

## 6 Introduction and Chapter Objectives

So far, we have considered circuits that have been governed by algebraic relations. These circuits have, in general, contained only power sources and resistive elements. All elements in these circuits, therefore, have either supplied power from external sources or dissipated power. For these resistive circuits, we can apply either time-varying or constant signals to the circuit without really affecting our analysis approach. Ohm's law, for example, is equally applicable to time-varying or constant voltages and currents:

$$V = I \cdot R \Leftrightarrow v(t) = i(t) \cdot R$$

Since the governing equation is algebraic, it is applicable at every point in time – voltages and currents at a point in time are affected only by voltages and currents at the same point in time.

We will now begin to consider circuit elements, which are governed by differential equations. These circuit elements are called *dynamic circuit elements* or *energy storage elements*. Physically, these circuit elements store energy, which they can later release back to the circuit. The response, at a given time, of circuits that contain these elements is not only related to other circuit parameters at the same time; it may also depend upon the parameters at other times.

This chapter begins with an overview of the basic concepts associated with energy storage. This discussion focuses not on electrical systems, but instead introduces the topic qualitatively in the context of systems with which the reader is already familiar. The goal is to provide a basis for the mathematics, which will be introduced subsequently. Since we will now be concerned with time-varying signals, section 6.2 introduces the basic signals that we will be dealing with in the immediate future. This chapter concludes with presentations of the two electrical energy storage elements that we will be concerned with: capacitors and inductors. The method by which energy is stored in these elements is presented in sections 6.3 and 6.4, along with the governing equations relating voltage and current for these elements.

**After completing this chapter, you should be able to:**

- Qualitatively state the effect of energy storage on the type of mathematics governing a system
- Define transient response
- Define steady-state response
- Write the mathematical expression for a unit step function
- Sketch the unit step function
- Sketch shifted and scaled versions of the unit step function
- Write the mathematical expression for a decaying exponential function
- Define the time constant of an exponential function
- Sketch a decaying exponential function, given the function's initial value and time constant
- Use a unit step function to restrict an exponential function to times greater than zero

- Write the circuit symbol for a capacitor
- State the mechanism by which a capacitor stores energy
- State the voltage-current relationship for a capacitor in both differential and integral form
- State the response of a capacitor to constant voltages and instantaneous voltage changes
- Write the mathematical expression describing energy storage in a capacitor
- Determine the equivalent capacitance of series and parallel combinations of capacitors
- Sketch a circuit describing a non-ideal capacitor
- Write the circuit symbol for an inductor
- State the mechanism by which an inductor stores energy
- State the voltage-current relationship for an inductor in both differential and integral form
- State the response of an inductor to constant voltages and instantaneous current changes
- Write the mathematical expression describing energy storage in an inductor
- Determine the equivalent inductance of series and parallel combinations of inductors
- Sketch a circuit describing a non-ideal inductor

## 6.2 Fundamental Concepts

This section provides a brief overview of what it meant by energy storage in terms of a system-level description of some physical process. Several examples of energy storage elements are presented, for which the reader should have an intuitive understanding. These examples are intended to introduce the basic concepts in a qualitative manner; the mathematical analysis of dynamic systems will be provided in later chapters.

We have previously introduced the concept of representing a physical process as a *system*. In this viewpoint, the physical process has an input and an output. The input to the system is generated from sources external to the system – we will consider the input to the system to be a known function of time. The output of the system is the system's response to the input. The *input-output equation* governing the system provides the relationship between the system's input and output. A general input-output equation has the form:

$$y(t) = f\{u(t)\} \quad \text{Eq. 6.1}$$

The process is shown in block diagram form in Fig. 6.1.

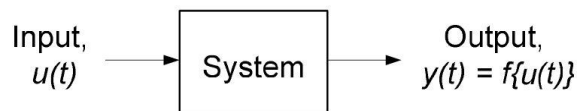


Figure 6.1. Block diagram representation of a system.

The system of Fig. 6.1 transfers the energy in the system input to the system output. This process transforms the input signal  $u(t)$  into the output signal  $y(t)$ . In order to perform this energy transfer, the system will, in general, contain elements that both store and dissipate energy. To date, we have analyzed systems which contain only energy dissipation elements. We review these systems briefly below in a systems context. Subsequently, we introduce systems that store energy; our discussion of energy storage elements is mainly qualitative in this chapter and presents systems for which the reader should have an intuitive understanding.

### 6.2.1 Systems with no Energy Storage

In previous chapters, we considered cases in which the input-output equation is algebraic. This implies that the processes being performed by the system involve only sources and components which dissipate energy. For example, output voltage of the inverting voltage amplifier of Fig. 6.2 is:

$$V_{OUT} = -\left(\frac{R_f}{R_{in}} V_{in}\right) \quad \text{Eq. 6.2}$$

This circuit contains only resistors (in the form of  $R_f$  and  $R_{in}$ ) and sources (in the form of  $V_{in}$  and the op-amp power supplies) and the equation relating the input and output is algebraic. Note that the op-amp power supplies do not appear in equation (6.2), since linear operation of the circuit of Fig. 6.2 implies that the output voltage is independent of the op-amp power supplies.

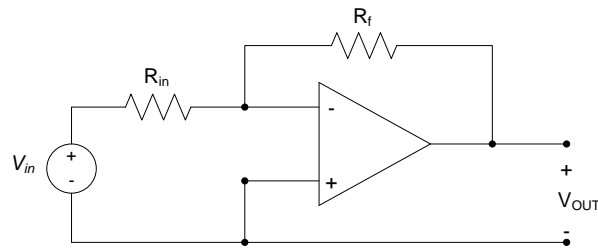


Figure 6.2. Inverting voltage amplifier.

One side effect of an algebraic input-output equation is that the output responds instantaneously to any changes in the input. For example, consider the circuit shown in Fig. 6.3. The input voltage is based on the position of a switch; when the switch closes, the input voltage applied to the circuit increases instantaneously from 0V to 2V. Fig. 6.3 indicates that the switch closes at time  $t = 5$  seconds; thus, the input voltage as a function of time is as shown in Fig. 6.4(a). For the values of  $R_f$  and  $R_{in}$  shown in Fig. 6.3, the input-output equation becomes:

$$V_{OUT}(t) = -5V_{in}(t) \quad \text{Eq. 6.3}$$

And the output voltage as a function of time is as shown in Fig. 6.4(b). The output voltage responds immediately to the change in the input voltage.

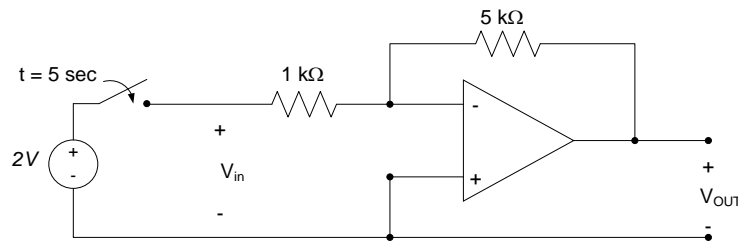


Figure 6.3. Switched voltage amplifier.

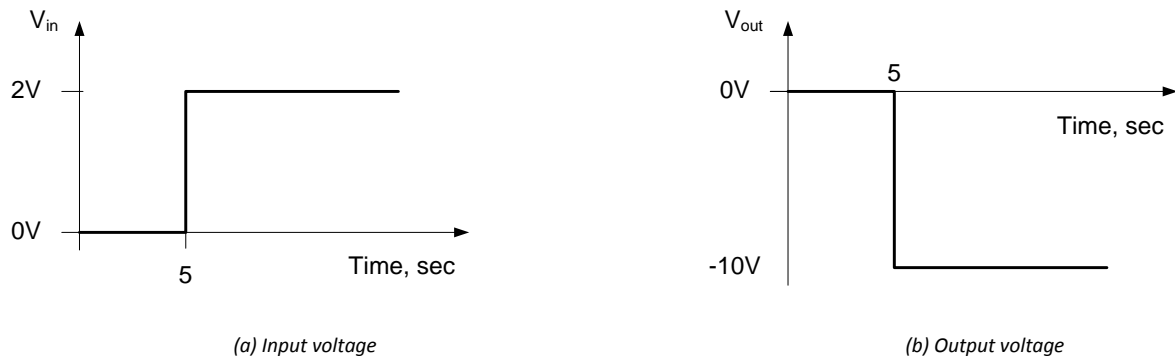


Figure 6.4. Input and output signals for circuit of Figure 3.

## 6.2.2 Systems with Energy Storage

We now consider systems, which contain energy storage elements. The inclusion of energy storage elements results in the input-output equation for the system, which is a differential equation. We present the concepts in terms of two examples for which the reader most likely has some expectations based on experience and intuition.

### Example 6.1: Mass-damper system

As an example of a system, which includes energy storage elements, consider the mass-damper system shown in Fig. 6.5. The applied force  $F(t)$  pushes the mass to the right. The mass's velocity is  $v(t)$ . The mass slides on a surface with sliding coefficient of friction  $b$ , which induces a force, which opposes the mass's motion. We will consider the applied force to be the input to our system and the mass's velocity to be the output, as shown by the block diagram of Fig. 6.6. This system models, for example, pushing a stalled automobile.

The system of Fig. 6.5 contains both energy storage and energy dissipation elements. Kinetic energy is stored in the form of the velocity of the mass. The sliding coefficient of friction dissipates energy. Thus, the system has a single energy storage element (the mass) and a single energy dissipation element (the sliding friction). In section 4.1, we determined that the governing equation for the system was the first order differential equation:

$$m \frac{dv(t)}{dt} + bv(t) = F(t) \quad \text{Eq. 6.4}$$

The presence of the energy storage element causes the input-output equation to be a differential equation.

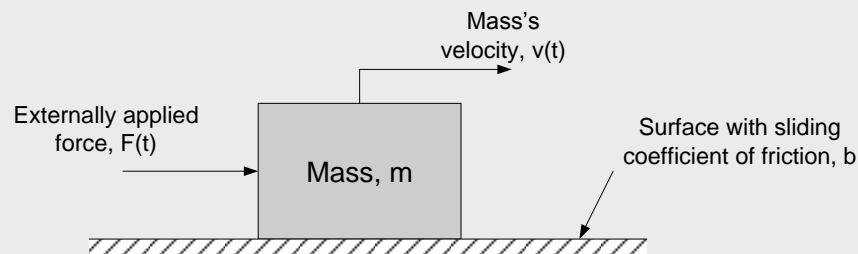
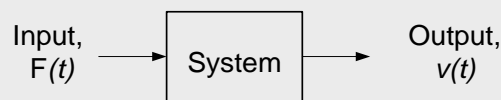
Figure 6.5. Sliding mass on surface with friction coefficient,  $b$ .

Figure 6.6. Mass-damper system represented as a block diagram.

We will examine the effect that the energy storage element has upon the system response in qualitative terms, rather than explicitly solving equation (6.4). If we increase the force applied to the mass, the mass will accelerate and the velocity of the mass increases. The system, therefore, is converting the energy in the input force to a kinetic energy of the mass. This energy transfer results in a change in the output variable, velocity.

The energy storage elements of the system of Fig. 6.5 do not, however, allow an instantaneous change in velocity to an instantaneous change in force. For example, say that before time  $t = 0$  no force is applied to the mass and the mass is at rest. At time  $t = 0$  we suddenly apply a force to the mass, as shown in Fig. 6.7(a) below. At time  $t = 0$  the mass begins to accelerate but it takes time for the mass to approach its final velocity, as shown in Fig. 6.7(b). This transitory stage, when the system is in transition from one constant operating condition to another is called the *transient response*. After a time, the energy input from the external force is balanced by the energy dissipated by the sliding friction, and the velocity of the mass remains constant. When the operating conditions are constant, the energy input is exactly balanced by the energy dissipation, and the system's response is said to be in *steady-state*. We will discuss these terms in more depth in later chapters when we perform the mathematical analysis of dynamic systems.

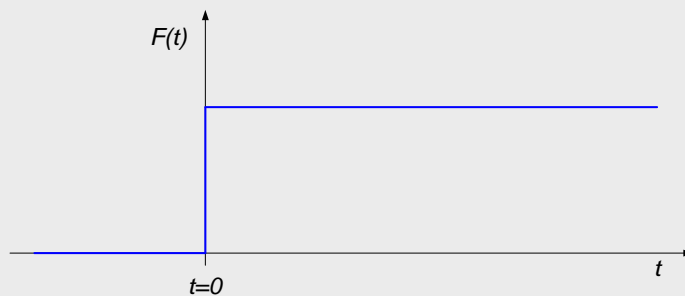


Figure 6.7(a). Force applied to mass.

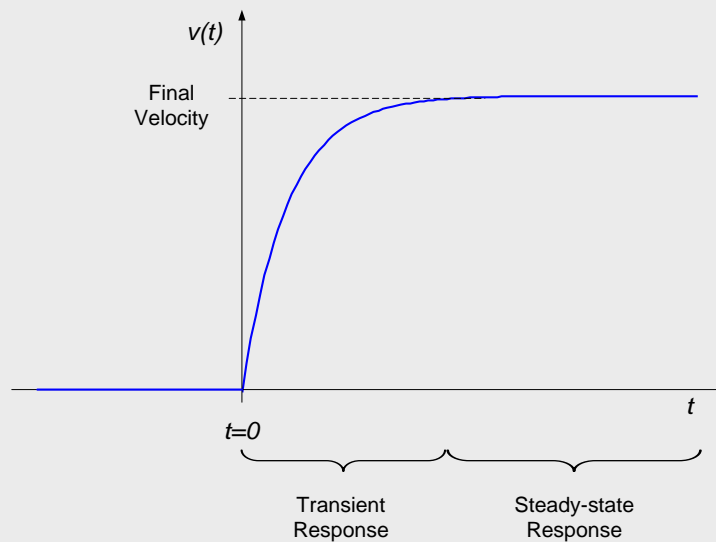


Figure 6.7(b). Velocity of mass.

## Example 6.2

Our second example of a system, which includes energy storage elements, is a body that is subjected to some heat input. The overall system is shown in Figure 6.8. The body being heated has some mass  $m$ , specific heat  $C_p$ , and temperature  $T_B$ . Some heat input  $q_{in}$  is applied to the body from an external source, and the body transfers heat  $q_{out}$  to its surroundings. The surroundings are at some ambient temperature  $T_0$ . We will consider the input to

our system to be the applied heat input  $q_{in}$  and the output to be the temperature of the body  $T_B$ , as shown in the block diagram of Fig. 6.9. This system is a model, for example, of the process of heating a frying pan on a stove. Heat input is applied by the stove burner and the pan dissipates heat by transferring it to the surroundings.

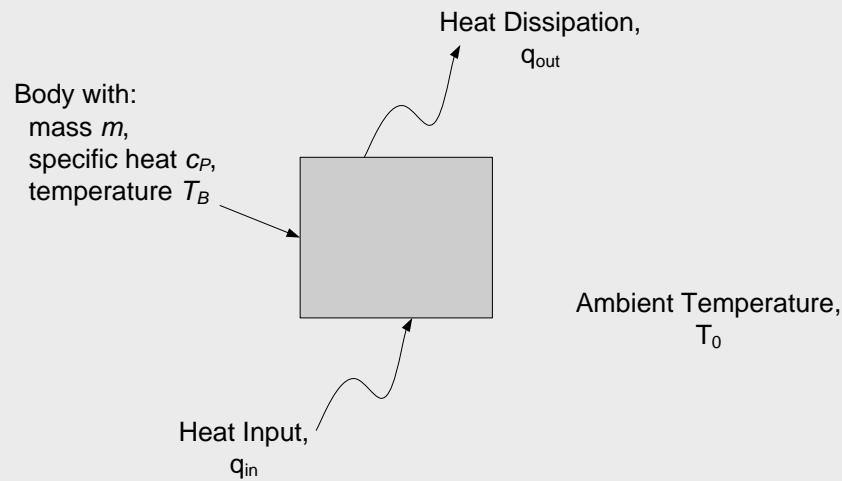


Figure 6.8. Body subjected to heating.

