacitor voltage by assuming that the voltage in the differential equation is constant, and setting its derivative to zero. Thus, the steady-state capacitor voltage can be determined from:

$$15v_{ss} = 100$$

So that $v_{ss} = \frac{100}{15} = 6.67V$ and agrees with the final value obtained by replacing the capacitor with an open circuit, as was done in example 7.8.

Example 7.12

In example 7.9, we determined (directly from the circuit behavior) that the final value of the inductor current was:

$$i_L(t \to \infty) = 4A$$

and that the governing differential equation for the circuit was:

$$\frac{di_L}{dt} + \frac{9}{4} i_L = 9$$

So that $i_{ss} = 9 \cdot \frac{4}{9} = 4A$ which agrees with the final value obtained by replacing the inductor with a short circuit, as was done in example 7.9.

7.5.2 DC Gain

The steady-state response of a circuit to a step input provides an important parameter which is often used to characterize the circuit's behavior. This parameter is called the DC gain, and is essentially the steady state response, normalized by the magnitude of the input step function. Mathematically, if the amplitude of the input step is A, the DC gain is given by:

$$DC\ gain = \frac{y_{ss}}{A}$$ Eq. 7.56

So the DC gain is simply the ratio of the output amplitude to the input amplitude, as $t \to \infty$. This parameter is of comparable importance to the characterization of first order circuits as the time constant. If we know the time constant and the DC gain of the circuit, we can immediately sketch the response of the circuit to <u>any</u> step input.

Example 7.13

Determine the DC gain for the circuit of example 7.8.

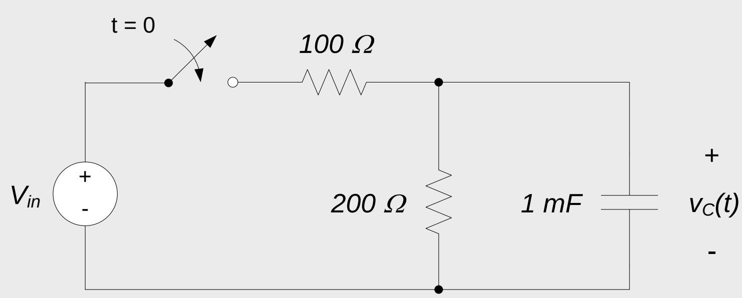
In example 7.8, the input voltage amplitude was 10V. The steady state output, the capacitor voltage, had an amplitude of:

$$v_C(t \to \infty) = 6.67V$$

Thus, the DC gain is simply the ratio of the input magnitude to the (steady-state) output magnitude:

$$DC\ gain = \frac{6.67V}{10V} = 0.67$$

The DC gain can also be determined from the governing differential equation. This is probably easiest to do if we replace the original 10V source with an arbitrary voltage, *Vin*, as shown below.



Re-deriving the governing differential equation, as was done in example 7.8, results in:

$$\frac{dv_C}{dt} + 15v_C = 10V_{IN}$$

If we are only concerned with the steady-state response, the derivative term can be set to zero and:

$$15v_{ss} = 10V_{IN}$$

So that $\frac{v_{ss}}{V_{in}} = \frac{10}{15} = 0.667$

Which agrees with the DC gain determined by calculating the steady state response to a specific input voltage and taking the same ratio.

Section Summary

- The step response of a first order circuit describes the response of a first order circuit to an applied step function. Typically, the term "step response" implies that all initial conditions in the circuit are zero, but this is not a requirement for application of any of the concepts presented in this section
- The differential equation describing the forced response of a first order circuit is of the form:

$$\frac{dy(t)}{dt}+\frac{1}{\tau}y(t)=Au_0(t)$$

Where $u_0(t)$ is the unit step function defined in section 6.2. The time constant, τ, of the circuit is readily obtained from the differential equation when it is written in the above form.

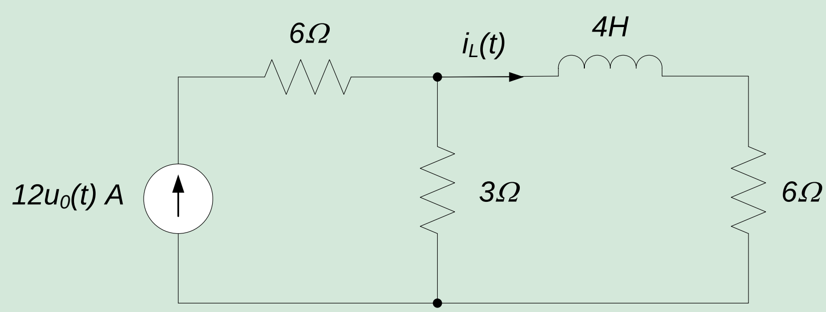
- The form of the step response of a first order system is:

$$y(t)=K_1e^{\frac{-1}{\tau}}+K_2$$

- The time constant in the solution above can be determined from either the governing differential equation or directly from the circuit itself.
- The unknown constants in the response are determined from initial, $y(0^+)$, and final, $y(t\to\infty)$, conditions. The initial conditions must be determined from the circuit itself. The final conditions can be determined from either the circuit itself or from the governing differential equation.
- Although both the time constant and the final value of the response can be determined from either the governing differential equation or the circuit itself, it is strongly recommended that both approaches be used and the results compared with one another to provide a cross-check of your analysis.
- The DC gain of a circuit provides the ratio of the output amplitude to the input amplitude, as $t\to\infty$, if the input is a constant value. The DC gain and the time constant are often used to characterize the response of a first order circuit.

7.5 Exercises

1. The initial current in the circuit below is zero. (e.g. $i_L(t=0^-)=0A$).)
   a. Write the differential equation governing $i_L(t)$.
   b. From your result in part 1, determine the time constant of the circuit.
   c. Write the form of the current $i_L(t)$.
   d. Use conditions at $t=0$ and $t\to\infty$ to determine the unknown constants in the expression for $i_L(t)$ in part 3.
   e. Determine the equivalent resistance seen by the inductor to check your answer from part b.

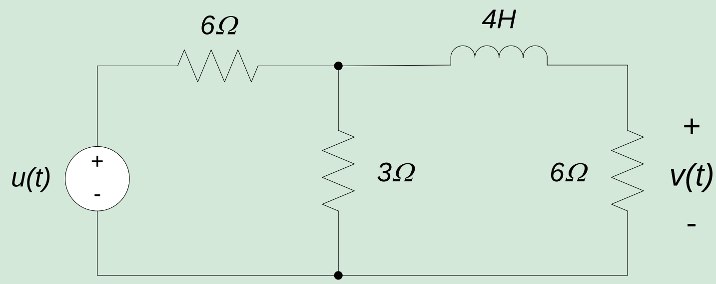


2. An input voltage input, $v(t)$, is applied to a first order electrical circuit. The differential equation governing the resulting current, $i(t)$, through an inductor is determined to be:

$$2\frac{di(t)}{dt}+3i(t)=5v(t)$$

What is the DC gain of the circuit? What are the units of DC gain for this circuit? What is the time constant of the circuit?

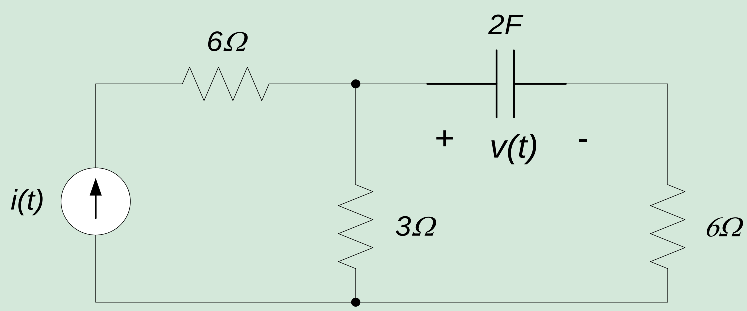
3. What is the DC gain of the circuit below? *u(t)* is the voltage input to the circuit and *v(t)* is the response. What are the units of the DC gain for this circuit?



4. The differential equation governing the circuit shown below is determined to be:

$$\frac{dv(t)}{dt} + \frac{1}{9}v(t) = i(t)$$

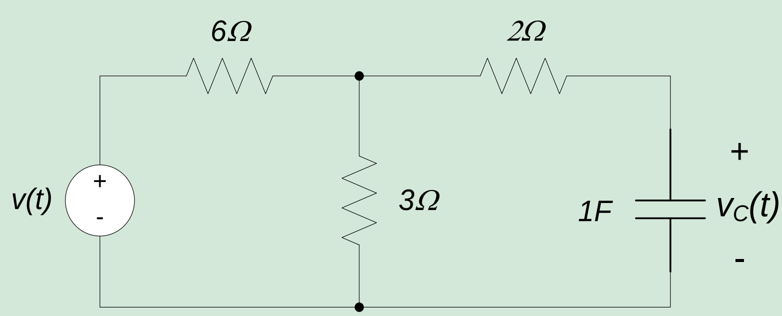
Where the current *i(t)* is the input to the circuit and the voltage *v(t)* is the circuit response. Without re-deriving the differential equation governing the circuit, do you feel that the given differential equation above accurately describes the circuit response?



5. The differential equation governing the circuit shown below is determined to be:

$$3\frac{dv_C(t)}{dt} + v_C(t) = v(t)$$

Where the voltage *v(t)* is the input to the circuit and the capacitor voltage $v_C(t)$ is the circuit response. Without re-deriving the differential equation governing the circuit, do you feel that the given differential equation above accurately describes the circuit response? Justify your answer.



Real Analog Chapter 7: Lab Projects

7.2.1: Passive RC Circuit Natural Response

In this lab assignment, we will examine the natural response of a simple RC circuit. We will use both a manual switching operation and a square wave voltage source to create our circuit's natural response. We will see that the method used to create the response affects the circuit being measured.

Before beginning this lab, you should be able to:

- Determine the time constant of exponential functions
- Determine the natural response of passive RC circuits
- Correctly implement an electrolytic capacitor (Lab 6.3.2)
- Use the Analog Discovery waveform generator to apply a time-varying voltage input to an electrical circuit (Lab 6.2.1)
- Use the Analog Discovery oscilloscope to measure and display time-varying waveforms

After completing this lab, you should be able to:

- Use a manual switching operation to create the natural response of a first order circuit
- Use the trigger on the Analog Discovery oscilloscope to acquire a signal
- Be able to explain in your own words the difference between continuous and single-sequence data acquisition
- Use the Analog Discovery waveform generator to create the natural response of a first order circuit.
- Measure the initial condition and time constant of a first order circuit natural response

This lab exercise requires:

- Analog Discovery module
- Digilent Analog Parts Kit

Symbol Key:

DEMO — Demonstrate circuit operation to teaching assistant; teaching assistant should initial lab notebook and grade sheet, indicating that circuit operation is acceptable.

ANALYSIS — Analysis; include principle results of analysis in laboratory report.

SIM — Numerical simulation (using PSPICE or MATLAB as indicated); include results of MATLAB numerical analysis and/or simulation in laboratory report.

DATA — Record data in your lab notebook.

General Discussion:

The basic RC circuit being used in this assignment is shown in Fig. 1. We will be interested primarily in the measured vs. expected behavior of the capacitor voltage, $v_c(t)$. Initially, the voltage applied to the RC circuit is 5V. We obtain the natural response of the RC circuit by changing the applied voltage to 0V instantaneously at time $t = 0$. The natural response of the capacitor voltage is $v_c(t)$, $t>0$.

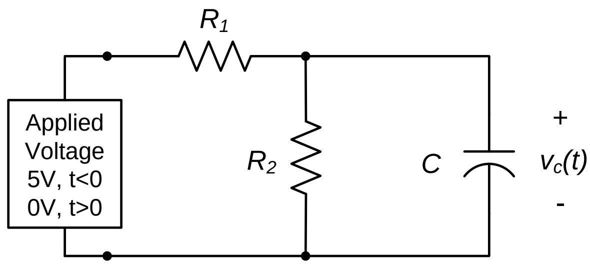
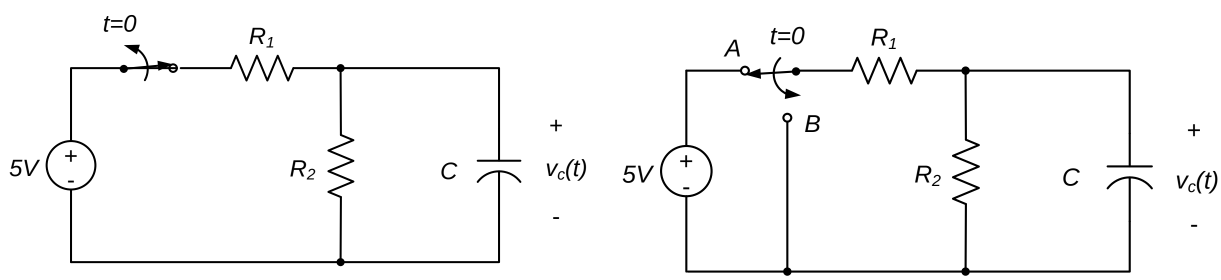


Figure 1. Basic RC circuit.

The way in which we reduce the applied voltage in Fig. 1 from 5V to 0V can have an effect on the circuit's natural response. In this lab assignment, we will use two different approaches to the switching process involved in changing the applied voltage:

a. We will use a voltage source to apply the initial 5V, and physically open a switch to reduce the applied voltage to 0V. This will result in the circuit as shown in Fig. 2(a). Notice that in Fig. 2(a), the voltage source is replaced by an open circuit.

b. A voltage source will be used to apply the 5V source, as above. However, in order to reduce the applied voltage to 0V, we will simply turn off the voltage source. This approach will result in the circuit shown in Fig. 2(b). Notice that in Fig. 2(b), the voltage source is replaced by a short circuit.



(a) Voltage change from switching

(b) Voltage change by turning "off" source.

Figure 2. Models of physical approaches to reducing applied voltage.

Pre-lab:

ANALYSIS

Estimate the initial capacitor voltage, $v_c(t<0)$, and the time constant for the circuits of Figs. 2(a) and 2(b). Your solutions may be functions of R_1, R_2, and C.

Lab Procedures:

DATA

a. Construct the circuit shown in Fig. 1, using R_1=1kΩ, R_2=2.2kΩ, and C = 22μF. (As always, measure the actual resistance value; measure the capacitance value if you have the appropriated instrument – some DMMs have a capacitance meter – otherwise, assume that the nominal capacitance value is correct.)

DATA

i. Use the Analog Discovery oscilloscope to measure the capacitor voltage $v_C(t)$ and use V+ on the Analog Discovery to apply a 5V source to the circuit. While acquiring data with the oscilloscope, quickly disconnect the power supply from your circuit. Record an image of the oscilloscope window, showing the response $v_C(t)$ of the capacitor voltage after the power is disconnected. <u>The data acquisition process can be difficult unless you use the oscilloscope's trigger to acquire a single sequence of the data. Brief instructions for doing this are in Appendix A of this lab assignment, more detailed instructions are provided in the on-line tutorials on Digilent's website</u>.

DEMO

ii. Demonstrate operation of your circuit to a teaching assistant and have them initial your lab notebook and the lab checklist.

ANALYSIS

iii. Estimate the time constant of the circuit from your measured data. Compare this result with your expectations based on your pre-lab analysis and the measured values of R_1, R_2, and C. Calculate a percent difference between the expected and measured time constants. Comment briefly on your results.

b. Still using the circuit shown in Fig. 1, (with R_1=1kΩ, R_2=2.2kΩ, and C = 22μF as in part a), use a square wave with an amplitude of 2.5V and an offset of 2.5V to create a square wave that oscillates between 0V and 5V. We will be using this square wave to implement a transition between 5V and 0V; use a very low frequency, 1Hz or so[1].

DATA

i. Record an image of the oscilloscope window, showing the response $v_C(t)$ of the capacitor voltage after the applied voltage goes to zero. Again, it is suggested that you use a trigger to assist in the data acquisition. The trigger settings from part (a) should also be appropriate for this section, but you may want to acquire the data differently. This is presented in Appendix B of this assignment.

DEMO

ii. Demonstrate operation of your circuit to a teaching assistant and have them initial your lab notebook and the lab checklist.

ANALYSIS

iii. Estimate the time constant of the circuit from your measured data. Compare this result with your expectations based on your pre-lab analysis and the measured values of R_1, R_2, and C. Calculate a percent difference between the expected and measured time constants. Comment briefly on your results.

Appendix A: Triggering and Single Acquisition

The *trigger* essentially defines where on the horizontal axis "zero" time occurs. The trigger point is commonly set by a particular feature on the waveform being measured. The basic trigger controls on the oscilloscope toolbar are shown in Fig. A1. These controls allow you to choose the trigger mode, the source, the condition, and the trigger level. Additional trigger controls are available by clicking on the **View** option on the oscilloscope menu bar and selecting the **Advanced Trigger** option.

Figure A1: Basic trigger controls.

Options for the primary trigger controls consist of the following:

- *Trigger mode:* basic options are **Normal**, **Auto**, or **None**. For this lab, we will use **Normal.**
- *Source:* Choose the channel which controls the trigger. A wave form feature on this channel will determine zero time.
- *Cond* and *Level:* These options specify the waveform feature used to set the trigger. *Cond* specifies a condition on the trigger – this is either **Rising** or **Falling**. If Rising is chosen, the trigger will set when the signal is increasing; Falling results in the trigger being set when the signal is decreasing. *Level* sets a

[1] Since we really just want to turn "off" the voltage once after charging the capacitor, we want our square wave to be "on" and "off" for long times relative the time required for the circuit to respond. Typically, a "long" time is considered to be at least five times the time constant of the circuit. You can use this fact, along with your calculated time constant based on the pre-lab, to choose a square wave frequency yourself if you want.

voltage level for the trigger. In Fig. A2, example settings for this lab are shown. In this example, the trigger is set to activate when the signal first reaches 1V and is decreasing[2] (**Falling**). Fig. 2 shows that zero time on the horizontal axis corresponds to this condition on the wave form.

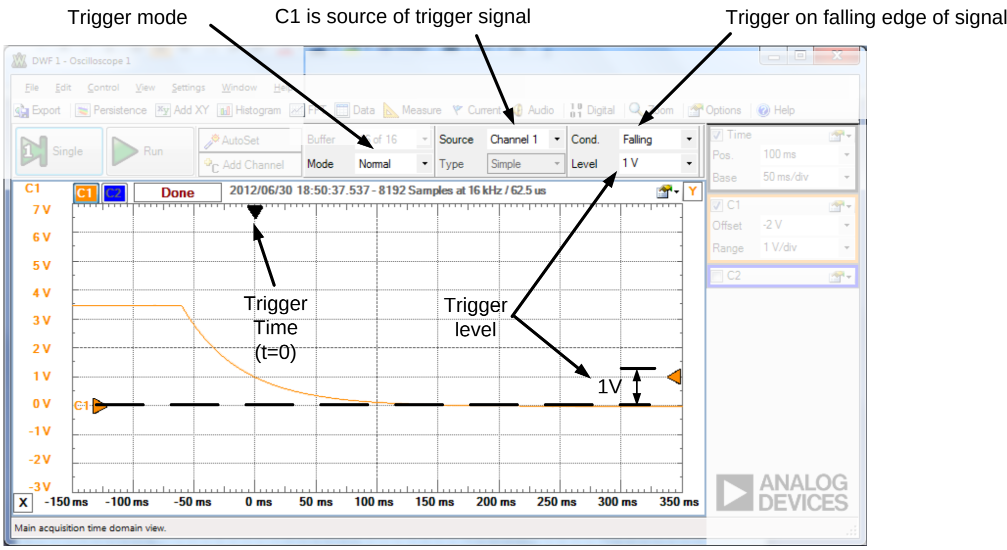


Figure 2. Example trigger settings and resulting waveform.

After you have set up the trigger, you can acquire the data. For part (a) of the lab procedures, we will generate a single natural response – once this response is generated, we want to display it in the oscilloscope window and freeze it there. We do not want to continue to display data after the response has decayed to zero. To generate a single screen of data and then stop acquiring additional data, click on the "Single" button on the scope instrument.

It will be worth your time, at this stage, to spend some time playing around with the trigger controls. Especially try changing the trigger point and the trigger level, as discussed below:

- Notice that the trigger point is denoted in the plot window by the black inverted triangle at the top of the plot window. The position of the trigger can be set by the *Pos* value in the time axis setting box, or by clicking on the trigger indicator (the black inverted triangle) with your left mouse button and dragging the trigger point to the desired position. Try it and observe its effect on the display.
- If the trigger source is one of the oscilloscope channels, the trigger level is shown on the plot window by the trigger level indicator – this is a triangular symbol of the same color as the trigger source channel on the right side of the plot window. For example, if the trigger source is channel 1, the indicator will be an orange triangle. The trigger level can be adjusted by clicking with your left mouse button on the trigger level indicator and dragging the indicator to the desired location or by changing the trigger level in the drop-down box.

Appendix B: Continuous Acquisition

In part (b) of these procedures, we are using the waveform generator to apply voltage to the circuit. The waveform generator continuously applies the selected waveform to your circuit, so the input to the circuit is a series of on/off steps which lasts as long as the waveform generator is running. In this case, we can use single-sequence acquisition

[2] Since the natural response will decrease, it would be useless to try to trigger on a rising condition. This will only cause the scope to trigger when – and if – we turn the power back on.

as we did in part (a) and display the result of one arbitrary sample of turning power to the circuit off. However, it is more typical to use continuous acquisition of the waveform.

At small time scales on the oscilloscope, the oscilloscope is essentially taking a series of “frames” of data and successively displaying them in the main scope window[3]. If the waveform is repetitive, triggering allows us to assign a “zero time” to a particular feature on the signal. That feature gets placed on the same point on the plot window every time the oscilloscope screen updates; if the signal repeats itself based on this feature, the oscilloscope will display the same section of the signal every time the screen updates, making the signal appear to be unchanging.

To continuously acquire data for part (b) of this lab, set your trigger point to be as you desire. (The same settings as you used in part (a) should work, though you may feel like modifying them.) Apply power to your circuit using the waveform generator and then click on “Run”. The data should be displayed in the window. The waveform displayed <u>should</u> appear to be unchanging, but it is actually being updated at rapid intervals – the successive waveforms simply lie directly on top of one another.