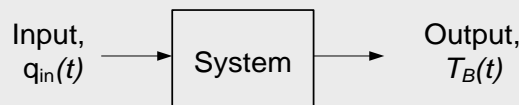


Figure 6.9. System block diagram.

The system of Fig. 6.8 contains both energy storage and energy dissipation elements. Energy is stored in the form of the temperature of the mass. Energy is dissipated in the form of heat transferred to the surroundings. Thus, the system has a single energy storage element (the mass) and a single energy dissipation element (the heat dissipation). The governing equation for the system is the first order differential equation:

$$mc_p \frac{d(T_B - T_0)}{dt} + q_{OUT} = q_{in} \quad \text{Eq. 6.5}$$

The presence of the energy storage element causes the input-output equation to be a differential equation.

We again examine the response of this system to some input in qualitative rather than quantitative terms in order to provide some insight into the overall process before immersing ourselves in the mathematics associated with analyzing the system quantitatively. If the heat input to the system is increased instantaneously (for example, if we suddenly turn up the heat setting on our stove burner) the mass's temperature will increase. As the mass's temperature increases, the heat transferred to the ambient surroundings will increase. When the heat input to the mass is exactly balanced by the heat transfer to the surroundings, the mass's temperature will no longer change and the system will be at a *steady-state* operating condition. Since the mass provides energy storage, the temperature of the mass will not respond instantaneously to a sudden change in heat input – the temperature will rise relatively slowly to its steady-state operating condition. (We know from experience that changing the burner setting on the stove does not immediately change the temperature of our pan, particularly if the pan is heavy.) The process of changing the body's temperature from one steady state operating condition to another is the system's *transient response*.

The process of changing the body's temperature by instantaneously increasing the heat input to the body is illustrated in Fig. 6.10. The signal corresponding to the heat input is shown in Fig. 6.10(a), while the resulting temperature response of the body is shown in Fig. 6.10(b).

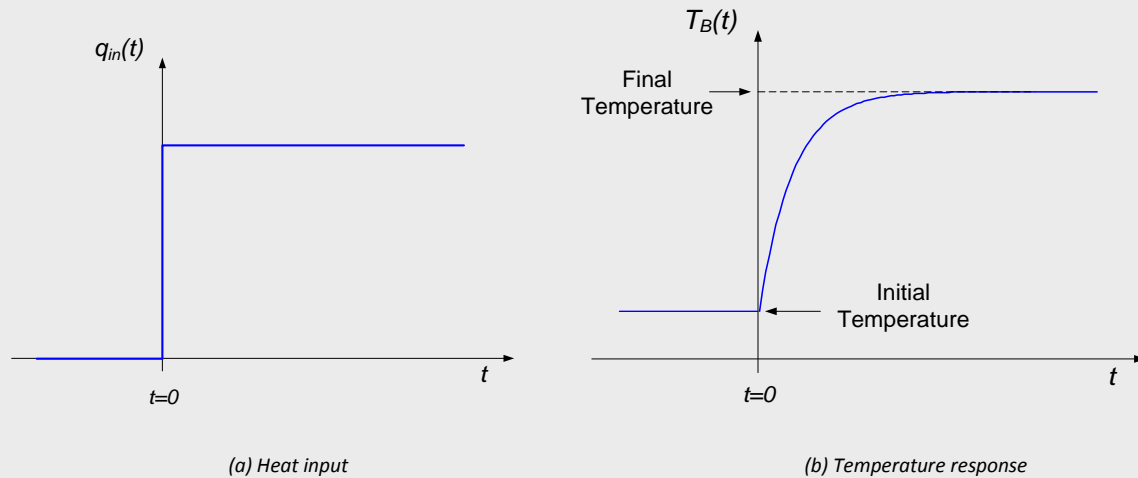


Figure 6.10. Temperature response to instantaneous heat input.

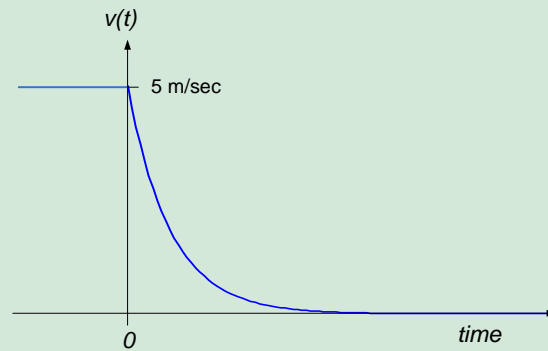
## Section Summary

- Systems with energy storage elements are governed by differential equations. Systems that contain only energy dissipation elements (such as resistors) are governed by algebraic equations.
- The responses of systems governed by algebraic equations will typically have the same “shape” as the input. The output at a given time is simply dependent upon the input at that same time – the system does not “remember” any previous conditions.
- The responses of systems governed by differential equations will not, in general, have the same “shape” as the forcing function applied to the system. The system “remembers” previous conditions – this is why the solution to a differential equation requires knowledge of initial conditions.
- The response of a system that stores energy is generally considered to consist of two parts: the *transient* response and the *steady-state* response. These are described as follows:
  - The transient response typically is shaped differently from the forcing function. It is due to initial energy levels stored in the system.
  - The steady-state response is the response of the system as  $t \rightarrow \infty$ . It is the same “shape” as the forcing function applied to the system.

In differential equations courses, the transient response corresponds (approximately) to the homogeneous solution of the governing differential equation, while the steady-state response corresponds to the particular solution of the governing differential equation.

### 6.1 Exercises

1. A mass is sliding on a surface with an initial velocity of 5 meters/seconds. All external forces (except for the friction force on the surface) are removed from the mass at time  $t = 0$  seconds. The velocity of the mass as a function of time is shown below. What is the steady-state velocity of the mass?



## 6.2 Basic Time-varying Signals

Since the analysis of dynamic systems relies upon time-varying phenomenon, this chapter section presents some common time-varying signals that will be used in our analyses. Specific signals that will be presented are step functions and exponential functions.

### 6.2.1 Step Function

We will use a *step function* to model a signal, which changes suddenly from one constant value to another. These types of signals can be very important. Examples include digital logic circuits (which switch between low and high voltage levels) and control systems (whose design specifications are often based on the system's response to a sudden change in input).

We define a *unit step function*,  $u_0(t)$  as follows:

$$u_0(t) = \begin{cases} 0, & t < 0 \\ 1, & t > 0 \end{cases} \quad \text{Eq. 6.6}$$

The unit step function is illustrated in Fig. 6.11 below. For now, it will not be necessary to define a value for the step function at time  $t=0$ .

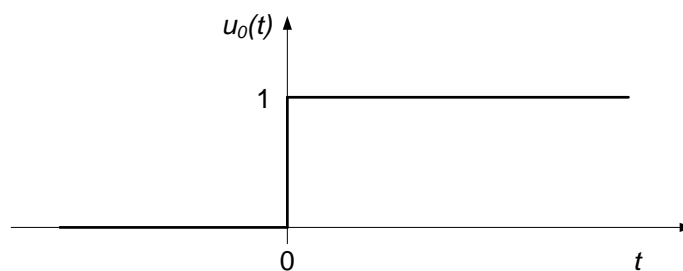


Figure 6.11. Unit step function.

Physically, the step function models a switching process. For example, the output voltage  $V_{out}$  of the circuit shown in Figure 6.12, in which a constant 1V source supplies voltage through a switch which closes at time  $t=0$ , is a unit step function.



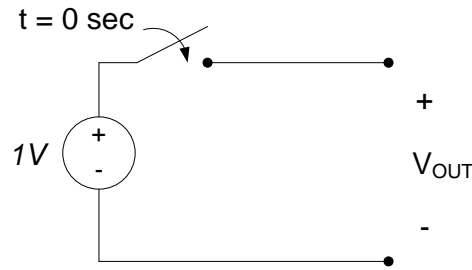
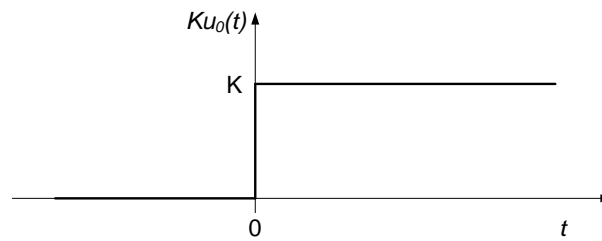


Figure 6.12. Circuit to realize a unit step function.

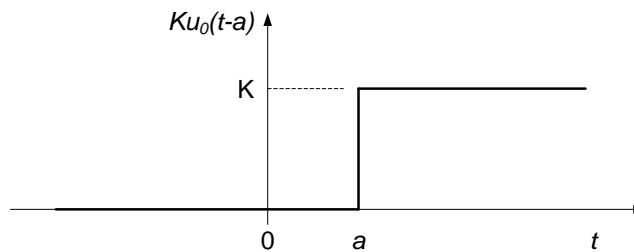
The unit step function can be *scaled* to provide different amplitudes. Multiplication of the unit step function by a constant  $K$  results in a signal which is zero for times less than zero and  $K$  for times greater than zero, as shown in Fig. 6.13.

Figure 6.13. Scaled step function  $Ku_0(t)$ ;  $K > 0$ .

The step function can also be *shifted* to model processes which switch at times other than  $t=0$ . A step function with amplitude  $K$  which occurs at time  $t=a$  can be written as  $Ku_0(t-a)$ :

$$Ku_0(t-a) = \begin{cases} 0, & t < a \\ K, & t > a \end{cases} \quad \text{Eq. 6.7}$$

The function is zero when the argument  $t-a$  is less than zero and  $K$  when the argument  $t-a$  is greater than zero, as shown in Fig. 6.14. If  $a > 0$ , the function is shifted to the right of the origin; if  $a < 0$ , the function is shifted to the left of the origin.

Figure 6.14. Shifted and scaled step function  $Ku_0(t-a)$ ;  $K > 0$  and  $a > 0$ .

Switching the sign of the above argument in equation (6.7) results in:

$$Ku_0(-t+a) = Ku_a(a-t) = \begin{cases} K, & t < a \\ 0, & t > a \end{cases} \quad \text{Eq. 6.8}$$

And the value of the function is  $K$  for  $t < a$  and zero for  $t > a$ , as shown in Fig. 6.15. As above, the transition from  $K$  to zero is to the right of the origin if  $a > 0$  and to the left of the origin if  $a < 0$ .

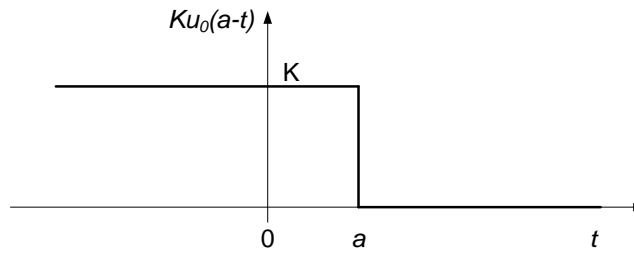


Figure 6.15. The function step function  $Ku_0(a-t)$ ;  $K>0$  and  $a>0$ .

Step functions can also be used to describe finite-duration signals. For example, the function:

$$f(t) = \begin{cases} 0, & t < 0 \\ 1, & 0 < t < 2 \\ 0, & t > 2 \end{cases}$$

Illustrated in Fig. 6.16, can be written in terms of sums or products of unit step functions as follows:

$$f(t) = u_0(t) - u_0(t - 1)$$

Or

$$f(t) = u_0(t) \cdot u_0(2 - t)$$

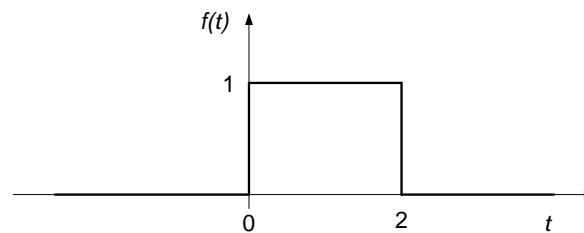


Figure 6.16. Finite-duration signal.

The step function can also be used to create other finite-duration functions. For example, the finite-duration *ramp* function:

$$f(t) = \begin{cases} 0, & t < 0 \\ t, & 0 < t < 1 \\ 0, & t > 1 \end{cases}$$

Shown in Fig. 6.17, can be written as a single function over the entire range  $-\infty < t < \infty$  by using unit step functions, as follows:

$$f(t) = t \cdot [u_0(t) - u_0(t - 1)]$$

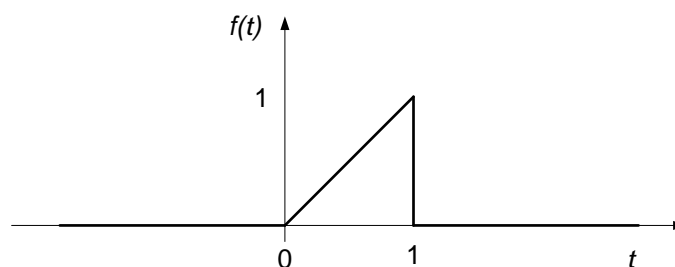


Figure 6.17. Finite-duration "ramp" signal.

## 6.2.2 Exponential Functions

A function that appears commonly in the analysis of linear systems is the *decaying exponential*:

$$f(t) = Ae^{-at}$$

Where  $a > 0$ . The function  $f(t)$  is illustrated in Fig. 6.18. The value of the function is  $A$  at  $t=0$  and decreases to zero as  $t \rightarrow \infty$ . As  $t \rightarrow -\infty$  the function increases without bound. The constant  $a$  dictates the rate at which the function decreases as time increases.

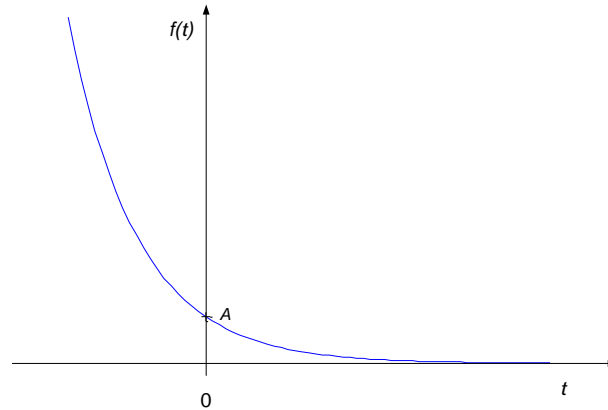


Figure 6.18. Decaying exponential function.

We will usually be interested in this function only for positive values of time. We will also commonly write our exponential function in terms of a time constant,  $\tau$ , rather than the constant  $a$ . Thus, the decaying exponential function we will generally use is:

$$f(t) = \begin{cases} 0, & t < 0 \\ Ae^{-\frac{t}{\tau}}, & t \geq 0 \end{cases} \quad \text{Eq. 6.10}$$

Or, using the unit step function to limit the function to positive values of time:

$$f(t) = Ae^{-\frac{t}{\tau}} \cdot u_0(t) \quad \text{Eq. 6.11}$$

The function of equations (6.10) and (6.11) is illustrated in Fig. 6.19. The time constant,  $\tau$ , is a positive number which dictates the rate at which the function will decay with time. When the time  $t = \tau$ ,  $f(t)Ae^{-1} = 0.368A$  and the function has decayed to 36.8% of its original value. In fact, the function decreases by 36.8% every  $\tau$  seconds. Therefore, a signal with a small time constant decays more rapidly than a signal with a large time constant, as illustrated in Fig. 6.20.

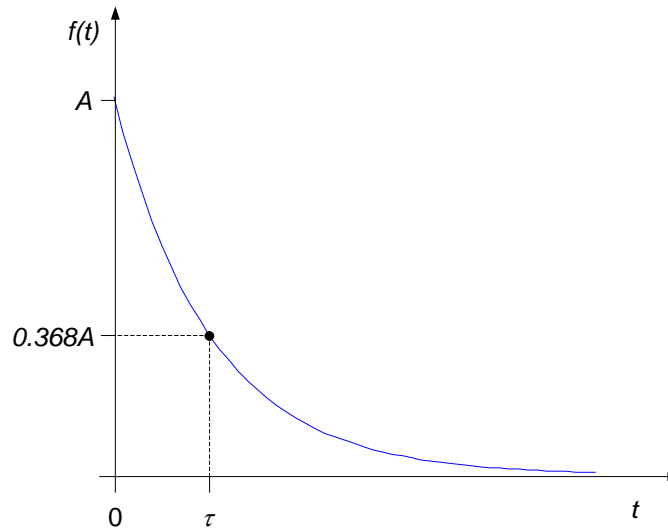


Figure 6.19. Exponential function  $f(t) = Ae^{\frac{-t}{\tau}}u_0(t)$ .

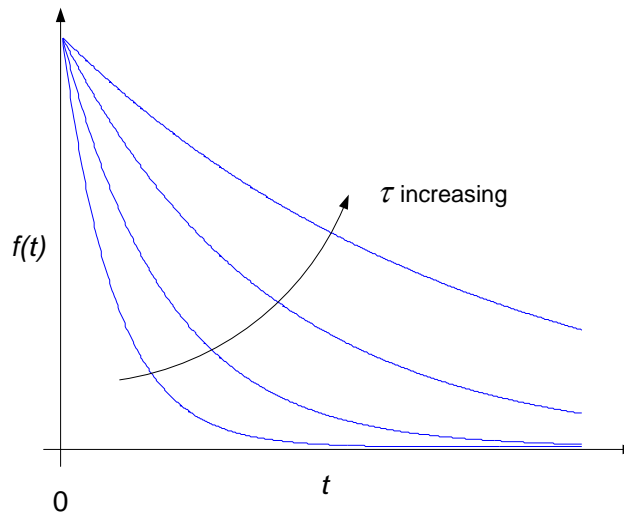


Figure 6.20. Exponential function variation with time cons.

## Section Summary

- Step functions are useful for representing conditions (generally inputs), which change from one value to another instantaneously. In electrical engineering, they are commonly used to model the opening or closing of a switch that connects a circuit to a source, which provides a constant voltage or current. Mathematically, an arbitrary step function can be represented by:

$$Ku_0(-t + a) = Ku_0(a - t) = \begin{cases} K, & t < a \\ 0, & t > a \end{cases}$$

So that the step function turns “on” at time  $t=a$ , and has an amplitude  $K$ .

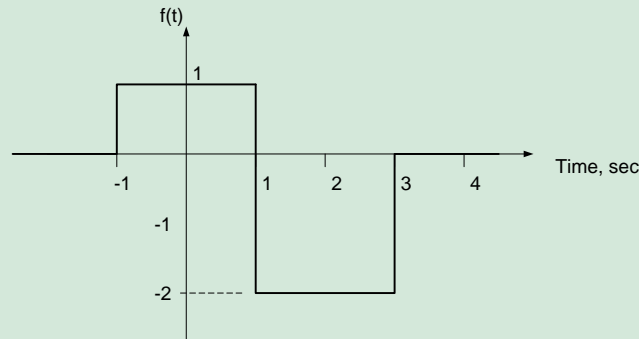
- An exponential function, defined for  $t>0$ , is mathematically defined as:

$$f(t) = Ae^{\frac{-t}{\tau}} \cdot u_0(t)$$

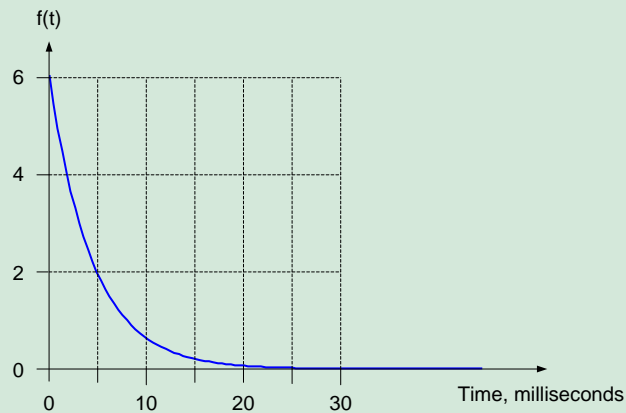
The function has an initial value,  $A$ , and a time constant,  $\tau$ . The time constant indicates how quickly the function decays; the value of the function decreases by 63.2% every  $\tau$  seconds. Exponential functions are important to use because the solutions of linear, constant coefficient, ordinary differential equations typically take the form of exponentials.

## 6.2 Exercises

- Express the signal below in terms of step functions.



- The function shown below is a decaying exponential. Estimate the function from the given graph.



## 6.3 Capacitors

We begin our study of energy storage elements with a discussion of capacitors. Capacitors, like resistors, are passive two-terminal circuit elements. That is, no external power supply is necessary to make them function. Capacitors consist of a non-conductive material (or *dielectric*) which separates two electrical conductors; capacitors store energy in the form of an electric field set up in the dielectric material.

In this section, we describe physical properties of capacitors and provide a mathematical model for an ideal capacitor. Using this ideal capacitor model, we will develop mathematical relationships for the energy stored in a capacitor and governing relations for series and parallel connections of capacitors. The section concludes with a brief discussion of practical (non-ideal) capacitors.

### 6.3.1 Capacitors

Two electrically conductive bodies, when separated by a non-conductive (or *insulating*) material, will form a *capacitor*. Figure 6.21 illustrates the special case of a *parallel plate capacitor*. The non-conductive material

between the plates is called a dielectric; the material property of the dielectric, which is currently important to us, is its *permittivity*,  $\epsilon$ . When a voltage potential difference is applied across the two plates, as shown in Fig. 6.21, charge accumulates on the plates – the plate with the higher voltage potential will accumulate positive charge  $q$ , while the plate with the lower voltage potential will accumulate negative charge,  $-q$ . The charge difference between the plates induces an *electric field* in the dielectric material; the capacitor stores energy in this electric field. The *capacitance* of the capacitor is a quantity that tells us, essentially, how much energy can be stored by the capacitor. Higher capacitance means that more energy can be stored by the capacitor. Capacitance has units of *Farads*, abbreviated F.

The amount of capacitance a capacitor has is governed by the geometry of the capacitor (the shape of the conductors and their orientation relative to one another) and the permittivity of the dielectric between the conductors. These effects can be complex and difficult to quantify mathematically; rather than attempt a comprehensive discussion of these effects, we will simply claim that, in general, capacitance is dependent upon the following parameters:

- The spacing between the conductive bodies (the distance  $d$  in Fig. 6.21). As the separation between the bodies increases, the capacitance decreases.
- The surface area of the conductive bodies. As the surface area of the conductors increases, the capacitance increases. The surface area referred to here is the area over which both the conductors and the dielectric overlap.
- The permittivity of the dielectric. As the permittivity increases, the capacitance increases.

The parallel-plate capacitor shown in Fig. 6.21, for example, has capacitance:

$$C = \frac{\epsilon \cdot A}{d}$$

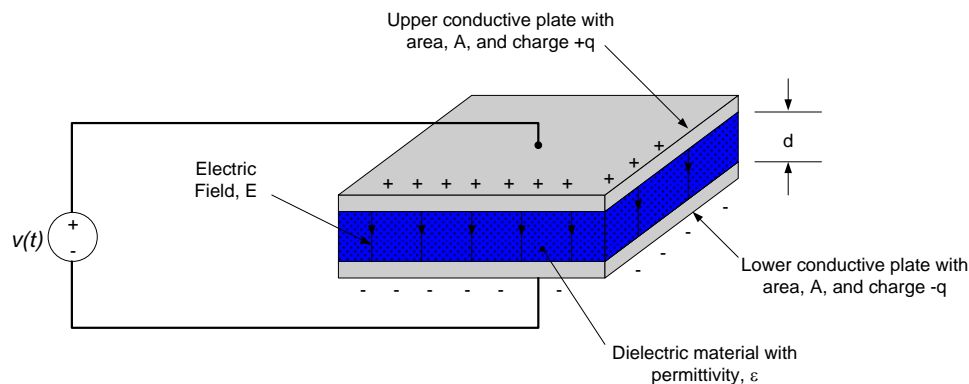


Figure 6.21. Parallel plate capacitor with applied voltage across conductors.

Mathematically, the capacitance of the device relates the voltage difference between the plates and the charge accumulation associated with this voltage:

$$q(t) = CV(t) \tag{Eq. 6.12}$$

Capacitors that obey the relationship of equation (6.12) are *linear capacitors*, since the potential difference between the conductive surfaces is linearly related to the charge on the surfaces. Please note that the charges on the upper and lower plate of the capacitor in Fig. 6.21 are equal and opposite – thus, if we increase the charge on one plate, the charge on the other plate must decrease by the same amount. This is consistent with our previous assumption electrical circuit elements cannot accumulate charge, and current entering one terminal of a capacitor must leave the other terminal of the capacitor.

Since current is defined as the time rate of change of charge,  $i(t) = \frac{dq(t)}{dt}$ , equation (6.12) can be re-written in terms of the current through the capacitor:

$$i(t) = \frac{d}{dt} [Dv(t)] \quad \text{Eq. 6.13}$$

Since the capacitance of a given capacitor is constant, equation (6.13) can be written as:

$$i(t) = C \frac{dv(t)}{dt} \quad \text{Eq. 6.14}$$

The circuit symbol for a capacitor is shown in Fig. 6.22, along with the sign conventions for the voltage-current relationship of equation (6.14). We use our passive sign convention for the voltage-current relationship – positive current is assumed to enter the terminal with positive voltage polarity.

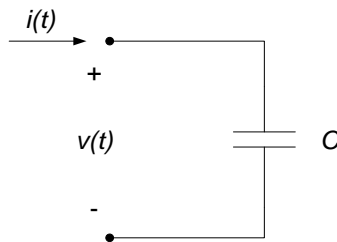


Figure 6.22. Capacitor circuit symbol and voltage-current sign convention.

Integrating both sides of equation (6.14) results in the following form for the capacitor's voltage-current relationship:

$$v(t) = \frac{1}{C} \int_{t_0}^t i(\xi) d\xi + v(t_0) \quad \text{Eq. 6.15}$$

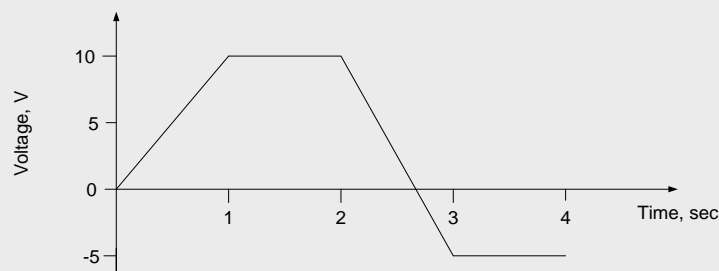
Where  $v(t_0)$  is a known voltage at some initial time,  $t_0$ . We use a dummy variable of integration,  $\xi$ , to emphasize that the only "t" which survives the integration process is the upper limit of the integral.

**Important result:** The voltage-current relationship for an ideal capacitor can be stated in either differential or integral form, as follows:

- $i(t) = C \frac{dv(t)}{dt}$
- $v(t) = \frac{1}{C} \int_{t_0}^t i(\xi) d\xi + v(t_0)$

### Example 6.3

If the voltage as a function of time across a capacitor with capacitance  $C=1\mu F$  is as shown below, determine the current as a function of time through the capacitor.



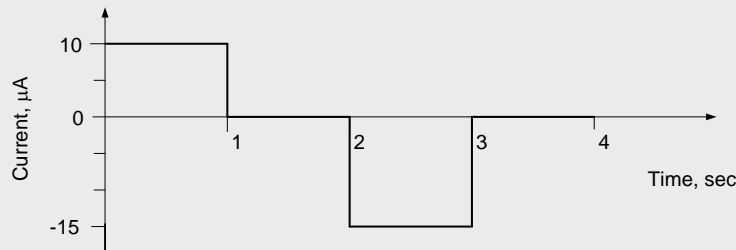
$0 < t < 1$ : The voltage rate of change is 10 V/s. Thus,  $C \frac{dv(t)}{dt} = (1 \times 10^{-6} F) \left(10 \frac{V}{s}\right) = 10 \mu A$ .

$1 < t < 2$ : The voltage is constant. Thus,  $C \frac{dv(t)}{dt} = 0 A$ .

$2 < t < 3$ : The voltage rate of change is -15V/s. Thus,  $C \frac{dv(t)}{dt} = (1 \times 10^{-6} F) \left(-15 \frac{V}{s}\right) = -15 \mu A$ .

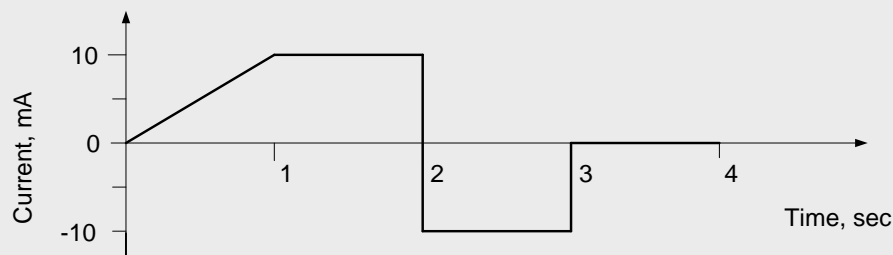
$3 < t < 4$ : The voltage is constant. Thus,  $C \frac{dv(t)}{dt} = 0 A$

A plot of the current through the capacitor as a function of time is shown below.



### Example 6.4

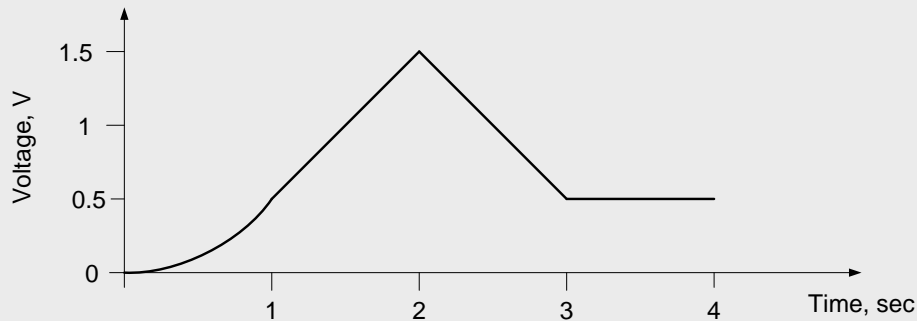
If the current as a function of time through a capacitor with capacitance  $C=10mF$  is as shown below, determine the voltage as a function of time across the capacitor. Assume that the voltage across the capacitor is 0V at time  $t=0$ .



- At time  $t=0$ , the voltage is given to be 0V.
- In the time period  $0 < t < 1$  second, the current increases linearly and the voltage will increase quadratically. The total voltage change during this time period is the integral of the current, which is simply the area under the current curve divided by the capacitance:  $\frac{1}{2} \frac{(10 \times 10^{-3} A)(1s)}{0.01F} = 0.5V$ .
- In the time period  $1 < t < 2$  seconds, the current is constant at 10 mA. The voltage change is the area under the current curve divided by the capacitance:  $(10 \times 10^{-3} A) \frac{(1s)}{0.01F} = 1V$ . The total voltage at  $t=2$  seconds is, then,  $0.5V + 1V = 1.5V$ .
- In the time period  $2 < t < 3$  seconds, the current is constant at -10 mA. The voltage change is the negative of the voltage change from  $1 < t < 2$  sec. The total voltage at  $t=3$  seconds is, then,  $1.5V - 1V = 0.5V$ .
- In the time period  $3 < t < 4$  seconds, the current is zero. The integral of zero over any time period is zero, so there is no change in voltage during this time range and the voltage remains constant at 0.5V.