[3] At time scales larger than about 100ms/div, the data scrolls from left to right across the window – things are happening slowly enough so that it makes sense to watch the signal evolve. If we try this approach at small time scales, the data scrolls by too quickly to be useful.

Real Analog Chapter 7: Lab Worksheets

7.2.1: Passive RC Circuit Natural Response (35 points total)

1. In the space below, provide (from your pre-lab results) the initial capacitor voltages and time constants for the circuits of Fig. 2. (3 pts)

2. Provide below a schematic of the circuit you implemented, including actual resistance and capacitance values used in your circuit. (2 pts)

3. Attach to this worksheet an image of the oscilloscope window, showing the capacitor voltage response for the circuit of Fig. 2(a), in which V+ is used as the voltage source. Briefly discuss differences between the measured data and your estimates from the pre-lab (as always, this should include a percent difference between the values). (5 pts)

4. In the space below, provide your estimate of the circuit's time constant from the measured response data. Briefly discuss differences between the measured data and your estimates from the pre-lab (as always, this should include a percent difference between the values). (5 pts)

5. **DEMO**: Have a teaching assistant initial this sheet, indicating that they have observed your circuits' operation. (5 pts)

   TA Initials: _______

6. Attach to this worksheet an image of the oscilloscope window, showing the capacitor voltage response for the circuit of Fig. 2(b), in which the waveform generator is used as the voltage source. Briefly discuss differences between the measured data and your estimates from the pre-lab (as always, this should include a percent difference between the values). (5 pts)

7. In the space below, provide your estimate of the circuit's time constant from the measured response data. Briefly discuss differences between the measured data and your estimates from the pre-lab (as always, this should include a percent difference between the values). (5 pts)

8. **DEMO**: Have a teaching assistant initial this sheet, indicating that they have observed your circuits' operation when using a square wave. (5 pts)

TA Initials: _______

Real Analog Chapter 7: Lab Projects

7.3.1: Passive RL Circuit Natural Response

In this lab assignment, we will examine the natural response of a simple RL circuit. We will use both a manual switching operation and a square wave voltage source to create our circuit's natural response. We will see that the method used to create the response affects the circuit being measured.

Before beginning this lab, you should be able to:

- Determine the time constant of exponential functions
- Determine the natural response of passive RL circuits
- Use the Analog Discovery waveform generator to apply a time-varying voltage input to an electrical circuit (Lab 6.2.1)
- Use the Analog Discovery oscilloscope to measure and display time-varying waveforms
- Create a math channel on the Analog Discovery scope (Labs 6.3.1, 6.4.1)

After completing this lab, you should be able to:

- Use a manual switching operation to create the natural response of a first order circuit
- Use the trigger on the Analog Discovery oscilloscope to acquire a signal
- Be able to explain in your own words the difference between continuous and single-sequence data acquisition
- Use the Analog Discovery waveform generator to create the natural response of a first order circuit.
- Measure the initial condition and time constant of a first order circuit natural response

This lab exercise requires:

- Analog Discovery module
- Digilent Analog Parts Kit

Symbol Key:

DEMO Demonstrate circuit operation to teaching assistant; teaching assistant should initial lab notebook and grade sheet, indicating that circuit operation is acceptable.

ANALYSIS Analysis; include principle results of analysis in laboratory report.

SIM Numerical simulation (using PSPICE or MATLAB as indicated); include results of MATLAB numerical analysis and/or simulation in laboratory report.

DATA Record data in your lab notebook.

General Discussion:

The basic RL circuit being used in this assignment is shown in Fig. 1. We will be interested primarily in the measured vs. expected behavior of the inductor current, $i_L(t)$. Initially, the voltage applied to the RL circuit is 5V. We obtain the natural response of the circuit by changing the applied voltage to 0V instantaneously at time $t = 0$. The natural response of the inductor current is $i_L(t)$, $t>0$.

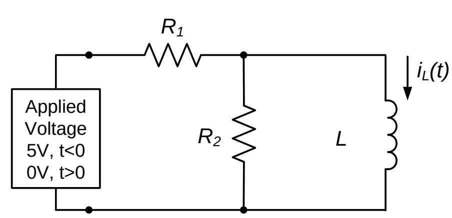


Figure 1. Basic RL circuit.

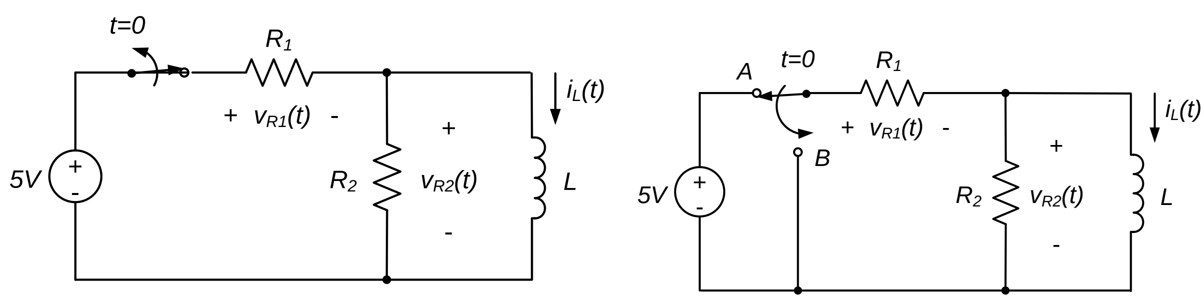
The way in which we reduce the applied voltage in Fig. 1 from 5V to 0V can have an effect on the circuit's natural response. In this lab assignment, we will use two different approaches to the switching process involved in changing the applied voltage:

a. We will use a voltage source to apply the initial 5V, and physically open a switch to reduce the applied voltage to 0V. This will result in the circuit as shown in Fig. 2(a). Notice that in Fig. 2(a), the voltage source is replaced by an open circuit.
b. A voltage source will be used to apply the 5V source, as above. However, in order to reduce the applied voltage to 0V, we will simply turn off the voltage source. This approach will result in the circuit shown in Fig. 2(b). Notice that in Fig. 2(b), the voltage source is replaced by a short circuit.

The implementation of the circuit of Fig. 1 is further complicated by the fact that we cannot directly measure a time-varying current; oscilloscopes will only measure voltages. Thus, we must infer the inductor current from parameters that we can actually measure. KCL tells us that the current through the inductor is the difference between the current through the resistor R_1 and the current through the resistor R_2. The currents through the resistors R_1 and R_2 can be determined from the voltages across and Ohm's law. Thus, the inductor current can be determined from the voltages across the resistors as follows:

$$i_L(t) = \frac{v_{R1}(t)}{R_1} - \frac{v_{R2}(t)}{R_2} \qquad \text{Eq. 1}$$

where the voltages $v_{R1}(t)$ and $v_{R2}(t)$ are as shown in Figs. 2. An oscilloscope can be used to measure these resistor voltages, and a math channel can be created to implement equation (1) and display the inductor current as a function of time.



(a) Voltage change from switching

(b) Voltage change by turning "off" source.

Figure 2. Models of physical approaches to reducing applied voltage.

Pre-lab:

ANALYSIS

Estimate the initial inductor current, $i_L(t<0)$, and the time constant for the circuits of Figs. 2(a) and 2(b). Your solutions may be functions of R_1, R_2, and L.

Lab Procedures:

DATA

a. Construct the circuit shown in Fig. 1, using $R_1=100\Omega$, $R_2=47\Omega$, and L = 1mH. (As always, measure the actual resistance values; you may assume that the nominal inductance value is correct.)

   i. Use the oscilloscope to measure the resistor voltages $v_{R1}(t)$ and $v_{R2}(t)$. Set up a math channel according to equation (1) above to display the inductor current, $i_L(t)$. Apply power to the circuit by turning on the V+ supply. While acquiring data with the oscilloscope, quickly disconnect the power supply from your circuit. Record an image of the oscilloscope window, showing the response $i_L(t)$ of the inductor current after the power is disconnected. <u>The data acquisition process can be difficult unless you use the oscilloscope's trigger to acquire a single sequence of the data. Brief instructions for doing this are in Appendix A of this lab</u>

DATA

assignment, more detailed instructions are provided in the on-line tutorials on Digilent's website.

ii. Demonstrate operation of your circuit to a teaching assistant and have them initial your lab notebook and the lab checklist.

iii. Estimate the time constant of the circuit from your measured data. Compare this result with your expectations based on your pre-lab analysis and the measured values of R_1, and R_2. Calculate a percent difference between the expected and measured time constants. Comment briefly on your results.

b. Construct the circuit of Fig. 2(b), still using R_1=1kΩ, R_2=2.2kΩ, and L = 1mH as in part a). This circuit is essentially the same as the circuit of part (a), except that the waveform generator is used to provide power to the circuit. Set up the waveform generator to apply a a square wave with an amplitude of 2.5V and an offset of 2.5V to your circuit. This square wave is used to implement the transition between 5V and 0V; use a very low frequency, 1Hz or so[4].

i. As in part a above, use the oscilloscope to measure the resistor voltages $v_{R1}(t)$ and $v_{R2}(t)$. Set up a math channel according to equation (1) above to display the inductor current, $i_L(t)$. Apply power to the circuit by turning on the waveform generator. Record an image of the oscilloscope window, showing the natural response $i_L(t)$ of the inductor current. Again, it is suggested that you use a trigger to assist in the data acquisition. The trigger settings from part (a) should also be appropriate for this section, but you may want to acquire the data differently. This is presented in Appendix B of this assignment.

ii. Demonstrate operation of your circuit to a teaching assistant and have them initial your lab notebook and the lab checklist.

iii. Estimate the time constant of the circuit from your measured data. Compare this result with your expectations based on your pre-lab analysis and the measured values of R_1 and R_2. Calculate a percent difference between the expected and measured time constants. Comment briefly on your results.

Appendix A: Triggering and Single Acquisition

The *trigger* essentially defines where on the horizontal axis "zero" time occurs. The trigger point is commonly set by a particular feature on the waveform being measured. The basic trigger controls on the oscilloscope toolbar are shown in Fig. A1. These controls allow you to choose the trigger mode, the source, the condition, and the trigger level. Additional trigger controls are available by clicking on the **View** option on the oscilloscope menu bar and selecting the **Advanced Trigger** option.

Figure A1: Basic trigger controls.

Options for the primary trigger controls consist of the following:

- *Trigger mode:* basic options are **Normal**, **Auto**, or **None**. For this lab, we will use **Normal.**

[4] Since we really just want to turn "off" the voltage once after charging the inductor, we want our square wave to be "on" and "off" for long times relative the time required for the circuit to respond. Typically, a "long" time is considered to be at least five times the time constant of the circuit. You can use this fact, along with your calculated time constant based on the pre-lab, to choose a square wave frequency yourself if you want.

- *Source:* Choose the channel which controls the trigger. A wave form feature on this channel will determine zero time.
- *Cond* and *Level:* These options specify the waveform feature used to set the trigger. *Cond* specifies a condition on the trigger – this is either **Rising** or **Falling**. If Rising is chosen, the trigger will set when the signal is increasing; Falling results in the trigger being set when the signal is decreasing. *Level* sets a voltage level for the trigger. In Fig. A2, example settings for this lab are shown. In this example, the trigger is set to activate when the signal first reaches 1V and is decreasing[5] (**Falling**). Fig. A2 shows that zero time on the horizontal axis corresponds to this condition on the wave form.

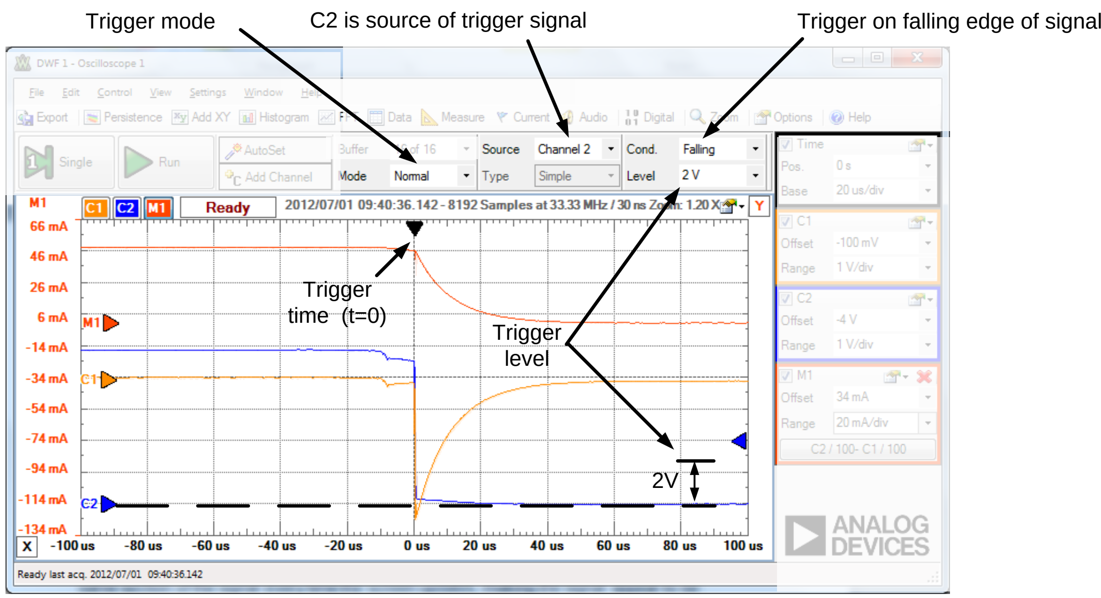


Figure A2. Example trigger settings and resulting waveform.

After you have set up the trigger, you can acquire the data. For part (a) of the lab procedures, we will generate a single natural response – once this response is generated, we want to display it in the oscilloscope window and freeze it there. <u>We do not want to continue to display data after the response has decayed to zero</u>. To generate a single screen of data and then stop acquiring additional data, click on the "Single" button on the scope instrument.

It will be worth your time, at this stage, to spend some time playing around with the trigger controls. Especially try changing the trigger point and the trigger level, as discussed below:

- Notice that the trigger point is denoted in the plot window by the black inverted triangle at the top of the plot window. The position of the trigger can be set by the *Pos* value in the time axis setting box, or by clicking on the trigger indicator with your left mouse button and dragging the trigger point to the desired position. Try it and observe its effect on the display.
- If the trigger source is one of the oscilloscope channels, the trigger level is shown on the plot window by the trigger level indicator – this is a triangular symbol of the same color as the trigger source channel on the right side of the plot window. For example, if the trigger source is channel 1, the indicator will be a blue triangle. The trigger level can be adjusted by clicking with your left mouse button on the trigger level indicator and dragging the indicator to the desired location or by changing the trigger level in the drop-down box.

[5] Since the natural response will decrease, it would be useless to try to trigger on a rising condition. This will only cause the scope to trigger when – and if – we turn the power back on.

Appendix B: Continuous Acquisition

In part (b) of these procedures, we are using the waveform generator to apply voltage to the circuit. The waveform generator continuously applies the selected waveform to your circuit, so the input to the circuit is a series of on/off steps which lasts as long as the waveform generator is running. In this case, we can use single-sequence acquisition as we did in part (a) and display the result of one arbitrary sample of turning power to the circuit off. However, it is more typical to use continuous acquisition of the waveform.

At small time scales on the oscilloscope, the oscilloscope is essentially taking a series of "frames" of data and successively displaying them in the main scope window[6]. If the waveform is repetitive, triggering allows us to assign a "zero time" to a particular feature on the signal. That feature gets placed on the same point on the plot window every time the oscilloscope screen updates; if the signal repeats itself based on this feature, the oscilloscope will display the same section of the signal every time the screen updates, making the signal appear to be unchanging.

To continuously acquire data for part (b) of this lab, set your trigger point to be as you desire. (The same settings as you used in part (a) should work, though you may feel like modifying them.) Apply power to your circuit using the waveform generator and then click on "Run". The data should be displayed in the window. The waveform displayed should appear to be unchanging, but it is actually being updated at rapid intervals – the successive waveforms simply lie directly on top of one another.

[6] At time scales larger than about 100ms/div, the data scrolls from left to right across the window – things are happening slowly enough so that it makes sense to watch the signal evolve. If we try this approach at small time scales, the data scrolls by too quickly to be useful.

Real Analog Chapter 7: Lab Worksheets

7.3.1: Passive RL Circuit Natural Response (35 points total)

1. In the space below, provide (from your pre-lab results) the initial inductor current and time constants for the circuits of Figure 2. (3 pts)

2. Provide below a schematic of the circuit you implemented, including actual resistance values used in your circuit. (2 pts)

3. Attach to this worksheet an image of the oscilloscope window, showing the inductor voltage, the resistor voltage, and the inductor current for the circuit of Figure 2(a), in which V+ is used as the voltage source. Briefly discuss differences between the measured data and your estimates from the pre-lab (as always, this should include a percent difference between the values). (5 pts)

4. In the space below, provide your estimate of the circuit's time constant from the measured response data. Briefly discuss differences between the measured data and your estimates from the pre-lab (as always, this should include a percent difference between the values). (5 pts)

5. **DEMO**: Have a teaching assistant initial this sheet, indicating that they have observed your circuits' operation. (5 pts)

   TA Initials: _______

6. Attach to this worksheet an image of the oscilloscope window, showing the inductor voltage, the resistor voltage, and the inductor current for the circuit of Figure 2(b), in which the waveform generator is used as the voltage source. Briefly discuss differences between the measured data and your estimates from the pre-lab (as always, this should include a percent difference between the values). (5 pts)

7. In the space below, provide your estimate of the circuit's time constant from the measured response data. Briefly discuss differences between the measured data and your estimates from the pre-lab (as always, this should include a percent difference between the values). (5 pts)

8. **DEMO**: Have a teaching assistant initial this sheet, indicating that they have observed your circuits' operation when using a square wave. (5 pts)

TA Initials: _______

Real Analog Chapter 7: Lab Projects

7.4.1: Inverting Differentiator

In this lab assignment, we will examine the forced response of a circuit which performs a differentiation – that is, the circuit output is the derivative with respect to time of the input to the circuit. We will apply sinusoids of various frequencies to the circuit and compare the output with our expectations based on analysis.

Before beginning this lab, you should be able to:

- Determine the input-output relationship for first order circuits
- Use the Analog Discovery waveform generator to apply a time-varying voltage input to an electrical circuit (Lab 6.2.1)
- Use the Analog Discovery oscilloscope to measure and display time-varying waveforms

After completing this lab, you should be able to:

- Measure the forced response of a circuit which performs a differentiation process

This lab exercise requires:

- Analog Discovery module
- Digilent Analog Parts Kit

Symbol Key:

DEMO Demonstrate circuit operation to teaching assistant; teaching assistant should initial lab notebook and grade sheet, indicating that circuit operation is acceptable.

ANALYSIS Analysis; include principle results of analysis in laboratory report.

SIM Numerical simulation (using PSPICE or MATLAB as indicated); include results of MATLAB numerical analysis and/or simulation in laboratory report.

DATA Record data in your lab notebook.

General Discussion:

The circuit we will be concerned with in this assignment is shown in Fig. 1. The output of the circuit, $V_{OUT}(t)$, is proportional to the inverse of the derivative of the input voltage, $V_{IN}(t)$. The constant of proportionality is determined by the values of the resistance and capacitance in the circuit. In this assignment, we will analytically estimate the relationship between the circuit's input and output and compare this with measured input and output signals.

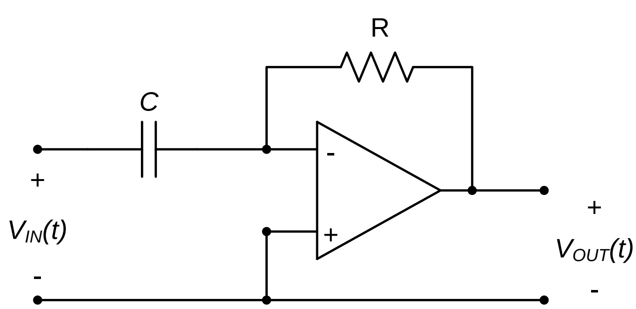


Figure 1. Differentiating circuit.

Pre-lab:

ANALYSIS

Determine the circuit output $V_{OUT}(t)$ as a funciton of the circuit input, $V_{IN}(t)$. (This is the circuit's input-output relationship.) Your relationship will be a function of the resistance, R, and the capacitance, C.

ANALYSIS

If the input function is a sinusoid of the form

$$V_{IN}(t) = Acos(\omega t)$$

determine the output function. (Your solution will be a function of R, C, A, and ω.)

Note:

The argument in the cosine function has units of radians. Thus, ω has units of radians/second. This is different from the units of Hertz (cycles/second) that are used by most waveform generators. The conversion between Hz and radians/second is:

$$\omega = 2\pi f$$

where the frequency f has units of Hz. The above conversion is essential, if you want to compare calculations with data.

Lab Procedures:

DATA

a. Construct the circuit shown in Fig. 1, using R=1.5kΩ and C = 100nF. Use V+ and V- as the positive and negative power supplies of the operational amplifier. Use the oscilloscope to measure both the input and output voltages $V_{IN}(t)$ and $V_{OUT}(t)$. Set the oscilloscope measurements to provide at least the amplitude of each of these waveforms. (As always, measure the actual resistance value; measure the capacitance value if you have the appropriated instrument – some DMMs have a capacitance meter – otherwise, assume that the nominal capacitance value is correct.)

   i. Apply a sinusoidal input voltage with frequency = 1kHz, amplitude = 1V, and offset = 0V to the circuit of Fig. 1. Use your oscilloscope to display the data listed above (waveforms corresponding to $V_{IN}(t)$ and $V_{OUT}(t)$; measurement window displaying amplitudes of $V_{IN}(t)$ and $V_{OUT}(t)$). Record the image of the oscilloscope window, showing the waveforms and their measured amplitudes.

DATA

   ii. Apply a sinusoidal input voltage with frequency = 2kHz, amplitude = 1V, and offset = 0V to the circuit of Fig. 1. Use your oscilloscope to display the data listed above (waveforms corresponding to $V_{IN}(t)$ and $V_{OUT}(t)$; measurement window displaying amplitudes of $V_{IN}(t)$ and $V_{OUT}(t)$). Record the image of the oscilloscope window, showing the waveforms and their measured amplitudes.