A plot of the voltage across the capacitor as a function of time is shown below.





It is often useful, when analyzing circuits containing capacitors, to examine the circuit's response to constant operating conditions and to instantaneous changes in operating condition. We examine the capacitor's response to each of these operating conditions below:

- Capacitor response to constant voltage:
  - If the voltage across the capacitor is constant, equation (6.14) indicates that the current through the capacitor is zero. Thus, if the voltage across the capacitor is constant, the capacitor is equivalent to an open circuit.
  - This property can be extremely useful in determining a circuit's steady-state response to constant inputs. If the inputs to a circuit change from one constant value to another, the transient components of the response will eventually die out and all circuit parameters will become constant. Under these conditions, capacitors can be replaced with open circuits and the circuit analyzed relatively easily. As we will see later, this operating condition can be useful in determining the response of circuits containing capacitors and in double-checking results obtained using other methods.
- Capacitor response to instantaneous voltage changes:
  - If the voltage across the capacitor changes instantaneously, the rate of change of voltage is infinite. Thus, by equation (6.14), if we wish to change the voltage across a capacitor instantaneously, we must supply infinite current to the capacitor. This implies that infinite power is available, which is not physically possible. Thus, in any practical circuit, the voltage across a capacitor cannot change instantaneously.
  - Any circuit that allows an instantaneous change in the voltage across an ideal capacitor is not physically realizable. We may sometimes assume, for mathematical convenience, that an ideal capacitor's voltage changes suddenly; however, it must be emphasized that this assumption requires an underlying assumption that infinite power is available and is thus not an allowable operating condition in any physical circuit.

#### Important Capacitor Properties:

- Capacitors can be replaced by open-circuits, under circumstances when all operating conditions are constant.
- Voltages across capacitors cannot change instantaneously. No such requirement is placed on currents.

### 6.3.2 Energy Storage

The power dissipated by a capacitor is:

$$p(t) = v(t) \cdot i(t) \quad \text{Eq. 6.16}$$

Since both voltage and current are functions of time, the power dissipation will also be a function of time. The power as a function of time is called the *instantaneous power*, since it provides the power dissipation at any instant in time.

Substituting equation (6.14) into equation (6.16) results in:

$$p(t) = C \cdot v(t) \frac{dv(t)}{dt} \quad \text{Eq. 6.17}$$

Since power is, by definition, the rate of change of energy, the energy is the time integral of power. Integrating equation (6.17) with respect to time gives the following expression for the energy stored in a capacitor:

$$W_C(t) = \int_{-\infty}^t C v(\xi) \frac{dv(\xi)}{dt} dt = \int_{-\infty}^t C v(\xi) dv(\xi) = \frac{1}{2} C v^2(\xi) \Big|_{-\infty}^t$$

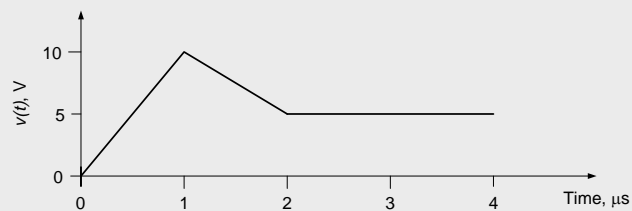
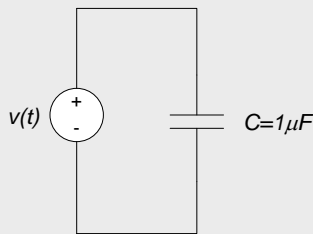
Where we have set our lower limits of integration at  $t = -\infty$  to avoid issues relative to initial conditions. We assume that no energy is stored in the capacitor at time  $t = -\infty$  so that:

$$W_C(t) = \frac{1}{2} C v^2(t) \quad \text{Eq. 6.18}$$

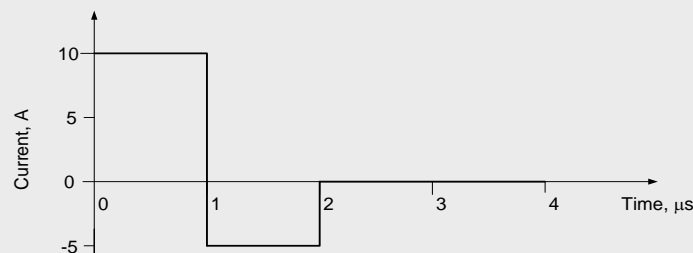
From equation (6.18) we see that the energy stored in a capacitor is always a non-negative quantity, so  $W_C(t) \geq 0$ . Ideal capacitors do not dissipate energy, as resistors do. Capacitors store energy when it is provided to them from the circuit; this energy can later be recovered and returned to the circuit.

### Example 6.5

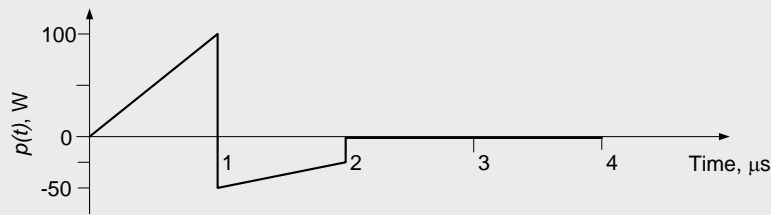
Consider the circuit shown below. The voltage applied to the capacitor by the source is as shown. Plot the power absorbed by the capacitor and the energy stored in the capacitor as functions of time.



Power is most readily computed by taking the product of voltage and current. The current can be determined from equation (6.14). The current as a function of time is plotted below.

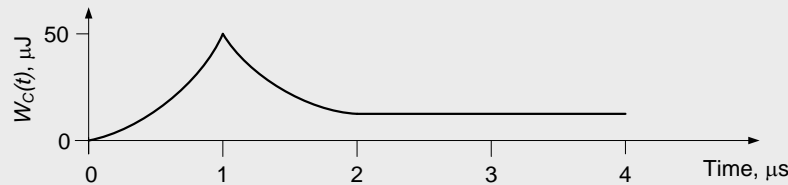


The power absorbed by the capacitor is determined by taking a point-by-point product between the voltage and current.



Recall that power is absorbed or generated based on the passive sign convention. If the relative signs between voltage and current agree with the passive sign convention, the circuit element is absorbing power. If the relative signs between voltage and current are opposite to the passive sign convention, the element is generating power. Thus, the capacitor in this example is absorbing power for the first microsecond. It generates power (returns power to the voltage source) during the second microsecond. After the second microsecond, the current is zero and the capacitor neither absorbs nor generates power.

The energy stored in the capacitor can be determined either from integrating the power or from application of equation (6.18) to the voltage curve provided in the problem statement. The energy in the capacitor as a function of time is shown below:



During the first microsecond, while the capacitor is absorbing power, the energy in the capacitor is increasing. The maximum energy in the capacitor is 50 μJ, at 1 μs. During the second microsecond, the capacitor is releasing power back to the circuit and the energy in the capacitor is decreasing. At 2 μs, the capacitor still has 12.5 μJ of stored energy. After 2 μs, the capacitor neither absorbs nor generates energy and the energy stored in the capacitor remains at 12.5 μJ.

### 6.3.3 Capacitors in Series

Consider the series connection of  $N$  capacitors shown in Fig. 6.23.

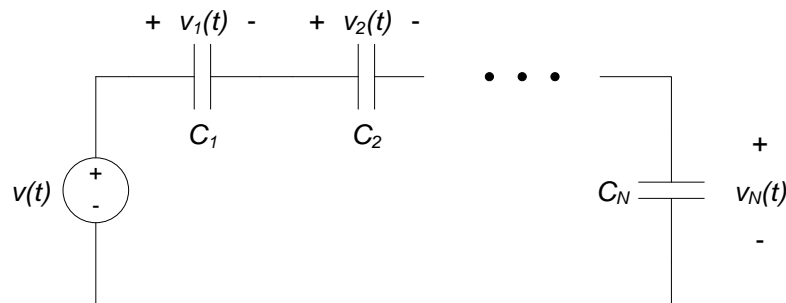


Figure 6.23. Series connection of  $N$  capacitors.

Applying Kirchhoff's voltage law around the loop results in:

$$v(t) = v_1(t) + v_2(t) + \cdots v_N(t) \quad \text{Eq. 6.19}$$

Using equation (6.15) to write the capacitor voltage drops in terms of the current through the loop gives:

$$\begin{aligned}
 v(t) &= \left[ \frac{1}{C_1} \int_{t_0}^t i(\xi) d\xi + v_1(t_0) \right] + \left[ \frac{1}{C_2} \int_{t_0}^t i(\xi) d\xi + v_2(t_0) \right] + \cdots + \left[ \frac{1}{C_N} \int_{t_0}^t i(\xi) d\xi + v_N(t_0) \right] \\
 &= \left[ \frac{1}{C_1} \int_{t_0}^t i(\xi) d\xi + \frac{1}{C_2} \int_{t_0}^t i(\xi) d\xi + \cdots + \frac{1}{C_N} \int_{t_0}^t i(\xi) d\xi \right] + [v_1(t_0) + v_2(t_0) + \cdots + v_N(t_0)] \\
 &= \left( \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N} \right) \int_{t_0}^t i(\xi) d\xi + v(t_0)
 \end{aligned}$$

This can be re-written using summation notation as:

$$v(t) = \left( \sum_{k=1}^N \frac{1}{C_k} \right) \int_{t_0}^t i(\xi) d\xi + v(t_0) \quad \text{Eq. 6.21}$$

Thus, the circuits of Fig. 6.23 and Fig. 6.24 are equivalent circuits, if the equivalent capacitance is chosen according to equation (6.21).

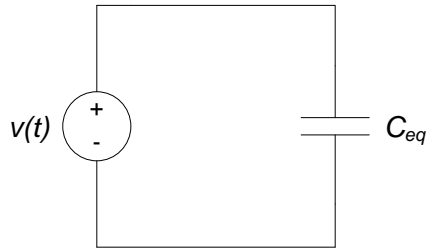


Figure 6.24. Equivalent circuit to Figure 3.

For the special case of two capacitors  $C_1$  and  $C_2$  in series, equation (6.21) simplifies to:

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} \quad \text{Eq. 6.22}$$

Equations (6.21) and (6.22) are analogous to the equations, which provide the equivalent resistance of parallel combinations of resistors.

### 6.3.4 Capacitors in Parallel

Consider the parallel combination of  $N$  capacitors, as shown in Fig. 6.25.

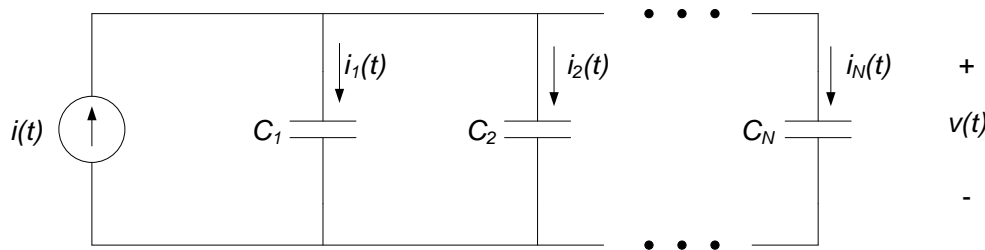


Figure 6.25. Series connection of  $N$  capacitors.

Applying Kirchhoff's current law at the upper node results in:

$$i(t) = i_1(t) + i_2(t) + \cdots + i_N(t) \quad \text{Eq. 6.23}$$

Using equation (6.14) to write the capacitor currents in terms of their voltage drop gives:

$$\begin{aligned} i(t) &= C_1 \frac{dv(t)}{dt} + C_2 \frac{dv(t)}{dt} + \cdots + C_N \frac{dv(t)}{dt} \\ &= (C_1 + C_2 + \cdots + C_N) \frac{dv(t)}{dt} \end{aligned}$$

Using summation notation results in:

$$i(t) = \left( \sum_{k=1}^N C_k \right) \frac{dv(t)}{dt} \quad \text{Eq. 6.24}$$

This is the same equation that governs the circuit of Fig. 6.26, if:

$$C_{eq} = \sum_{k=1}^N C_k \quad \text{Eq. 6.25}$$

Thus, the equivalent capacitance of a parallel combination of capacitors is simply the sum of the individual capacitances. This result is analogous to the equations, which provide the equivalent resistance of a series combination of resistors.

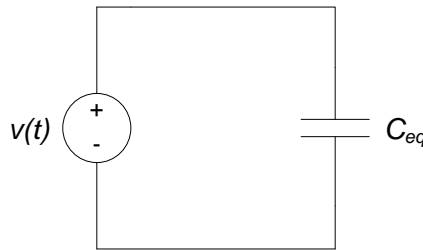


Figure 6.26. Equivalent circuit to Figure 5.

### Summary: Series and Parallel Capacitors

- The equivalent capacitance of a series combination of capacitors  $C_1, C_2, \dots, C_N$  is governed by a relation which is analogous to that providing the equivalent resistance of a parallel combination of resistors:

$$\frac{1}{C_{eq}} = \sum_{k=1}^N \frac{1}{C_k}$$

- The equivalent of a parallel combination of capacitors  $C_1, C_2, \dots, C_N$  is governed by a relation which is analogous to that providing the equivalent resistance of a series combination of resistors:

$$C_{eq} = \sum_{k=1}^N C_k$$

### 6.3.5 Practical Capacitors

Commercially available capacitors are manufactured in a wide range of both conductor and dielectric materials and are available in a wide range of capacitances and voltage ratings. The voltage rating of the device is the maximum voltage, which can be safely applied to the capacitor; using voltages higher than the rated value will damage the capacitor. The capacitance of commercially available capacitors is commonly measured in microfarads ( $\mu\text{F}$ ; one microfarad is  $10^{-6}$  of a Farad) or pico-farads ( $\text{pF}$ ; one picofarad is  $10^{-12}$  of a Farad). Large capacitors are available, but are relatively infrequently used. These are generally called “super-capacitors” or “ultra-capacitors”

and are available in capacitances up to tens of Farads. For most applications, however, using one would be comparable to buying a car with a 1000 gallon gas tank.

Several approaches are used for labeling a capacitor with its capacitance value. Large capacitors often have their value printed plainly on them, such as “10 $\mu$ F” (for 10 microfarads). Smaller capacitors, appearing as small disks or wafers, often have their values printed on them in an encoded manner. For these capacitors, a three-digit number indicates the capacitor value in pico-farads. The first two digits provides the “base” number, and the third digit provides an exponent of 10 (so, for example, “104” printed on a capacitor indicates a capacitance value of  $10 \times 10^4$  or 100000 pF). Occasionally, a capacitor will only show a two-digit number, in which case that number is simply the capacitor value in pF. (For completeness, if a capacitor shows a three-digit number and the third digit is 8 or 9, then the first two digits are multiplied by .01 and .1 respectively).

Capacitors are generally classified according to the dielectric material used. Common capacitor types include mica, ceramic, Mylar, paper, Teflon and polystyrene. An important class of capacitors which require special mention are *electrolytic* capacitors. Electrolytic capacitors have relatively large capacitances relative to other types of capacitors of similar size. However, some care must be exercised when using electrolytic capacitors – they are *polarized* and must be connected to a circuit with the correct polarity. The positive lead of the capacitor must be connected to the positive lead of the circuit. Connecting the positive lead of the capacitor to the negative lead of a circuit can result in unwanted current “leakage” through the capacitor or, in extreme cases, destroy the capacitor. Polarized capacitors either have a dark stripe near the pin that must be kept at the higher voltage, or a “-” near the pin that must be kept at a lower voltage.