DATA

   iii. Apply a sinusoidal input voltage with frequency = 500Hz, amplitude = 1V, and offset = 0V to the circuit of Fig. 1. Use your oscilloscope to display the data listed above (waveforms corresponding to $V_{IN}(t)$ and $V_{OUT}(t)$; measurement window displaying amplitudes of $V_{IN}(t)$ and $V_{OUT}(t)$). Record the image of the oscilloscope window, showing the waveforms and their measured amplitudes.

DATA

DEMO

   iv. Demonstrate operation of your circuit to a teaching assistant and have them initial your lab notebook and the lab checklist.

Post-lab Exercises:

ANALYSIS

For the three cases in the lab procedures (1kHz sinusoid, 2kHz sinusoid, 500Hz sinusoid), calculate the expected output voltage based on your pre-lab analysis. Make a table comparing listing the measured and expected amplitudes of the output voltage, and the percent difference between the measured and expected amplitudes.

ANALYSIS

Comment briefly on the overall shapes of the expected vs. measured waveforms. Is the phase difference (the time delay between the input and output sinusoids) of the measured voltages consistent with what you would expect from your pre-lab analysis?

Real Analog Chapter 7: Lab Worksheets

7.4.1: Passive RL Circuit Natural Response (30 points total)

1. In the space below, provide (from your pre-lab results) the input-output relation for the circuit of Figure 1 and the expected output of the circuit resulting from the given sinusoidal forcing function. (3 pts)

2. Provide below a schematic of the circuit you implemented, including actual resistance (and, if possible, capacitance) values used in your circuit. (2 pts)

3. Attach to this worksheet an image of the oscilloscope window, showing waveforms corresponding to $V_{IN}(t)$ and $V_{OUT}(t)$ and a measurement window displaying amplitudes of $V_{IN}(t)$ and $V_{OUT}(t)$ for the 1kHz sinusoidal input. (5 pts)

4. Attach to this worksheet an image of the oscilloscope window, showing waveforms corresponding to $V_{IN}(t)$ and $V_{OUT}(t)$ and a measurement window displaying amplitudes of $V_{IN}(t)$ and $V_{OUT}(t)$ for the 2kHz sinusoidal input. (5 pts)

5. Attach to this worksheet an image of the oscilloscope window, showing waveforms corresponding to $V_{IN}(t)$ and $V_{OUT}(t)$ and a measurement window displaying amplitudes of $V_{IN}(t)$ and $V_{OUT}(t)$ for the 500Hz sinusoidal input. (5 pts)

6. **DEMO**: Have a teaching assistant initial this sheet, indicating that they have observed your circuits' operation. (5 pts)

   TA Initials: _______

7. In the space below, provide a table showing the expected output voltage amplitudes, the measured voltage output amplitudes, and a percent difference between the two for each of the above test cases (500Hz, 1kHz, and 2kHz frequencies). Briefly discuss differences between the measured and expected values. (5 pts)

Real Analog Chapter 7: Lab Projects

7.5.1: Passive RC Circuit Step Response

In this lab assignment, we will examine the step response of a simple RC circuit. We will use a square wave voltage source to emulate a step input to the system. An oscilloscope will be used to monitor both the applied input voltage and the response voltage from the circuit. Both the voltage across the capacitor and the voltage across the resistor will be measured and the waveforms across both inputs compared. We will also examine the effects of loading a passive RC circuit in this lab assignment.

Before beginning this lab, you should be able to:

- Determine the time constant of exponential functions
- State voltage-current relationships for inductors and capacitors
- Determine the natural response of passive first order (RL, RC) circuits
- Determine the step response of passive first order (RL, RC)

After completing this lab, you should be able to:

- Use a function generator to apply a square wave voltage input to an electrical circuit.
- Measure the time constant and steady-state response of first order passive electrical circuits
- State the potential effects of loading on a passive RC circuit

This lab exercise requires:

- Analog Discovery module
- Digilent Analog Parts Kit
- Digital multimeter (optional)

Symbol Key:

DEMO Demonstrate circuit operation to teaching assistant; teaching assistant should initial lab notebook and grade sheet, indicating that circuit operation is acceptable.

ANALYSIS Analysis; include principle results of analysis in laboratory report.

SIM Numerical simulation (using PSPICE or MATLAB as indicated); include results of MATLAB numerical analysis and/or simulation in laboratory report.

DATA Record data in your lab notebook.

General Discussion:

This lab assignment will be concerned with the simple series RC circuit shown in Fig. 1. We will be interested primarily in the measured vs. expected behavior of the capacitor voltage, but a secondary goal of this assignment is to compare the voltage differences across the capacitor and resistor. Thus, we will measure both the voltage differences $v_R(t)$ and $v_C(t)$ as shown in Fig. 1[7].

[7] The oscilloscope instrument on the Analog Discovery will allow you to take "double-sided" or "differential" voltage measurements – this allows you to directly measure the voltage difference across any component in the same way you measure a voltage difference using a DMM. Thus, we can measure the voltage differences in Figure 1 exactly as they are indicated on that figure. Many oscilloscopes, however, make single-sided measurements, in which voltage differences are all measured with respect to a "common" or ground node. In order to measure

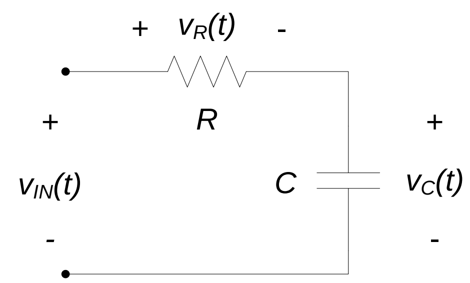


Figure 1. RC circuit being tested.

Pre-lab:

ANALYSIS

Estimate the time constant for the circuit shown in Fig. 1 if R=470 Ω, and C = 1μF. Also determine the steady state response of $v_C(t)$ if v_{IN} is a step input with amplitude 5V. (e.g. $v_{IN} = 5u_0(t)$ V.) Note: you do <u>not</u> need to write or solve a differential equation for either of the circuits in order to do this.

Lab Procedures:

DATA

a. Construct the circuit shown in Fig. 1, using R=470 Ω and C = 1μF. (As always, measure the actual resistance value; measure the capacitance value if you have the appropriated instrument – some DMMs have a capacitance meter – otherwise, assume that the nominal capacitance value is correct.)

DATA

i. Apply a 4V peak-to peak square wave input with period = 10 ms (frequency = 100Hz) as shown in Fig. 2 to the circuit[8]. Display both $v_{IN}(t)$ and $v_{OUT}(t)$ on your oscilloscope window. Record the image of the oscilloscope window, showing the waveforms <u>and save the signals as a data file for later plotting</u>[9].

ANALYSIS

ii. Calculate the time constant and steady state response of the output voltage. Compare your results with your expectations based on the pre-lab analysis and comment on any differences.

DEMO

iii. Demonstrate operation of your circuit to a teaching assistant and have them initial your lab notebook and the lab checklist.

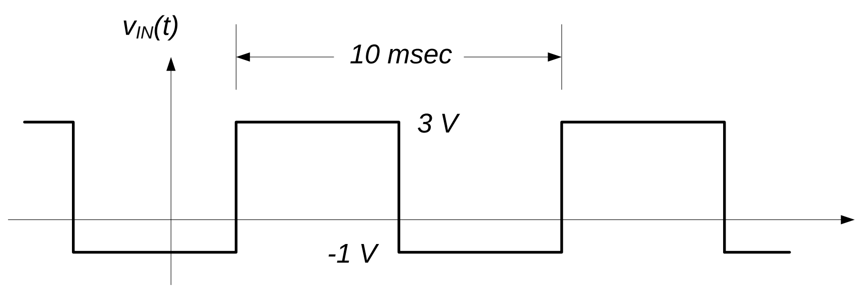


Figure 2. Input voltage signal.

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the voltage across the resistor in Figure 1 using a single-sided oscilloscope, we would typically measure $v_{in}(t)$ and $v_C(t)$ and then using a math channel on the oscilloscope to take the difference between the two to get the voltage across the resistor

[8] In the terminology used in the Analog Discovery waveform generators, we would say that the waveform of Figure 1 has an amplitude (zero-to-peak) of 2V and an average value (or offset) 1V.

[9] The "Export" button on the oscilloscope toolbar allows you to save measured data as a .csv file.

DATA

ANALYSIS

DEMO

b. RC circuits are often used for signal conditioning. The conditioned signal will, in general, be applied to a load in order to be useful.

i. Apply a load to the RC circuit of Fig. 1 by constructing the circuit shown in Fig. 3. Use R = R_L = 470 Ω, and C = 1μF. Measure the output voltage across the load resistor, for the input voltage shown in Fig. 2. Record the image of the oscilloscope window, showing the waveforms and their measured amplitudes, <u>and save the signals as a data file for later plotting</u>.

ii. Determine a time constant and steady state response from your measured data. How do these parameters compare with those of the unloaded circuit of part (a)? Do the loaded circuit parameters agree with your expectations? (Hint: calculate an equivalent resistance seen by the capacitor.)

iii. Demonstrate operation of your circuit to a teaching assistant and have them initial your lab notebook and the lab checklist.

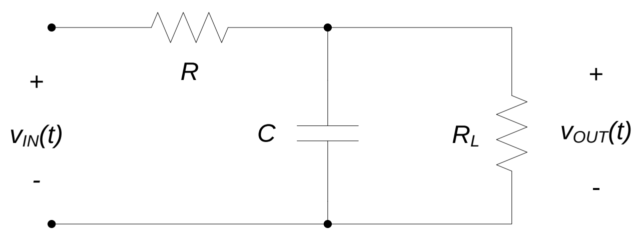

Figure 3. Loaded passive RC circuit.

Post-lab Exercises:

ANALYSIS

a. Import the oscilloscope data you acquired in part (a) of the lab procedures into Excel, Matlab, or any similar software package which provides basic mathematics and plotting capabilities. Plot the resistor and capacitor voltages. Use your software to sum the capacitor and resistor voltages. Plot the result and compare it to the input waveform you applied to the circuit. Comment on differences or similarities.

ANALYSIS

b. Import the voltage $v_{OUT}(t)$ you acquired in part (b) of the lab procedures. Create a plot displaying this data overlayed with the capacitor voltage of the unloaded circuit acquired in part(a) of the lab procedures. Comment on the differences between the two, including qualitative comparisons of the time constants and steady-state responses.

Real Analog Chapter 7: Lab Worksheets

7.5.1: Passive RC Circuit Step Response (40 points total)

1. In the space below, provide (from your pre-lab results) the time constant for the circuit shown in Fig. 1 if R=470 Ω, C = 1μF. Also provide your estimate steady state response of $v_C(t)$ if v_{IN} is a step input with amplitude 5V. (3 pts)

2. Provide below a schematic of the circuit you implemented, including actual resistance and capacitance values used in your circuit. (2 pts)

3. Attach to this worksheet an image of the oscilloscope window, showing the input and output voltages. In the space below, provide your estimate of the time constant of the circuit. Briefly discuss differences between the measured data and your estimates from the pre-lab (as always, this should include a percent difference between the values). (5 pts)

4. **DEMO**: Have a teaching assistant initial this sheet, indicating that they have observed your circuits' operation. (5 pts)

   TA Initials: _______

5. In the space below, provide a circuit schematic for the "loaded" circuit with measured resistor values (both the RC circuit resistance and the load resistance). (3 pts)

6. Attach, to this worksheet, a plot of your measured input and output data for the loaded circuit. Annotate the plot to show the time constant and steady-state response. (3 pts)

7. In the space below, provide the time constant and steady-state response of the loaded circuit. Comment below on the differences between the loaded and unloaded circuit responses. Do the results of the loaded circuit agree with your expectations based on analysis? (5 pts)

8. **DEMO**: Have a teaching assistant initial this sheet, indicating that they have observed your circuits' operation. (4 pts)

   TA Initials: _______

9. Attach to this worksheet, a plot overlaying the capacitor voltage, the resistor voltage, and the sum of the two from part (a) of the post-lab exercises. In the space below, comment on the relationship between the sum of the capacitor and resistor voltages and the waveform applied to the circuit. (5 pts)

10. Attach to this worksheet, a plot overlaying the capacitor voltage for both the loaded and unloaded cases from part (b) of the post-lab exercises. In the space below, comment on differences between the two (you may refer to your quantitative results in items 3 and 7 above. (5pts)

Real Analog Chapter 7: Lab Projects

7.5.2: Passive RL Circuit Step Response

In this lab assignment, we will examine the step response of a simple RL circuit. We will use a square wave voltage source to emulate a step input to the system. An oscilloscope will be used to monitor both the applied input voltage and the response voltage from the circuit. Both the voltage across the inductor and the voltage across the resistor will be measured and the waveforms across both inputs compared.

Before beginning this lab, you should be able to:

- Determine the time constant of exponential functions
- State voltage-current relationships for inductors and capacitors
- Determine the natural response of passive first order (RL, RC) circuits
- Determine the step response of passive first order (RL, RC)

After completing this lab, you should be able to:

- Use a function generator to apply a square wave voltage input to an electrical circuit.
- Measure the time constant and steady-state response of first order passive electrical circuits
- State the potential effects of loading on a passive RL circuit

This lab exercise requires:

- Analog Discovery module
- Digilent Analog Parts Kit
- Digital multimeter (optional)

Symbol Key:

DEMO Demonstrate circuit operation to teaching assistant; teaching assistant should initial lab notebook and grade sheet, indicating that circuit operation is acceptable.

ANALYSIS Analysis; include principle results of analysis in laboratory report.

SIM Numerical simulation (using PSPICE or MATLAB as indicated); include results of MATLAB numerical analysis and/or simulation in laboratory report.

DATA Record data in your lab notebook.

General Discussion:

This lab assignment will be concerned with the simple series RL circuit shown in Fig. 1. We will be interested primarily in the measured vs. expected behavior of the resistor voltage, but a secondary goal of this assignment is to compare the voltage differences across the inductor and resistor. Thus, we will measure both the voltage differences $v_R(t)$ and $v_L(t)$ as shown in Fig. 1[10].

[10] The oscilloscope instrument on the Analog Discovery will allow you to take “double-sided” or “differential” voltage measurements – this allows you to directly measure the voltage difference across any component in the same way you measure a voltage difference using a DMM. Thus, we can measure the voltage differences in Figure 1 exactly as they are indicated on that figure. Many oscilloscopes, however, make single-sided measurements, in which voltage differences are all measured with respect to a “common” or ground node. In order to measure the voltage across the inductor in Fig. 1 using a single-sided oscilloscope, we would typically measure $v_{in}(t)$ and $v_R(t)$ and then using a math channel on the oscilloscope to take the difference between the two to get the voltage across the resistor

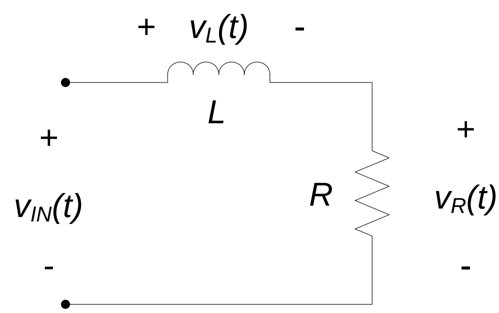

Figure 1. RL circuit being tested.

Pre-lab:

ANALYSIS

Estimate the time constant for the circuit shown in Figs. 1 if R=200 Ω and L = 1mH. Also determine the steady state response of v_C(t) if v_{IN} is a step input with amplitude 2V. (e.g. $v_{IN} = 2u_0(t)$ V.) Note: you do not need to write or solve a differential equation for either of the circuits in order to do this.

Lab Procedures:

DATA

a. Construct the circuit shown in Fig. 1, using R=200 Ω and L = 1mH. (As always, measure the actual resistance value; assume that the nominal inductance value of the inductor in your parts kit is correct.)

   i. Use a square wave input to the circuit to emulate a step function. The input step should have an amplitude which goes from zero to 2V (as shown in Fig. 2) and have a period which is long enough to allow the circuit to reach steady-state[11]. Display both $v_{IN}(t)$ and $v_{OUT}(t)$ on your oscilloscope window. Record the image of the oscilloscope window, showing the waveforms and save the signals as a data file for later plotting[12].

DATA

ANALYSIS

   ii. Calculate the time constant and steady state response of the resistor voltage. Compare your results with your expectations based on the pre-lab analysis and comment on any differences.

DEMO

   iii. Demonstrate operation of your circuit to a teaching assistant and have them initial your lab notebook and the lab checklist.

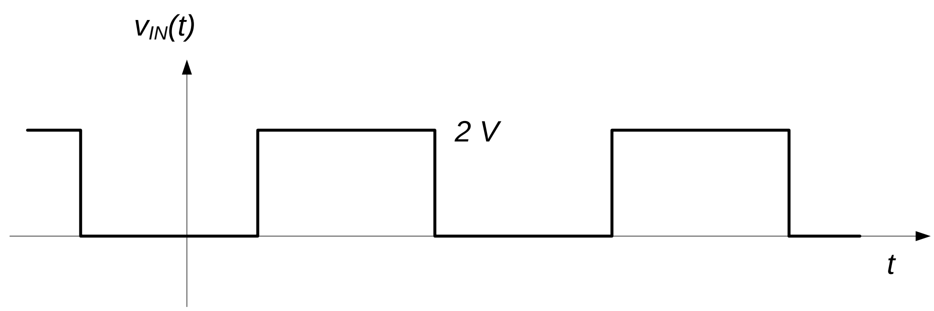

Figure 2. Input voltage signal.

b. First order circuits are often used for signal conditioning. The conditioned signal will, in general, be applied to a load in order to be useful.

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[11] It is generally assumed that a first order circuit has reached steady state after a time corresponding to approximately five time constants.

[12] The "Export" button on the oscilloscope toolbar allow you to save measured data as a .csv file.

DATA

i. Apply a load to the RL circuit of Fig. 1 by constructing the circuit shown in Fig. 3. Use R = R_L = 200 Ω, and L = 1mH. Measure the output voltage across the load resistor, for the input voltage shown in Fig. 2. Sketch the input and output voltage signals in your lab notebook, <u>and save the signals as a data file for later plotting</u>.

ANALYSIS

ii. Determine a time constant and steady state response from your measured data. How do these parameters compare with those of the unloaded circuit of part (a)? Do the loaded circuit parameters agree with your expectations? (Hint: calculate an equivalent resistance seen by the capacitor.)

DEMO

iii. Demonstrate operation of your circuit to a teaching assistant and have them initial your lab notebook and the lab checklist.

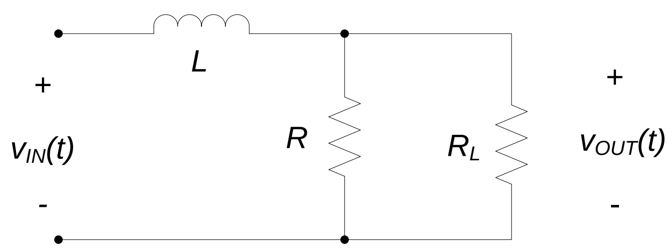


Figure 3. Loaded passive RL circuit.

Post-lab Exercises:

ANALYSIS

c. Import the oscilloscope data you acquired in part (a) of the lab procedures into Excel, Matlab, or any similar software package which provides basic mathematics and plotting capabilities. Plot the resistor and inductor voltages. Use your software to sum the inductor and resistor voltages. Plot the result and compare it to the input waveform you applied to the circuit. Comment on differences or similarities.

ANALYSIS

d. Import the voltage $v_{OUT}(t)$ you acquired in part (b) of the lab procedures. Create a plot displaying this data overlayed with the capacitor voltage of the unloaded circuit acquired in part(a) of the lab procedures. Comment on the differences between the two, including qualitative comparisons of the time constants and steady-state responses.

Real Analog Chapter 7: Lab Worksheets

7.5.2: Passive RL Circuit Step Response (40 points total)

1. In the space below, provide (from your pre-lab results) the time constant for the circuit shown in Fig. 1 if R=200 Ω and L = 1mH. Also provide your estimate steady state response of $v_R(t)$ if v_{IN} is a step input with amplitude 5V. (3 pts)

2. Provide below a schematic of the circuit you implemented, including actual resistance value used in your circuit. (2 pts)

3. Attach to this worksheet an image of the oscilloscope window, showing the input voltage, the inductor voltage, and the resistor voltage. In the space below, provide your estimate of the time constant of the circuit. Briefly discuss differences between the measured data and your estimates from the pre-lab (as always, this should include a percent difference between the values). (5 pts)

4. **DEMO**: Have a teaching assistant initial this sheet, indicating that they have observed your circuits' operation. (5 pts)

   TA Initials: _______

5. In the space below, provide a circuit schematic for the "loaded" circuit with measured resistor values (both the RL circuit resistance and the load resistance). (3 pts)

6. Attach, to this worksheet, a plot of your measured input and output data (the output data for this circuit is the voltage across the load resistor) for the loaded circuit. Annotate the plot to show the time constant and steady-state response. (3 pts)

7. In the space below, provide the time constant and steady-state response of the loaded circuit. Comment below on the differences between the loaded and unloaded circuit responses. Do the results of the loaded circuit agree with your expectations based on analysis? (5 pts)

8. **DEMO**: Have a teaching assistant initial this sheet, indicating that they have observed your circuits' operation. (4 pts)

   TA Initials: _______

9. Attach to this worksheet, a plot overlaying the inductor voltage, the resistor voltage, and the sum of the two from part (a) of the post-lab exercises. In the space below, comment on the relationship between the sum of the inductor and resistor voltages and the waveform applied to the circuit. (5 pts)

10. Attach to this worksheet, the plot overlaying the resistor voltage for both the loaded and unloaded cases from part (b) of the post-lab exercises. In the space below, comment on differences between the two (you may refer to your quantitative results in items 3 and 7 above. (5 pts)

Real Analog Chapter 7: Lab Projects

7.5.3: Active RC Circuit Step Response

In lab assignments 7.5.1 and 7.5.2, we examined the response of passive first order circuits. These circuits can be very useful, for example, in signal conditioning. However, passive first order circuits have similar drawbacks to passive resistive circuits. One major problem is that the addition of a load to the circuit can significantly modify the circuit's behavior, which may necessitate a re-design of the circuit anytime a different load is applied to the circuit. Another drawback is the inability to amplify any input signal – the energy out of a passive circuit cannot exceed the energy provided to the circuit.