Practical capacitors, unlike ideal capacitors, will dissipate some power. This power loss is primarily due to *leakage currents*. These currents are due to the fact that real dielectric materials are not perfect insulators – some small current will tend to flow through them. The overall effect is comparable to placing a high resistance in parallel with an ideal capacitor, as shown in Fig. 6.27. Different types of capacitors have different leakage currents. Mica capacitors tend to have low leakage currents, the leakage currents of ceramic capacitors vary according to the type of capacitor, and electrolytic capacitors have high leakage currents.

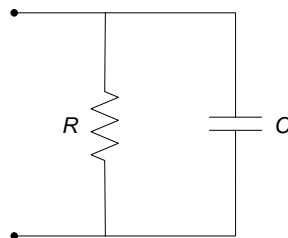


Figure 6.27. Model of practical capacitor including leakage current path.

## Section Summary

- Capacitors store electrical energy. This energy is stored in an electric field between two conductive elements, separated by an insulating material.
- Capacitor energy storage is dependent upon the voltage across the capacitor, if the capacitor voltage is known, the energy in the capacitor is known.
- The voltage-current relationship for a capacitor is:

$$i(t) = C \frac{dv(t)}{dt}$$

Where  $C$  is the capacitance of the capacitor. Units of capacitance are Farads (abbreviated F). The capacitance of a capacitor, very roughly speaking, gives an indication of how much energy it can store

- The above voltage-current relation results in the following important properties of capacitors:
  - If the capacitor voltage is constant, the current through the capacitor is zero. Thus, if the capacitor voltage is constant, the capacitor can be modeled as an open circuit.
  - Changing the capacitor voltage instantaneously requires infinite power. Thus (for now, anyway) we will assume that capacitors cannot instantaneously change their voltage.
- Capacitors placed in series or parallel with one another can be modeled as a single equivalent capacitance. Thus, capacitors in series or in parallel are not “independent” energy storage elements.

### 6.3 Exercises

1. Determine the maximum and minimum capacitances that can be obtained from four  $1\mu\text{F}$  capacitors. Sketch the circuit schematics that provide these capacitances.
2. Determine voltage divider relationships to provide  $v_1$  and  $v_2$  for the two uncharged series capacitors shown below. Use your result to determine  $v_2$  if  $C_1=C_2=10\mu\text{F}$ .

## 6.4 Inductors

We continue our study of energy storage elements with a discussion of *inductors*. Inductors, like resistors and capacitors, are passive two-terminal circuit elements. That is, no external power supply is necessary to make them function. Inductors commonly consist of a conductive wire wrapped around a core material; inductors store energy in the form of a magnetic field set up around the current-carrying wire.

In this section, we describe physical properties of inductors and provide a mathematical model for an ideal inductor. Using this ideal inductor model, we will develop mathematical relationships for the energy stored in an inductor and governing relations for series and parallel connections of inductors. The section concludes with a brief discussion of practical (non-ideal) inductors.

### 6.4.1 Inductors

Passing a current through a conductive wire will create a *magnetic field* around the wire. This magnetic field is generally thought of in terms of as forming closed loops of *magnetic flux* around the current-carrying element. This physical process is used to create *inductors*. Figure 6.28 illustrates a common type of inductor, consisting of a coiled wire wrapped around a core material. Passing a current through the conducting wire sets up lines of magnetic flux, as shown in Fig. 6.28; the inductor stores energy in this magnetic field. The *inductance* of the inductor is a quantity, which tells us how much energy can be stored by the inductor. Higher inductance means that the inductor can store more energy. Inductance has units of *Henrys*, abbreviated H.

The amount of inductance an inductor has is governed by the geometry of the inductor and the properties of the core material. These effects can be complex; rather than attempt a comprehensive discussion of these effects, we will simply claim that, in general, inductance is dependent upon the following parameters:

- The number of times the wire is wrapped around the core. More coils of wire results in a higher inductance.
- The core material's type and shape. Core materials are commonly ferromagnetic materials, since they result in higher magnetic flux and correspondingly higher energy storage. Air, however, is a fairly commonly used core material – presumably because of its ready availability.
- The spacing between turns of the wire around the core.

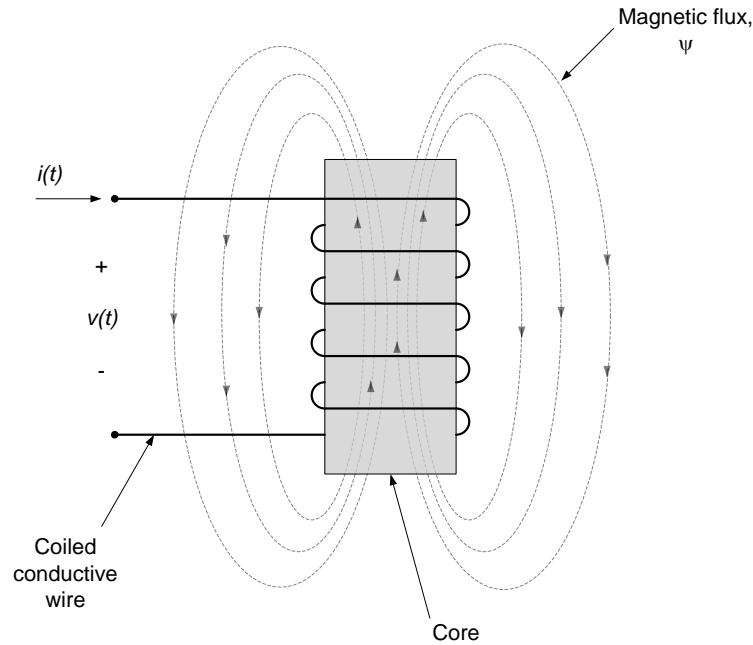


Figure 6.28. Wire-wrapped inductor with applied current through conductive wire.

We will denote the total magnetic flux created by the inductor by  $\psi$ , as shown in Fig. 6.28. For a linear inductor, the flux is proportional to the current passing through the wound wires. The constant of proportionality is the inductance,  $L$ :

$$\psi(t) = Li(t) \quad \text{Eq. 6.26}$$

Voltage is the time rate of change of magnetic flux, so:

$$v(t) = \frac{d\psi(t)}{dt} \quad \text{Eq. 6.27}$$

Combining equations (6.26) and (6.27) results in the voltage-current relationship for an ideal inductor:

$$v(t) = L \frac{di(t)}{dt} \quad \text{Eq. 6.28}$$

The circuit symbol for an inductor is shown in Fig. 6.29, along with the sign conventions for the voltage-current relationship of equation (6.28). The passive sign convention is used in the voltage-current relationship, so positive current is assumed to enter the terminal with positive voltage polarity.

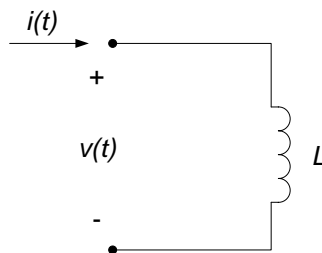


Figure 6.29. Inductor circuit symbol and voltage-current sign convention.

Integrating both sides of equation (6.28) results in the following form for the inductor's voltage-current relationship:



$$i(t) = \frac{1}{L} \int_{t_0}^t v(\xi) d\xi + i(t_0) \quad \text{Eq. 6.29}$$

In equation (6.29),  $i(t_0)$  is a known current at some initial time  $t_0$  and  $\xi$  is used as a dummy variable of integration to emphasize that the only “ $t$ ” which survives the integration process is the upper limit of the integral.

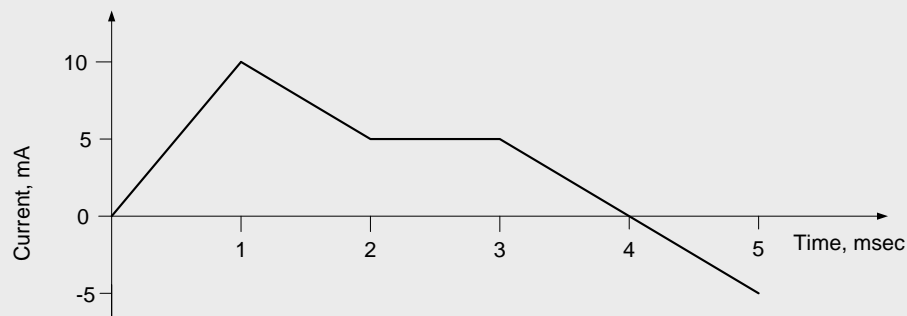
### Important Result

The voltage-current relationship for an ideal inductor can be stated in either differential or integral form, as follows:

- $v(t) = L \frac{di(t)}{dt}$
- $i(t) = \frac{1}{L} \int_{t_0}^t v(\xi) d\xi + i(t_0)$

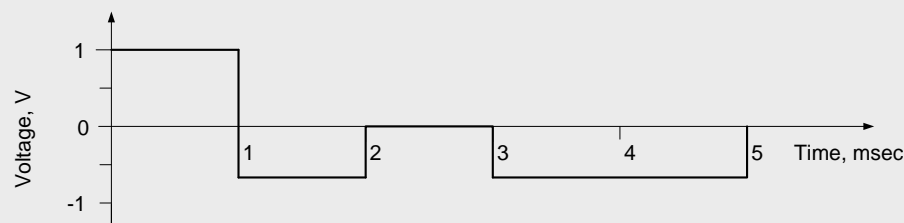
### Example 6.6

A circuit contains a 100mH inductor. The current as a function of time through the inductor is measured and shown below. Plot the voltage across the inductor as a function of time.

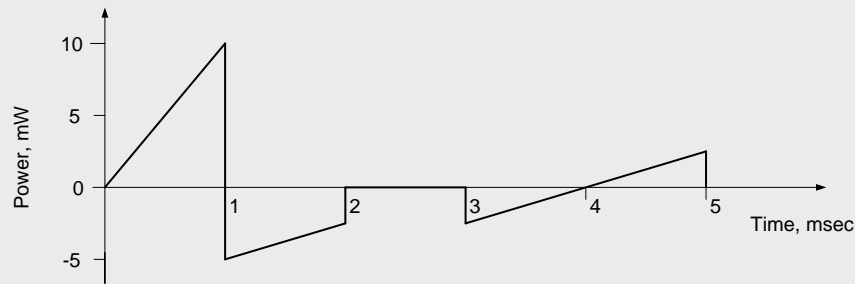


- In the time range  $0 < t < 1\text{ms}$ , the rate of change of current is 10 A/sec. Thus, from equation (3), the voltage is  $v(t) = (0.1\text{H})(10\text{A} / \text{s}) = 1\text{V}$ .
- In the time range  $1\text{ms} < t < 2\text{ms}$ , the rate of change of current is -5A/sec. The voltage is -0.5V.
- In the time range  $2\text{ms} < t < 3\text{ms}$ , the current is constant and there is no voltage across the inductor.
- In the time range  $3\text{ms} < t < 5\text{ms}$ , the rate of change of current is -5A/sec. The voltage is -0.5V

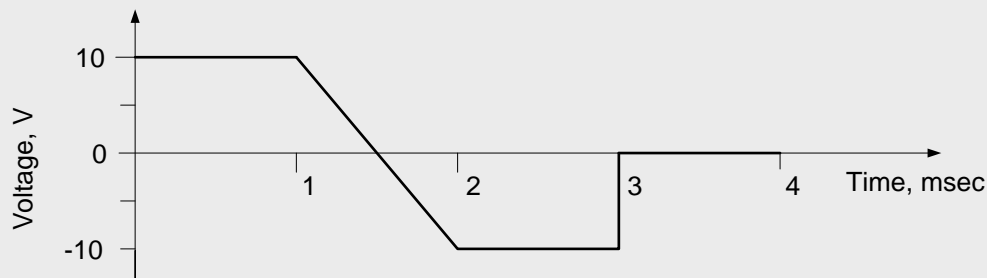
The plot of voltage vs. time is shown below:



Power is the product of voltage and current. If the signs of voltage and current are the same according to the passive sign convention, the circuit element absorbs power. If the signs of voltage and current are not the same, the circuit element generates power. From the above voltage and current curves, the inductor is absorbing power from the circuit during the times  $0 < t < 1\text{ms}$  and  $4\text{ms} < t < 5\text{ms}$ . The inductor returns power to the circuit during the times  $1\text{ms} < t < 2\text{ms}$  and  $3\text{ms} < t < 4\text{ms}$ .

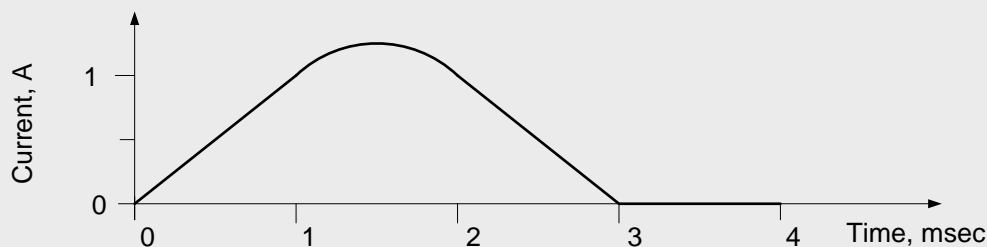


Example 2: If the voltage as a function of time across an inductor with inductance  $L = 10 \text{ mH}$  is as shown below, determine the current as a function of time through the capacitor. Assume that the current through the capacitor is  $0\text{A}$  at time  $t=0$ .



- At time  $t=0$ , the current is given to be  $0\text{A}$ .
- In the time period  $0 < t < 1 \text{ msec}$ , the voltage is constant and positive so the current will increase linearly. The total current change during this time period is the area under the voltage curve, divided by the inductance:  $\frac{1}{0.01} (10\text{V})(1 \times 10^{-3}\text{s}) = 1\text{A}$
- In the time period  $1 < t < 2 \text{ msec}$ , the voltage is decreasing linearly. The current during this time period is a quadratic curve, concave downward. The maximum value of current is  $1.25\text{A}$ , at  $t=1.5 \text{ msec}$ . The current at the end of this time period is  $1\text{A}$ .
- In the time period  $2 < t < 3 \text{ seconds}$ , the voltage is constant at  $-10\text{V}$ . The current change during this time period is the area under the voltage curve, divided by the inductance:  $\frac{1}{0.01} (-10\text{V})(1 \times 10^{-3}\text{s}) = -1\text{A}$ . The total current at  $t=3 \text{ seconds}$  is, then,  $1\text{A} - 1\text{A} = 0\text{A}$ .
- In the time period  $3 < t < 4 \text{ seconds}$ , the voltage is zero. The integral of zero over any time period is zero, so there is no change in current during this time range and the current remains constant at  $0\text{A}$ .

A plot of the current through the inductor as a function of time is shown below.



It is often useful, when analyzing circuits containing inductors, to examine the circuit's response to constant operating conditions and to instantaneous changes in operating condition. We examine the inductor's response to each of these operating conditions below:

- Inductor response to constant current:
  - If the current through the inductor is constant, equation (6.28) indicates that the voltage across the inductor is zero. Thus, if the current through the inductor is constant, the inductor is equivalent to a short circuit.
- Inductor response to instantaneous current changes:
  - If the current through the inductor changes instantaneously, the rate of change of current is infinite. Thus, by equation (6.28), if we wish to change the current through an inductor instantaneously, we must supply infinite voltage to the inductor. This implies that infinite power is available, which is not physically possible. Thus, in any practical circuit, the current through an inductor cannot change instantaneously.
  - Any circuit that allows an instantaneous change in the current through an ideal inductor is not physically realizable. We may sometimes assume, for mathematical convenience, that an ideal inductor's current changes suddenly; however, it must be emphasized that this assumption requires an underlying assumption that infinite power is available and is thus not an allowable operating condition in any physical circuit.