Active circuits can resolve these issues. Active circuits, since the power they supply comes from external sources, are somewhat immune to loading effects. The external sources of an active circuit also allow these circuits to amplify input signals – the output from these circuits can contain considerably more energy than is being provided by the input signal. In this lab assignment, we construct an active RC circuit and note that loading of the circuit – unlike loading of the circuit of lab 7.5.1 – does not significantly affect the circuit's behavior.

Before beginning this lab, you should be able to:

- Analyze, design, and build operational amplifier-based circuits
- State voltage-current relationships for inductors and capacitors
- Determine the natural and step responses of active first order circuits
- State the potential effects of loading on passive first order circuits (Labs 7.5.1, 7.5.2)

After completing this lab, you should be able to:

- Use a function generator to apply a square wave voltage input to an electrical circuit.
- Measure the time constant and steady-state response of first order active electrical circuits

This lab exercise requires:

- Analog Discovery module
- Digilent Analog Parts Kit
- Digital multimeter (optional)

Symbol Key:

DEMO — Demonstrate circuit operation to teaching assistant; teaching assistant should initial lab notebook and grade sheet, indicating that circuit operation is acceptable.

ANALYSIS — Analysis; include principle results of analysis in laboratory report.

SIM — Numerical simulation (using PSPICE or MATLAB as indicated); include results of MATLAB numerical analysis and/or simulation in laboratory report.

DATA — Record data in your lab notebook.

General Discussion:

In lab assignment 7.5.1, we (hopefully) noted that loading passive circuits can have a significant effect on their response. Active circuits tend to be less susceptible to loading effects. In this part of the assignment, we construct an active RC circuit with the same time constant as the circuit of lab 7.5.1. We then apply the load of lab 7.5.1 to this active circuit and observe the response.

The circuit shown in Fig. 1 has an input-output relationship:

$$\frac{dV_{OUT}(t)}{dt} + \frac{1}{RC}V_{OUT}(t) = -\frac{1}{RC}V_{IN}(t)$$

which, with the exception of a sign change, is the same as that of the circuit of Fig. 1(a) of Lab 7.5.1. Thus, the response of the circuit of Fig. 1 will have the same time constant and steady state gain as the circuit of lab 7.5.1, but will be of opposite sign.

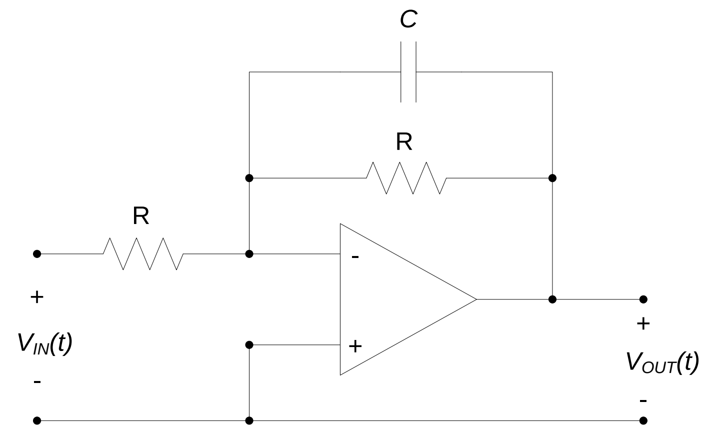


Figure 1. (Inverting) active RC circuit.

Pre-lab:

None

Lab Procedures:

DATA

a. Construct the circuit shown in Fig. 1, using R=470 Ω, and C = 1μF (As always, measure the actual resistance and capacitance values (if possible) and record them in your lab notebook.)

DATA

i. Apply a 4V peak-to-peak square wave input with period = 10 ms (frequency = 100Hz) as shown in Fig. 2 to the circuit. (Make sure the supply voltage ranges applied to the operational amplifier are adequate to provide the full range of output voltage.) Measure both $V_{IN}(t)$ and $V_{OUT}(t)$; record an image of the oscilloscope window, showing the waveforms.

ANALYSIS

ii. Calculate the time constant and steady state output voltage. Compare your results with your results from lab 7.5.1 and comment on any differences.

DEMO

iii. Demonstrate operation of your circuit to a teaching assistant and have them initial your lab notebook and the lab checklist.

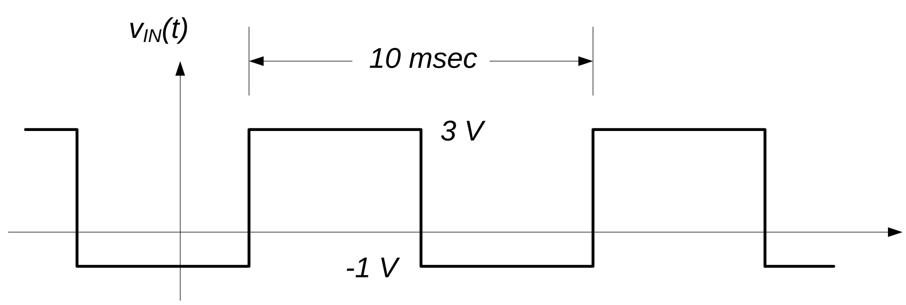


Figure 2. Input voltage signal.

b. Gradually increase the frequency of the input square wave and note the output voltage response.

DATA

i. Tabulate the peak-to-peak input and output voltages for (at least) frequencies of 300Hz, 500Hz, 1000Hz, and 2000Hz. Comment on the trends between the peak-to-peak input and output voltages and the input frequency. Comment on the reasons for this behavior. (Hint: as the input function changes faster than the circuit time constant, the capacitor in the circuit does not have time to fully charge or discharge before the input voltage changes.)

DEMO

ii. Demonstrate operation of your circuit to a teaching assistant and have them initial your lab notebook and the lab checklist.

c. Apply a load to the RC circuit of Fig. 1 by constructing the circuit shown in Fig. 2. Use $R = R_L = 470\ \Omega$.

DATA

i. Measure the V_{IN} and V_{OUT}, for the input voltage shown in Fig. 2. Record an image of the oscilloscope window, showing the waveforms.

ANALYSIS

ii. Determine a time constant and steady state response from your measured data. How do these parameters compare with those of the unloaded circuit of part (a) of Lab 7.5.1? How do they agree with the loaded passive circuit of part (c) of Lab 7.5.1?

DEMO

iii. Demonstrate operation of your circuit to a teaching assistant and have them initial your lab notebook and the lab checklist.

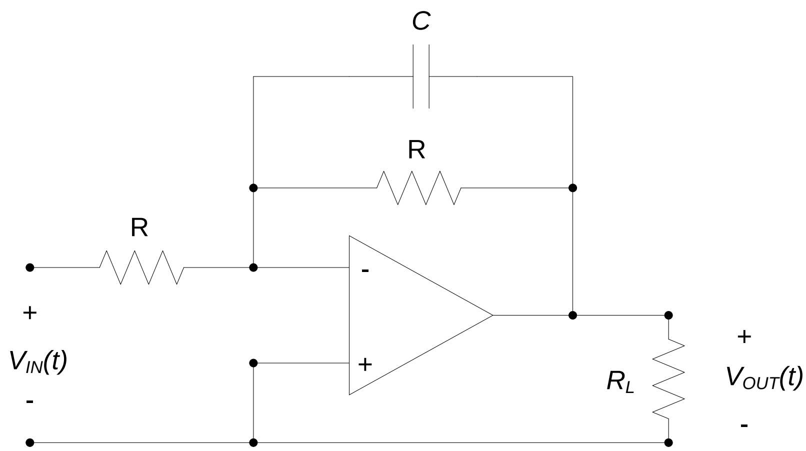


Figure 3. Loaded active RC circuit.

Real Analog Chapter 7: Lab Worksheets

7.5.3: Active RC Circuit Step Response (50 points total)

1. Capacitor voltage measurement (15 pts)
   a. Provide below a schematic of the circuit and measured resistance and (if possible) capacitance values. (3 pts)

   b. Attach, to this worksheet, the image of your oscilloscope time window, showing the measured input and output data. (4 pts)

   c. Provide below your estimates of the time constant and steady state response estimates (4 pts)

   d. In the space below, provide a discussion comparing the response of the active RC circuit with the passive RC circuit of Lab 7.5.1. (4 pts).

2. Circuit response vs. input voltage frequency (15 pts)
   a. Provide below a table of input and output voltage amplitudes and frequencies (5 pts)

b. Discuss your observations relative to peak-to-peak input and output voltage amplitudes vs. frequency and comment on possible reasons for these trends. (5 pts)

c. **DEMO**: Have a teaching assistant initial this sheet, indicating that they have observed your circuits' operation. (5 pts)

TA Initials: _______

3. Loaded circuit response (20 pts)
   a. Provide below a schematic of the loaded active RC circuit with measured resistance and (if possible) capacitance values. (3 pts)

   b. Attach, to this worksheet, the image of your oscilloscope time window, showing the measured input and output data. (4 pts)

   c. Provide your estimates of the active RC circuit time constant and steady-state response. (3 pts)

   d. Compare the response of the loaded active RC circuit with the unloaded active RC circuit of part (a). Also compare the response of the loaded active RC circuit with the unloaded passive circuit of Lab 7.5.1. Discuss your observations. (5 pts)

4. **DEMO**: Have a teaching assistant initial this sheet, indicating that they have observed your circuits' operation. (5 pts)

TA Initials: _______

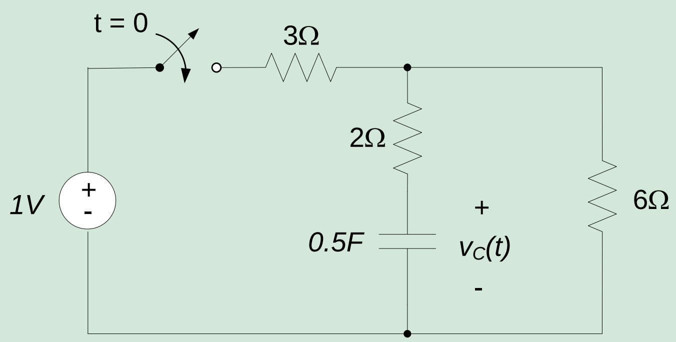
Real Analog Chapter 7: Homework

7.1 For the circuit below, determine

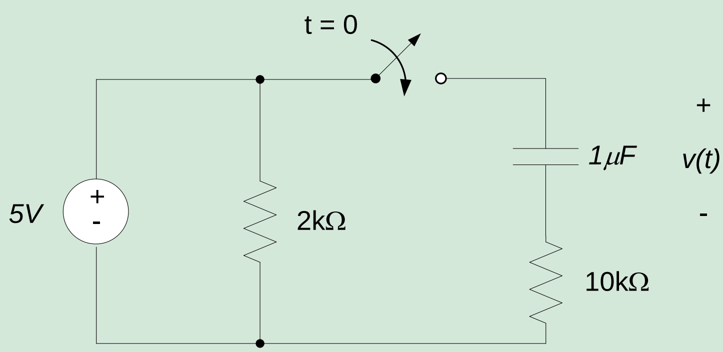
a. $v_C(t)$, $t>0$

b. $v_C(t)$, $t>0$ if the capacitance is $1F$

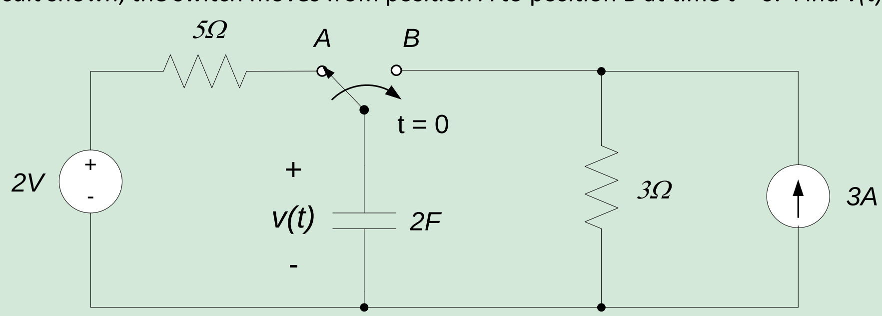
c. $v_C(t)$, $t>0$ if the capacitance is $0.25F$



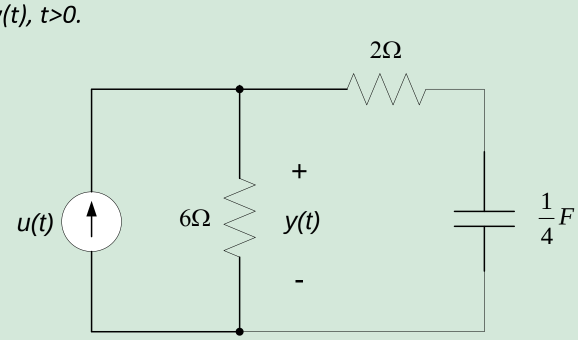
7.2 Find $v(t)$, $t>0$, in the circuit below.



7.3 For the circuit shown, the switch moves from position A to position B at time t = 0. Find $v(t)$, $t>0$.



7.4 For the circuit shown, the input is the current source $u(t)=2u_0(t)A$ and the output is the voltage across the 6Ω resistor, $y(t)$. Find $y(t)$, $t>0$.

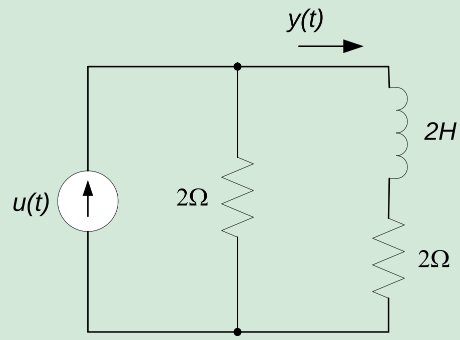


7.5 For the circuit shown, the input is the current source *u(t)* and the output is the current through the inductor, *y(t)*.

a. Determine the differential equation relating *u(t)* and *y(t)*.

b. If $u(t) = 3u_0(t)$, determine *y(t), t>0*.

c. Does your answer to part (b) agree with your expectations as to the circuit's physical behavior as $t\rightarrow\infty$? Why or why not?

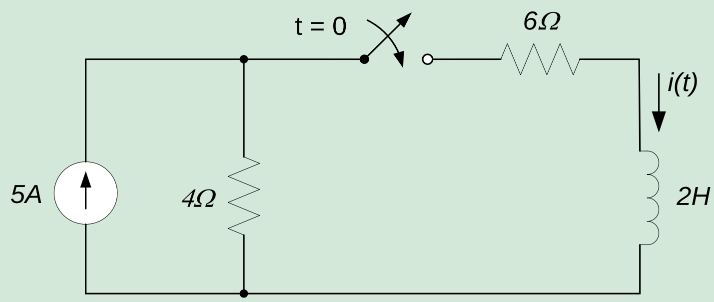


7.6 For the circuit shown, the switch closes at time *t = 0*.

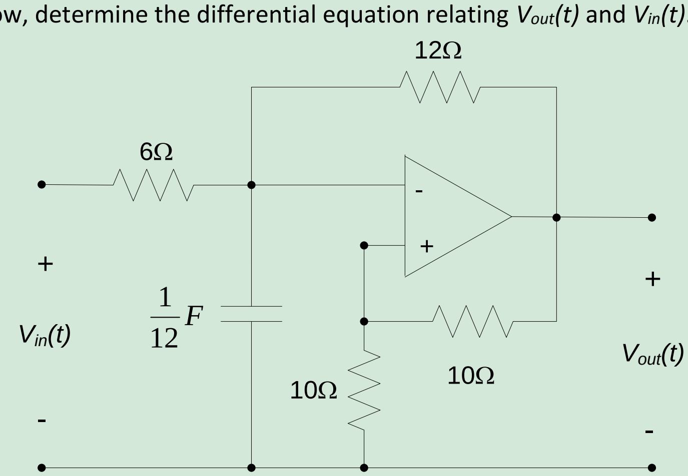
a. Write the differential equation governing *i(t), t>0*.

b. Determine initial (*t = 0*) and final ($t\rightarrow\infty$) conditions on the current *i(t)*. You may assume that no energy is stored in the inductor before *t=0*.

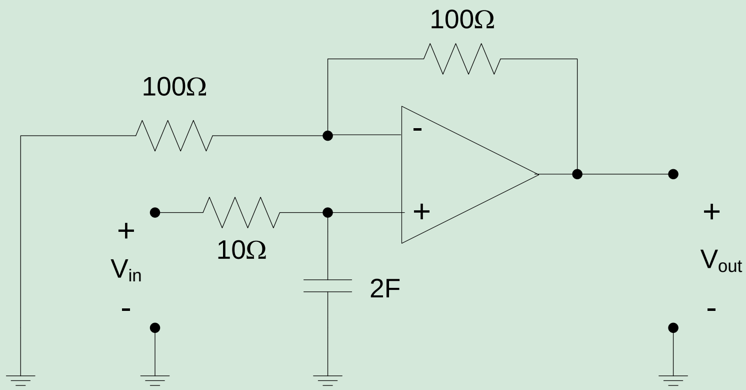
c. Find *i(t), t>0*.



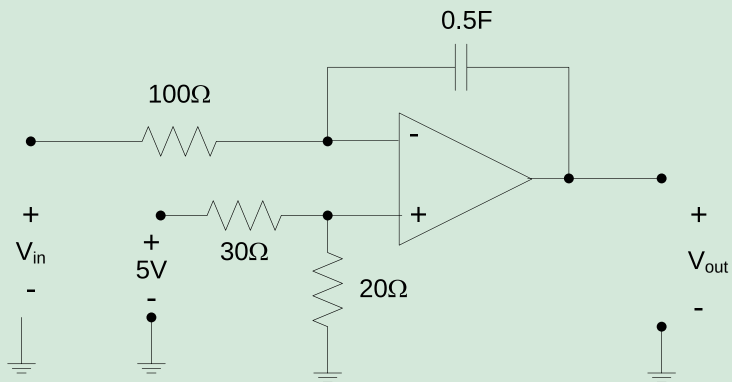
7.7 For the circuit below, determine the differential equation relating $V_{out}(t)$ and $V_{in}(t)$.



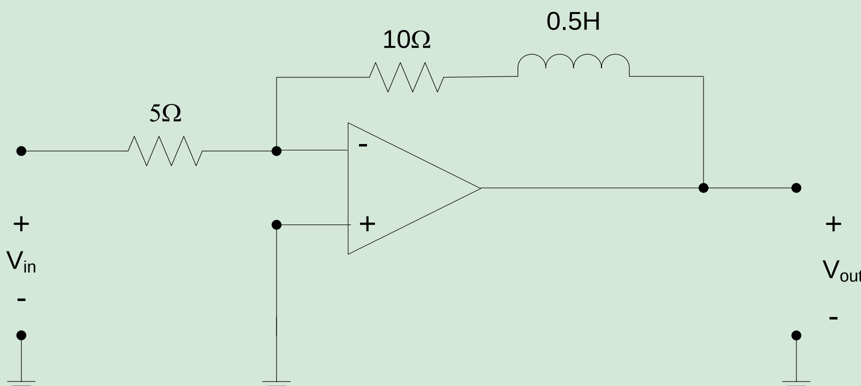
7.8 Determine the differential equations relating Vout and Vin for the circuit below.



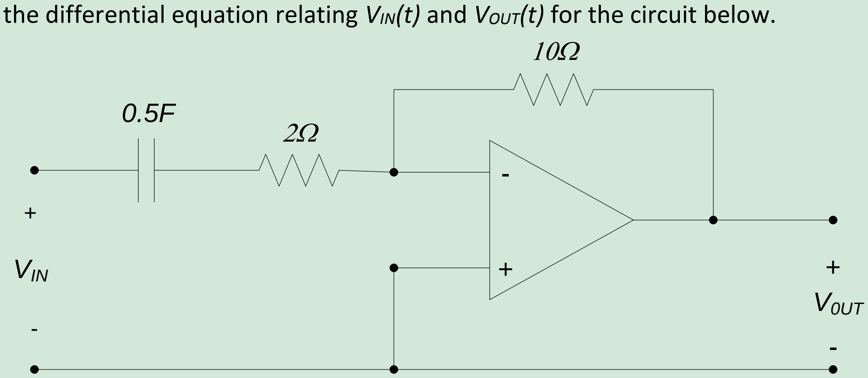
7.9 Determine the differential equations relating Vout and Vin for the circuit below.



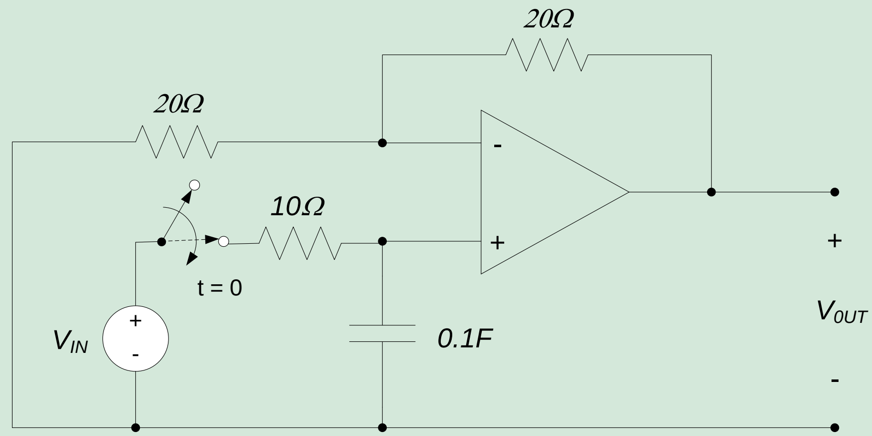
7.10 Determine the differential equations relating V_{out} and V_{in} for the circuit below.



7.11 Determine the differential equation relating $V_{IN}(t)$ and $V_{OUT}(t)$ for the circuit below.



7.12 Find $V_{OUT}(t)$, $t>0$, in the circuit below.



7.13 For the circuit below, determine

a. the differential equation governing $V_{OUT}(t)$, $t>0$

b. $V_{OUT}(t)$, $t>0$.

c. Does your solution for part (b) agree with your expectations, based on the circuit's behavior as $t \rightarrow \infty$?