### Important Inductor Properties

- Inductors can be replaced by short-circuits, under circumstances when all operating conditions are constant.
- Currents through inductors cannot change instantaneously. No such requirement is placed on voltages.

## 6.4.2 Energy Storage

The instantaneous power dissipated by an electrical circuit element is the product of the voltage and current:

$$p(t) = v(t) \cdot i(t) \quad \text{Eq. 6.30}$$

Using equation (6.28) to write the voltage in equation (6.30) in terms of the inductor's current:

$$p(t) = L \cdot i(t) \frac{di(t)}{dt} \quad \text{Eq. 6.31}$$

As was previously done for capacitors, we integrate the power with respect to time to get the energy stored in the inductor:

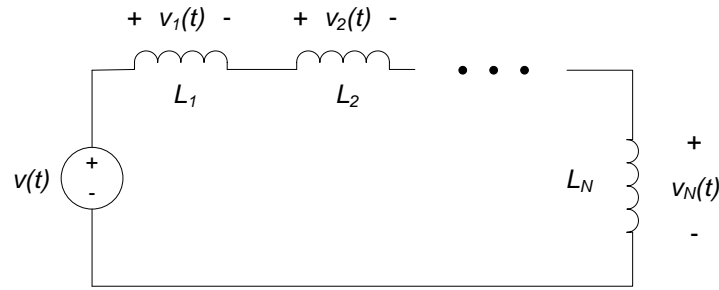
$$W_L(t) = \sum_{-\infty}^t Li(\xi) \frac{di(\xi)}{dt} dt$$

Which, after some manipulation (comparable to the approach taken when we calculated energy storage in capacitors), results in the following expression for the energy stored in an inductor:

$$W_L(t) = \frac{1}{2} Li^2(t) \quad \text{Eq. 6.32}$$

## 6.4.3 Inductors in Series

Consider the series connection of  $N$  inductors shown in Fig. 6.30.

Figure 6.30. Series connection of  $N$  inductors.

Applying Kirchhoff's voltage law around the loop results in:

$$v(t) = v_1(t) + v_2(t) + \cdots v_N(t) \quad \text{Eq. 6.33}$$

Using equation (6.28) to write the inductor voltage drops in terms of the current through the loop gives:

$$\begin{aligned} v(t) &= L_1 \frac{di(t)}{dt} + L_2 \frac{di(t)}{dt} + \cdots + L_N \frac{di(t)}{dt} \\ &= (L_1 + L_2 + \cdots L_N) \frac{di(t)}{dt} \end{aligned}$$

Using summation notation results in:

$$v(t) = \left( \sum_{k=1}^N L_k \right) \frac{di(t)}{dt} \quad \text{Eq. 6.34}$$

This is the same equation that governs the circuit of Fig. 6.31, if:

$$L_{eq} = \sum_{k=1}^N L_k$$

Thus, the equivalent inductance of a series combination of inductors is simply the sum of the individual inductances. This result is analogous to the equations which provide the equivalent resistance of a series combination of resistors.

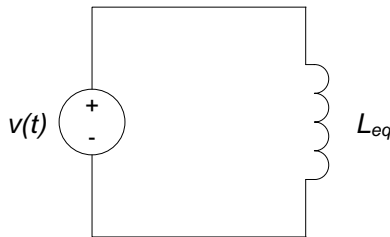
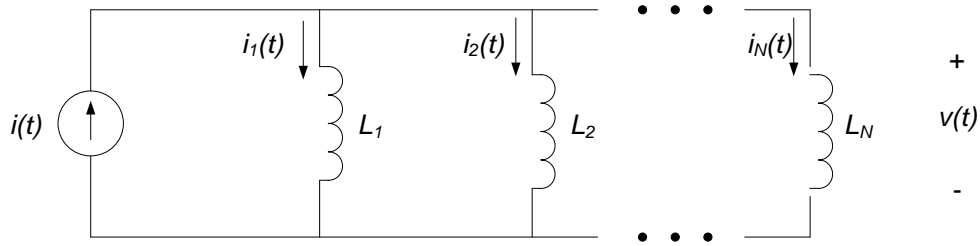


Figure 6.31. Equivalent circuit to Figure 3.

## 6.4.4 Inductors in Parallel

Consider the parallel combination of  $N$  inductors, as shown in Fig. 6.32.

Figure 6.32. Parallel combination of  $N$  inductors.

Applying Kirchhoff's current law at the upper node results in:

$$i(t) = i_1(t) + i_2(t) + \cdots i_N(t) \quad \text{Eq. 6.36}$$

Using equation (6.29) to write the inductor currents in terms of their voltage drops gives:

$$\begin{aligned} i(t) &= \left[ \frac{1}{L_1} \int_{t_0}^t v(\xi) d\xi + i_1(t_0) \right] + \left[ \frac{1}{L_2} \int_{t_0}^t v(\xi) d\xi + i_2(t_0) \right] + \cdots + \left[ \frac{1}{L_N} \int_{t_0}^t v(\xi) d\xi + i_N(t_0) \right] \\ &= \left[ \frac{1}{L_1} \int_{t_0}^t v(\xi) d\xi + \frac{1}{L_2} \int_{t_0}^t v(\xi) d\xi + \cdots \frac{1}{L_N} \int_{t_0}^t v(\xi) d\xi \right] + [i_1(t_0) + i_2(t_0) + \cdots + i_N(t_0)] \\ &= \left( \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_N} \right) \int_{t_0}^t v(\xi) d\xi + i(t_0) \end{aligned}$$

This can be re-written using summation notation as:

$$i(t) = \left( \sum_{k=1}^N \frac{1}{L_k} \right) \int_{t_0}^t v(\xi) d\xi + i(t_0) \quad \text{Eq. 6.37}$$

This is the same equation that governs the circuit of Fig. 6.31, if:

$$\frac{1}{L_{eq}} = \frac{L_1 L_2}{L_1 + L_2} \quad \text{Eq. 6.39}$$

Equations (6.38) and (6.39) are analogous to the equations which provide the equivalent resistance of parallel combinations of resistors.

## Summary: Series and Parallel Inductors

- The equivalent inductance of a series combination of inductors  $L_1, L_2, \dots, L_N$  is governed by a relation which is analogous to that providing the equivalent resistance of a series combination of resistors:

$$L_{eq} = \sum_{k=1}^N L_k$$

- The equivalent inductance of a parallel combination of inductors  $L_1, L_2, \dots, L_N$  is governed by a relation which is analogous to that providing the equivalent resistance of a parallel combination of resistors:

$$\frac{1}{L_{eq}} = \sum_{k=1}^N \frac{1}{L_k}$$

## 6.5 Practical Inductors

Most commercially available inductors are manufactured by winding wire in various coil configurations around a core. Cores can be a variety of shapes; Fig. 6.28 in this chapter shows a core, which is basically a cylindrical bar. Toroidal cores are also fairly common – a closely wound toroidal core has the advantage that the magnetic field is confined nearly entirely to the space inside the winding.

Inductors are available with values from less than 1 micro-Henry ( $1\mu H = 10^{-6}$  Henries) up to tens of Henries. A 1H inductor is very large; inductances of most commercially available inductors are measured in millihenries ( $1mH = 10^{-3}$  Henries) or microhenries. Larger inductors are generally used for low-frequency applications (in which the signals vary slowly with time).

Attempts at creating inductors in integrated-circuit form have been largely unsuccessful; therefore many circuits that are implemented as integrated circuits do not include inductors. Inclusion of inductance in the analysis stage of these circuits may however, be important. Since any current-carrying conductor will create a magnetic field, the *stray inductance* of supposedly non-inductive circuit elements can become an important consideration in the analysis and design of a circuit.

Practical inductors, unlike the ideal inductors discussed in this chapter, dissipate power. An equivalent circuit model for a practical inductor is generally created by placing a resistance in series with an ideal inductor, as shown in Fig. 6.33.

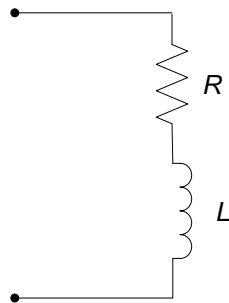


Figure 6.33. Equivalent circuit model for a practical inductor.

### Section Summary

- Inductors store magnetic energy. This energy is stored in a magnetic field (typically) generated by a coiled wire wrapped around a core material.
- Inductor energy storage is dependent upon the current through the inductor, if the inductor current is known, the energy in the inductor is known.
- The voltage-current relationship for an inductor is:

$$v(t) = L \frac{di(t)}{dt}$$

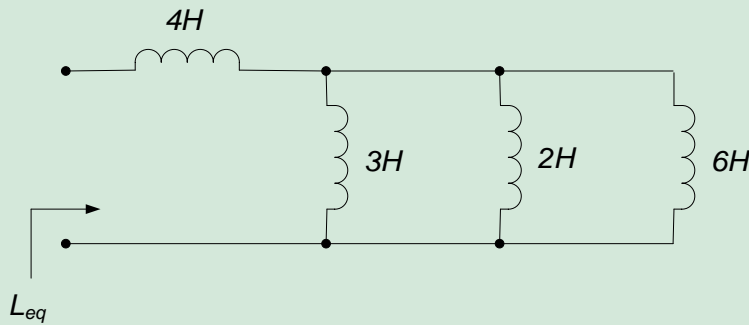
Where L is the inductance of the inductor. Units of inductance are Henries (abbreviated H). The inductance of an inductor, very roughly speaking, gives an indication of how much energy it can store.

- The above voltage-current relation results in the following important properties of inductors:
  - If the inductor current is constant, the voltage across the inductor is zero. Thus, if the inductor current is constant, the inductor can be modeled as a short circuit.

- Changing the inductor current instantaneously requires infinite power. Thus (for now, anyway) we will assume that inductors cannot instantaneously change their current.
- Inductors placed in series or parallel with one another can be modeled as a single equivalent inductance. Thus, inductors in series or in parallel are not “independent” energy storage elements.

## 6.4 Exercises

1. Determine the equivalent inductance of the network below:



## Real Analog Chapter 6: Lab Projects

### 6.2.1: Time-varying Signals

This assignment will focus on using an arbitrary waveform generator to generate time-varying signals and using an oscilloscope to measure time varying signals.

In chapter 6 of the text book, we deal analytically only with step functions and exponential functions. This lab will, however introduce us to a larger class of time-varying waveforms.

The ability to apply and measure time varying signals will be crucial throughout the remainder of your career. It is strongly recommended that you not only complete the specific steps outlined in this assignment, but that you spend some additional time “playing with” the tools we introduce in this assignment – it is guaranteed to be time well spent!

#### Before beginning this lab, you should be able to:

- Define a step function.
- State Ohm’s law for time-varying signals

#### After completing this lab, you should be able to:

- Use a switch to create a step function
- Use the Analog Discovery waveform generator to apply square, triangular, and sinusoidal waveforms
- Use the Analog Discovery oscilloscope to measure and display time-varying waveforms

#### This lab exercise requires:

- Analog Discovery module
- Digilent Analog Parts Kit

#### Symbol Key:

**DEMO**

Demonstrate circuit operation to teaching assistant; teaching assistant should initial lab notebook and grade sheet, indicating that circuit operation is acceptable.

**ANALYSIS**

Analysis; include principle results of analysis in laboratory report.

**SIM**

Numerical simulation (using PSPICE or MATLAB as indicated); include results of MATLAB numerical analysis and/or simulation in laboratory report.

**DATA**

Record data in your lab notebook.

#### General Discussion:

Once we begin to deal in earnest with systems which include energy storage elements, it will be crucial apply time-varying power to our electrical circuits and measure the circuits’ responses as functions of time. This lab introduces the concepts necessary for application, measurement, and interpretation of time-varying signals.

Since we have not yet been introduced to dynamic systems, the electrical circuit of interest in this assignment will be the voltage divider shown in Figure 1.



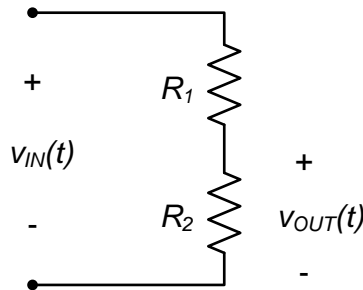


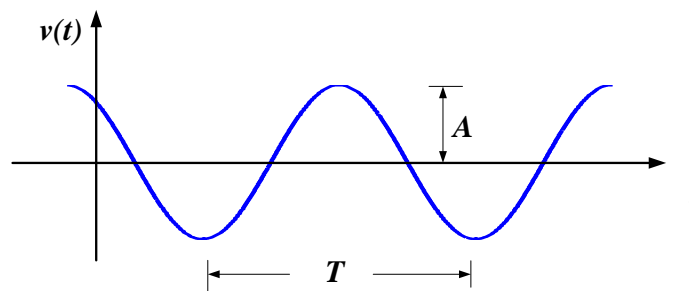
Figure 1. Voltage divider circuit.

In Figure 1, the output voltage,  $v_{OUT}(t)$  is related to the input voltage  $v_{IN}(t)$  via the voltage divider relation:

$$v_{OUT}(t) = v_{IN}(t) \frac{R_2}{R_1 + R_2} \quad \text{Eq. 1}$$

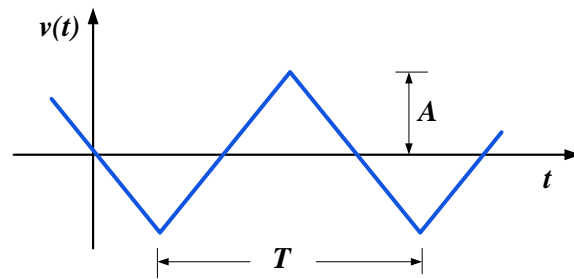
Notice that the relationship between  $v_{IN}(t)$  and  $v_{OUT}(t)$  is *algebraic* – the value of  $v_{OUT}$  at a particular time depends only upon the value of  $v_{IN}$  at that same time.

In order to familiarize ourselves with the fundamentals of applying and measuring time-varying signals, we will restrict ourselves to some of the most common signals encountered in engineering applications: sinusoidal waves, square waves, and triangular waves. The basic shapes of these signals are shown in Fig. 2. The signals of Fig. 2 are all *periodic* signals – that is, they repeat themselves at regular intervals. This interval is called the *period* (commonly denoted mathematically as  $T$ ). The period of each of the signals of interest to us is indicated on Fig. 2. The other primary attribute of the signals we will be dealing with is their amplitude (which we will denote as  $A$ ). The amplitude of the signal is essentially the maximum (and minimum) value that the signal achieves<sup>1</sup>.

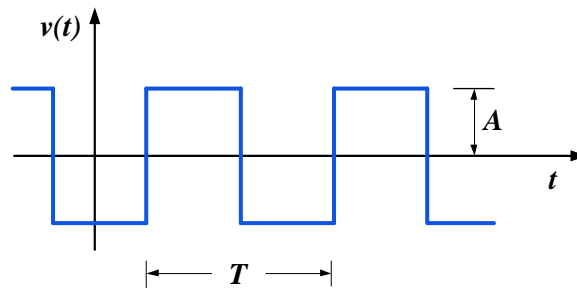


(a) Sinusoidal wave.

<sup>1</sup> For now, our signals will be symmetric with respect to the time axis. That is, their average value (also called the *offset*) will be zero. For the signals of immediate interest to us, this means that their minimum value will be the negative of their maximum value. Later labs will explore the effects of a non-zero offset to the signal, and signals which are not symmetric with respect to the time axis.



(b) Triangular wave



(c.) Square wave

Figure 2. Basic signal shapes.

Although we have used the signal's period as a fundamental parameter defining the signal, it is more common for electrical instruments to use the *frequency* of the signal as a defining characteristic. The frequency provides essentially the same information as the period; the frequency is just the inverse of the period:

$$f = \frac{1}{T} \quad \text{Eq. 2}$$

As defined in equation (2), the units of frequency are in Hertz (abbreviated Hz) or cycles per second. Sinusoidal signals, however, are more accurately defined mathematically in terms of their *radian frequency*, denoted as  $\omega$ . Since there are  $2\pi$  radians in one cycle, the conversion between frequency and radian frequency is:

$$\omega = 2\pi f = \frac{2\pi}{T} \quad \text{Eq. 3}$$

Mathematically, the sinusoidal wave of Fig. 2(a) can be represented as:

$$v(t) = A\cos(\omega t + \theta) = A\cos(2\pi f t + \theta) \quad \text{Eq. 4}$$

Where  $\theta$  is the *phase angle* of the signal; it translates the sinusoid in time. We will concern ourselves with phase later in the course.


### Pre-lab:

In the circuit of Fig. 1, if  $R_1 = R_2$ , overlay sketches using the input and output voltages ( $v_{IN}(t)$  and  $v_{OUT}(t)$ ) for the following cases:

- ANALYSIS** (a)  $v_{IN}(t)$  is a sinusoidal wave with amplitude A and period T.
- ANALYSIS** (b)  $v_{IN}(t)$  is a triangular wave with amplitude A and period T.
- ANALYSIS** (c)  $v_{IN}(t)$  is a square wave with amplitude A and period T.

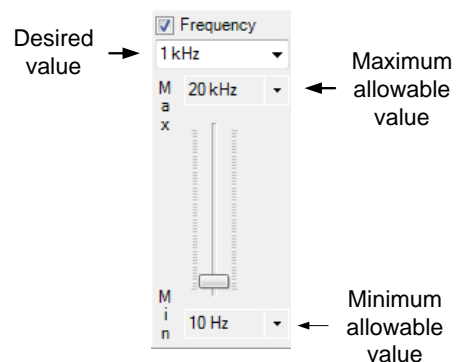
Label the amplitude and period of both the input and output waveforms on your sketch. These values may be functions of  $A$ ,  $T$ ,  $R_1$  and  $R_2$ .

### Lab Procedures:

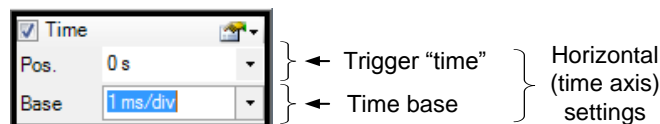
- (a) Test the response of the circuit to a sinusoidal input voltage with 2kHz frequency and 2V amplitude. Details are below:
  - i. Set  $v_{IN}(t)$  in the circuit of Fig. 1 to be a sinusoidal voltage with amplitude 2V and frequency 1kHz across the voltage divider. The average value of the sinusoid should be zero volts. To do this, open the *WaveGen* instrument in the waveforms file. Click on the **Basic** tab (if it is not already selected) and then click on the **Standard** option. There should be a series of icons in a column below this option, indicating the shape of the associated waveform. Click on the  icon to select a sinusoidal waveform. Choose 1kHz as the frequency (you can choose the desired frequency by selecting it from the drop-down menu, typing the desired value in the text box, or using the slider bar) and 2V as the amplitude<sup>2</sup>. The plot window on the waveform generator instrument will display one period the waveform you have set. Use this plot window to double check that your signal has the correct frequency and amplitude.

### Note on selecting parameters:

When choosing parameters describing signals (e.g. frequency, amplitude, offset, and symmetry) the allowable values are limited to the range specified by the values above and below the slider bar, as indicated on the figure to the right for the frequency parameter. When selecting a value, the desired value must be between the maximum and minimum values shown. If you want a value outside the displayed range, simply re-set the range using the appropriate drop-down menus. If the waveform generator will not let you set a desired value, be sure to check that the desired value is within the allowable range.



- ii. Use the oscilloscope to display the voltages  $v_{IN}(t)$  and  $v_{OUT}(t)$  of Figure 1. To do this, open the *Scope* instrument. Set the horizontal scale (or the time axis scale) to 1msec/div. Horizontal axis settings are set in the time axis settings box on the oscilloscope window; this box and the desired settings for this lab are shown below:

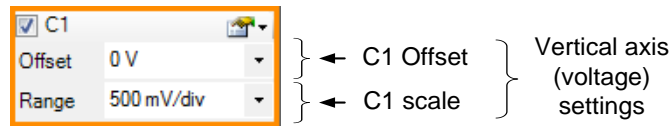



Set the vertical axis settings on both channel 1 and channel 2 (C1 and C2) to 500mv/div.

Vertical axis settings are set in the channel axis settings boxes on the oscilloscope window;


<sup>2</sup> The offset should be zero, the symmetry 50%, and the phase 0 degrees. These are the default values, and should not need to be re-set.

the settings box for channel 1 and its desired settings are shown below. Use the same settings for channel 2.

**DATA**

Click on  to acquire and display the data. Record an image of the oscilloscope main time window to a file for later documentation.

- iii. From the time plots displayed in the oscilloscope window, determine the period and amplitude of  $v_{IN}(t)$  and  $v_{OUT}(t)$ . From your measured period, calculate the signal's frequency in Hertz. Create a table, showing the expected amplitude and frequency of  $v_{IN}(t)$  and  $v_{OUT}(t)$  and your measured amplitude and frequency of  $v_{IN}(t)$  and  $v_{OUT}(t)$ .

- iv. Click on the  Measure button on the oscilloscope window to open a measurements window. Use the measurement window to measure the amplitude, period, and frequency of  $v_{IN}(t)$  and  $v_{OUT}(t)$ . Record the image of the oscilloscope window, showing the waveforms and their measured amplitudes, periods, and frequencies<sup>3</sup>. Comment on the agreement between the oscilloscope's measurements and the measurements you made in part iii above.

**DATA****ANALYSIS****DEMO**

- v. Demonstrate operation of your circuit to the Teaching Assistant. Have the TA initial the appropriate page(s) of your lab notebook and the lab checklist.
- vi. Vary the amplitude and frequency of the sinusoidal waveform using the waveform generator. Change the horizontal and vertical axis scales in the oscilloscope. Verify that the changes result in data that agree with your expectations. Familiarizing yourself with these instruments now will be rewarded in later experiments – you can only interpret the results of future experiments if you are comfortable with measuring the data upon which the results depend!

**ANALYSIS**

- (b) Test the response of the circuit to a triangular input voltage with 1kHz frequency and 3V amplitude.

- i. Perform all the steps you did above for the sinusoidal input.

**DEMO**


- ii. Demonstrate operation of your circuit to the Teaching Assistant. Have the TA initial the appropriate page(s) of your lab notebook and the lab checklist.

- (c) Test the response of the circuit to a square wave input voltage with 500Hz frequency and 2.5V amplitude.

- i. Perform all the steps you did above for the sinusoidal input.

**DEMO**

- ii. Demonstrate operation of your circuit to the Teaching Assistant. Have the TA initial the appropriate page(s) of your lab notebook and the lab checklist.

<sup>3</sup> Holding down the “Alt” key and pressing “Print Screen” (commonly labeled as “PrtScn” or “PrtSc” on computer keyboards) will copy the currently active window to the clipboard. You can then paste this image to a document. The  Export button on the oscilloscope instrument allows you to copy an image of the main time window to the clipboard or save it to a file in a variety of formats. This option will not, however, display the measurement window – if you use this approach, you will want to record the measured values elsewhere.

## Real Analog Chapter 6: Lab Worksheets

### 6.2.1: Time-varying Signals (40 points total)

1. Attach to this worksheet the input and output voltage sketches you created in the pre-lab for sinusoidal, triangular, and square waves. (7 pts)
2. Attach to this worksheet an image of the oscilloscope window, showing the sinusoidal waveforms and their measured amplitudes, periods, and frequencies. In the space below, provide the amplitudes, periods, and frequencies determined directly from the time plot in the oscilloscope window. Comment on the agreement between the two sets of data. (8 pts)

3. **DEMO:** Have a teaching assistant initial this sheet, indicating that they have observed your circuit's operation for sinusoidal inputs. (3 pts)

**TA Initials:** \_\_\_\_\_

4. Attach to this worksheet an image of the oscilloscope window, showing the triangular waveforms and their measured amplitudes, periods, and frequencies. In the space below, provide the amplitudes, periods, and frequencies determined directly from the time plot in the oscilloscope window. Comment on the agreement between the two sets of data. (8 pts)

5. **DEMO:** Have a teaching assistant initial this sheet, indicating that they have observed your circuit's operation for triangular inputs. (3 pts)

**TA Initials:** \_\_\_\_\_

6. Attach to this worksheet an image of the oscilloscope window, showing the square waveforms and their measured amplitudes, periods, and frequencies. In the space below, provide the amplitudes, periods, and frequencies determined directly from the time plot in the oscilloscope window. Comment on the agreement between the two sets of data. (8 pts)

7. **DEMO:** Have a teaching assistant initial this sheet, indicating that they have observed your circuit's operation for square wave inputs. (3 pts)

**TA Initials:** \_\_\_\_\_

## Real Analog Chapter 6: Lab Projects

### 6.3.1: Capacitor Voltage-current Relations

In this assignment, we will measure the relationship between the voltage difference across a capacitor and the current passing through it. We will apply several types of time-varying signals to a series combination of a resistor and a capacitor. The voltage difference across the resistor, in conjunction with Ohm's law, will provide an estimate of the current through the capacitor. This current can be related to the voltage difference across the capacitor.

The results of our voltage-current measurements will be compared to analytical expectations.

#### Before beginning this lab, you should be able to:

- State voltage-current relationships for capacitors in both differential and integral form
- Apply the capacitor voltage-current relations to calculate a capacitor's voltage from its current and vice-versa
- Use the Analog Discovery's arbitrary waveform generator and oscilloscope to apply and measure time-varying waveforms (Lab 6.2.1)

#### After completing this lab, you should be able to:

- Use the Analog Discovery oscilloscope's math function to calculate the current through a known resistor from the measured voltage difference.
- Verify a capacitor's voltage-current relations using measured data

#### This lab exercise requires:

- Analog Discovery module
- Digiilent Analog Parts Kit

#### Symbol Key:

##### DEMO

Demonstrate circuit operation to teaching assistant; teaching assistant should initial lab notebook and grade sheet, indicating that circuit operation is acceptable.

##### ANALYSIS

Analysis; include principle results of analysis in laboratory report.

##### SIM

Numerical simulation (using PSPICE or MATLAB as indicated); include results of MATLAB numerical analysis and/or simulation in laboratory report.

##### DATA

Record data in your lab notebook.

#### General Discussion:

We will use the circuit of Figure 1 in this lab assignment. Both the voltage difference across the capacitor and the resistor ( $v_C(t)$  and  $v_R(t)$ ) will be measured. From this data, we can readily compare the voltage across the capacitor with the current through the capacitor. Since the voltage across the resistor is measured, we can readily infer the current through the resistor via Ohm's law:

$$i_R(t) = \frac{v_R(t)}{R} \quad \text{Eq. 1}$$

The resistor and capacitor are in series, so the current through the capacitor is the same as the current through the resistor, so:

$$i_C(t) = \frac{v_R(t)}{R} \quad \text{Eq. 2}$$

Since we are also measuring the voltage difference across the capacitor,  $v_C(t)$ , we can readily compare these parameters with our expectations based on our mathematical models of the capacitor voltage-current relationships.

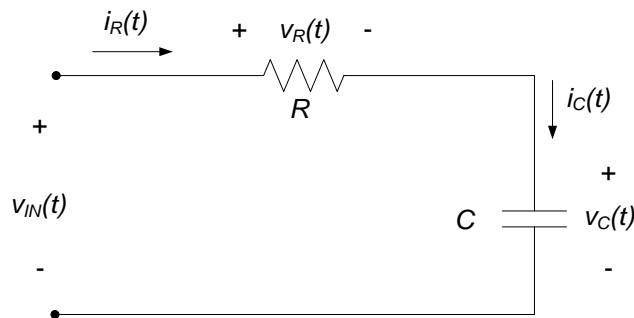


Figure 1. Series RC circuit.

### Pre-lab:

For the circuit of Figure 1, use the inductor voltage-current relations to overlay sketches of the capacitor voltage and the capacitor current ( $v_C(t)$  and  $i_C(t)$ ) if the capacitor voltage is:

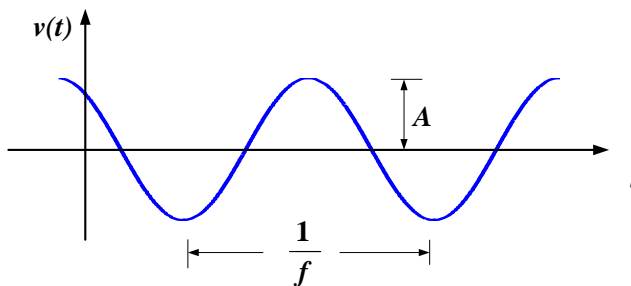
#### ANALYSIS

(a) A sinusoidal wave,  $v(t)$ , with frequency ( $f$ ) and amplitude ( $A$ ) as shown in Figure 2 (a)

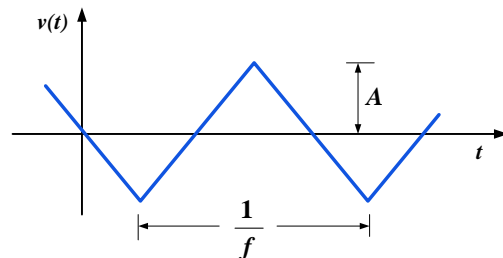
#### ANALYSIS

(b) A triangular wave,  $v(t)$ , with frequency ( $f$ ) and amplitude ( $A$ ) as shown in Figure 2 (b).

Label your sketch to show the amplitude and period of the capacitor current for both of the above cases. Your results may be dependent up on the parameters  $A$ ,  $f$ ,  $R$ , and  $C$ . Be sure that your sketches of voltage and current share the same time axis!



(a) Sinusoidal waveform



(b) Triangular waveform

### Lab Procedures:

Construct the circuit of Figure 1, using  $R = 100\Omega$  and  $C = 1\mu\text{F}$ . Use channel 1 of your oscilloscope to measure the resistor voltage difference, and channel 2 of your oscilloscope to measure the capacitor voltage difference. Use channel 1 of your waveform generator (W1) to apply the voltage  $v_{in}(t)$  in Figure 1. Set up a math channel to calculate the current through the capacitor per equation (2) in the

pre-lab<sup>4</sup>. Set the oscilloscope measurements to provide at least the amplitude of each of the three displayed waveforms.

**DATA**

1. Apply a sinusoidal input voltage with frequency = 1kHz, amplitude = 2V, and offset = 0V to the circuit of Figure 1. Use your oscilloscope to display the data listed above (waveforms corresponding to C1, C2, and M1; measurement window displaying amplitudes of C1, C2, and M1). Record the image of the oscilloscope window, showing the waveforms and their measured amplitudes.

**DATA**

2. Apply a sinusoidal input voltage with frequency = 2 kHz, amplitude = 2V, and offset = 0V to the circuit of Figure 1. Use your oscilloscope to display the data listed above (waveforms corresponding to C1, C2, and M1; measurement window displaying amplitudes of C1, C2, and M1). Record the image of the oscilloscope window, showing the waveforms and their measured amplitudes.

**DATA**

3. Apply a triangular input voltage with frequency = 100 Hz, amplitude = 4V, and offset = 0V to the circuit of Figure 1. Use your oscilloscope to display the data listed above (waveforms corresponding to C1, C2, and M1; measurement window displaying amplitudes of C1, C2, and M1). Record the image of the oscilloscope window, showing the waveforms and their measured amplitudes.

**DEMO**

4. Demonstrate operation of your circuit to the Teaching Assistant. Have the TA initial the appropriate page(s) of your lab notebook and the lab checklist.

**Post-lab Exercises:****ANALYSIS**

For the three cases in the lab procedures (1kHz sinusoid, 2kHz sinusoid, 100Hz triangular wave), use your pre-lab results to sketch the expected capacitor current waveforms corresponding to the capacitor voltage waveforms you measured in the lab procedures. Comment briefly on the agreement between the measured and expected capacitor currents for each of these cases. In your comments, be sure to include a quantitative comparison (including percent difference) between the expected and measured amplitudes of the capacitor

---

<sup>4</sup> Detailed instructions for doing this are provided in Appendix A.



## Real Analog Chapter 6: Lab Worksheets

### 6.3.1: Capacitor Voltage-current Relations (35 points total)

1. Attach to this worksheet the sketches of the capacitor voltage and current for both sinusoidal and triangular inputs. (6 pts)
2. Attach to this worksheet an image of the oscilloscope window, showing the capacitor voltage and current waveforms and the measured amplitudes of the waveforms for a 1kHz sinusoidal input. (8 pts)
3. Attach to this worksheet an image of the oscilloscope window, showing the capacitor voltage and current waveforms and the measured amplitudes of the waveforms for a 2kHz sinusoidal input. (8 pts)
4. Attach to this worksheet an image of the oscilloscope window, showing the capacitor voltage and current waveforms and the measured amplitudes of the waveforms for a 100Hz triangular input. (8 pts)
5. **DEMO:** Have a teaching assistant initial this sheet, indicating that they have observed your circuit's operation for the triangular input. (5 pts)

TA Initials: \_\_\_\_\_

## Real Analog Chapter 6: Lab Projects

### 6.3.2: Leakage Currents and Electrolytic Capacitors

Voltage-current relationships for ideal capacitors do not always adequately explain measured capacitor behavior. In this assignment, we will focus on the effects of *leakage currents* on capacitor behavior. As we saw in our discussion of non-ideal capacitors in section 6.3 of the text, models for realistic capacitors often include a resistor in parallel with an ideal capacitor; this resistor allows us to model leakage currents, which explain – among other effects – the inability of a capacitor to hold a charge indefinitely, even if the capacitor terminals are open-circuited.

An important effect of leakage currents is in the case of *electrolytic* capacitors. These capacitors are attractive in many cases, since a relatively large capacitance can be provided in a small package. However, one must be aware, when using electrolytic capacitors, that their leakage currents can be significant and that they are not symmetric relative to the capacitor's polarity. Thus, reversing the polarity of the capacitor in a circuit can alter the behavior of the capacitor – this makes electrolytic capacitors undesirable in some applications (such as filtering) in which the behavior of the capacitor should be independent of the polarity of the voltage applied to the capacitor.

#### Before beginning this lab, you should be able to:

- State voltage-current relationships for capacitors in both differential and integral form
- Define the time constant of an exponential waveform
- Use the Analog Discovery to apply and measure time-varying waveforms (Lab 6.2.1)





#### After completing this lab, you should be able to:

- Model a non-ideal capacitor as an ideal capacitor in parallel with a resistor
- Identify some effects of non-ideal capacitors
- Describe how the polarity of an electrolytic capacitor affects the capacitor's leakage current.

#### This lab exercise requires:

- Analog Discovery module
- Diligent Analog Parts Kit

#### Symbol Key:

|   |   |
|---|---|
|  | Demonstrate circuit operation to teaching assistant; teaching assistant should initial lab notebook and grade sheet, indicating that circuit operation is acceptable. |
|  | Analysis; include principle results of analysis in laboratory report.   |
|  | Numerical simulation (using PSPICE or MATLAB as indicated); include results of MATLAB numerical analysis and/or simulation in laboratory report.                      |
|  | Record data in your lab notebook.   |

#### General Discussion:

In large part, this lab will be concerned with *electrolytic* capacitors. Electrolytic capacitors are polarized – that is, one of their terminals is intended to always be at a higher voltage than the other. The terminal which is intended to be at the higher voltage is called the *anode*, while the terminal which is to be at the lower polarity is the *cathode*. A symbol for an electrolytic capacitor is shown in Figure 1 – the cathode side is indicated as a curved line. Physically, electrolytic capacitors are readily identifiable: the lead connected to the anode is a longer wire than

that of the cathode, and (if the capacitor is physically large enough) a bar is printed on the cathode side of the capacitor<sup>5</sup>.

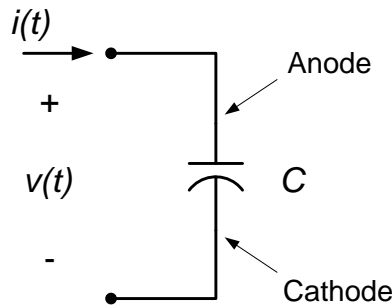


Figure 1. Electrolytic capacitor circuit symbol.

Electrolytic capacitors are desirable in that their capacitance can be large relative to their volume. However, they also have some undesirable qualities. Chief among these is that they can fail rather spectacularly if the cathode voltage is significantly higher than the anode voltage for an extended period of time. More subtle drawbacks include the fact that leakage currents can be large if the polarity of the capacitor is reversed<sup>6</sup>. It is this latter characteristic that we will explore in this assignment.

### Caution:

In order to explore leakage effects, we will be applying voltages with the opposite polarity as required by the electrolytic capacitor. Due to the voltage levels we will use, it is unlikely that we will cause a failure of the capacitor. However, it is recommended that you wear eye protection while doing this lab assignment.

In this lab assignment, we will measure the voltage across an electrolytic capacitor for both of the cases shown in Fig. 2. In Fig. 2(a), the polarity of the capacitor is correct; the anode is always at the higher voltage. In Fig. 2(b), the polarity of the capacitor is reversed – the cathode is now at the higher voltage. We will use a “switch” to change the voltage applied to the capacitor – our switch will be implemented simply by unplugging the positive voltage terminal of our power supply from the rest of the circuit.

The resistor  $R$  in the circuit of Fig. 2 simply limits the amount of current the capacitor demands when it is being initially charged. Capacitors require a large amount of current to charge rapidly; without the resistor, the capacitor will attempt to draw more current from the power supply than is available.

<sup>5</sup> In addition, the bar may have a negative sign printed on it, which further indicates that the cathode is to be at the lower (or negative) voltage.

<sup>6</sup> The non-symmetry of the leakage currents relative to capacitor voltage makes electrolytic capacitors poor choices for filter circuits. Filter circuits typically require their operation to be identical for positive and negative voltage inputs, unless special biasing schemes are introduced to ensure that the voltage polarity does not change.

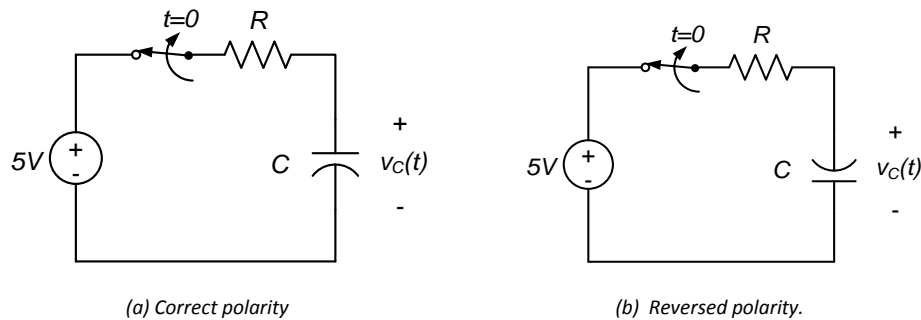




Figure 2. Capacitor configurations used in this lab.

**Pre-lab:**

None

**Lab Procedures:**

1. Using  $R = 100\Omega$  and  $C = 10\mu\text{F}$ , implement the circuit of Figure 2(a). (Recall that, in Figure 2(a), the anode is at the higher voltage. Thus, the capacitor terminal with the longer lead is connected to the resistor  $R$  and the shorter lead is connected to ground.) Use  $V+$  to apply the 5V supply.
  - i. Use channel 1 of your oscilloscope to measure the voltage across the capacitor,  $v_C(t)$ . We will be monitoring the amount of time required for the capacitor to discharge once we open the switch in Figure 2(a); this will take a relatively long time, so set the time scale on your oscilloscope to 5 s/div. Set the vertical scale of your scope to 1 V/div, with a -2V offset.
  - ii. Turn on the power supply and click  to start acquisition of data. The oscilloscope should indicate a +5V voltage across the capacitor.
  - iii. Open the “switch” in Figure 2(a) by unplugging the power supply terminal from the circuit. (Simply pull the  $V+$  connector out of the breadboard.) The capacitor voltage displayed on the oscilloscope screen should decay exponentially. Measure the time constant of the waveform<sup>7</sup>. Record the image of the oscilloscope window, showing the waveform.
  - iv. Demonstrate operation of your circuit to the Teaching Assistant. Have the TA initial the appropriate page(s) of your lab notebook and the lab checklist.
2. Still using  $R = 100\Omega$  and  $C = 10\mu\text{F}$ , implement the circuit of Figure 2(b). (In Figure 2(b), the cathode is at the higher voltage. Thus, the capacitor terminal with the shorter lead is connected to the resistor  $R$  and the longer lead is connected to ground.) Use  $V+$  to apply the 5V supply. Note that this circuit can be easily created from the circuit of Figure 2(a) by removing the capacitor, reversing the leads, and replacing it again.
  - i. Measure the voltage across the capacitor,  $v_C(t)$ , as in part 1. Turn on the power supply and click  to start acquisition of data. The oscilloscope should indicate a +5V voltage across the capacitor.
  - ii. Open the “switch” in Figure 2(b) by unplugging the power supply terminal from the circuit. (Simply pull the  $V+$  connector out of the breadboard.) The capacitor voltage displayed on the oscilloscope screen should decay approximately exponentially. Measure the time constant of the waveform. Record the image of the oscilloscope window, showing the waveform.

**DATA****DEMO****DATA**

<sup>7</sup> Recall that the time constant is the amount of time required for an exponential waveform to decay to 36.8% of its initial value.

**DEMO**

- iii. Demonstrate operation of your circuit to the Teaching Assistant. Have the TA initial the appropriate page(s) of your lab notebook and the lab checklist.

3. In the circuits of Figure 2, there is no way for the capacitor's voltage to decay – the charge difference on the plates cannot “leak” away, since there is (in our model, anyway) no path for the charge to get from one plate to the other. The circuits of Figure 2 are not realistic, based on the capacitor behavior we observed in parts 1 and 2 above. We must modify how we think about capacitors' behavior in order to explain our data!

In section 6.3 of the textbook, we modeled non-ideal capacitors as a resistance in parallel with an ideal capacitor. Using this model, the circuits of Figure 2 can be modified to become Figure 3 below. Now there is a path – the resistor  $R_C$  – which allows the capacitor voltage to decay after the switch opens, allowing us to explain our previous data!

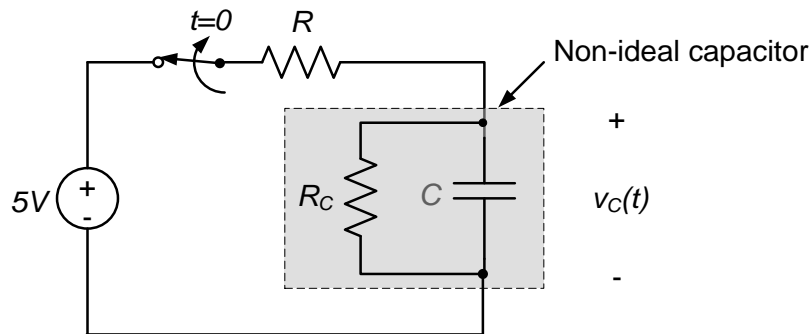


Figure 3. Circuit of Figures 2, with non-ideal capacitor.

As we observed in parts 1 and 2 above, the leakage rate in electrolytic capacitors changes, based on the polarity of the capacitor voltage. In essence, this means that the capacitor resistance,  $R_C$ , in the model of Figure 3 depends on the polarity of the capacitor voltage<sup>8</sup>!

To obtain an idea as to the variation in the capacitor resistance when the capacitor polarity is reversed, let's normalize the time constants we measured in parts 1 and 2 above. To do this, simply divide the time constant by the capacitance value:

$$R_C \approx \frac{\tau}{C} \quad \text{Eq. 1}$$

As we will see later, the units of equation (1) are consistent.

**ANALYSIS**

- i. Tabulate the results you obtained in parts 1, 2, and 3 above – for each polarity, list the measured time constant, and the estimated capacitor resistance as determined by equation (1). Briefly comment on the magnitude of these resistances and the differences between the capacitor resistances for the two cases. (Include a percent change in resistance induced by changing the capacitor polarity.)

**Post-lab Exercises:****ANALYSIS**

Suppose that we modify the circuit of Figure 2(a) so that there is a path for the capacitor to dissipate its voltage after the switch opens, as shown in Figure 4. Based on the capacitor resistances you estimated in part (c) of the lab procedures, how large would the resistor  $R$  need to be in order for the capacitor resistance  $R_C$  to change the rate at which the capacitor voltage dissipates by about 10%?

<sup>8</sup> This makes modeling electrolytic capacitors whose voltage changes polarity tedious, to say the least.

(E.g. how large would  $R$  need to be before the measured time constant of the capacitor voltage decay changes by 10% when the resistance  $R_C$  is included?)

Hint: the resistances  $R$  and  $R_C$  are in parallel. They can be combined to a single equivalent resistance seen by the capacitor.

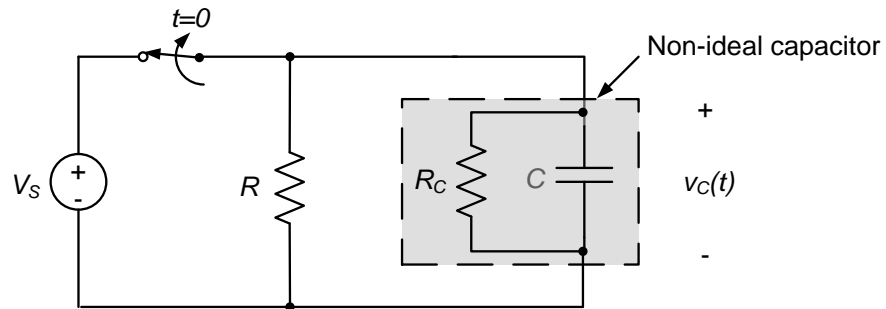


Figure 4. Circuit with external resistor connected across non-ideal capacitor.

## Real Analog Chapter 6: Lab Worksheets

### 6.3.2: Electrolytic Capacitor Leakage Currents (35 points total)

1. Attach to this worksheet an image of the oscilloscope window, showing the capacitor voltage, resulting from opening the switch in Fig. 2(a). (5 pts)
2. **DEMO:** Have a teaching assistant initial this sheet, indicating that they have observed your circuit's operation when the electrolytic capacitor is connected with the correct polarity. (5 pts)

TA Initials: \_\_\_\_\_

3. Attach to this worksheet an image of the oscilloscope window, showing the capacitor voltage, resulting from opening the switch in Figure 2(b). In the space below, provide your estimate of the time constant of the circuit. (5 pts)
4. **DEMO:** Have a teaching assistant initial this sheet, indicating that they have observed your circuit's operation when the electrolytic capacitor is connected in reversed polarity. (5 pts)

TA Initials: \_\_\_\_\_

5. In the space below, provide a table giving your estimated time constants and the calculated capacitor resistance for both capacitor polarities. Comment on the differences between the two cases, including a percent change in resistance. (8 pts)
6. In the space below, provide your estimate of the resistance required in the circuit of Figure 4 which changes the time constant of the capacitor by 10%. (7 pts)

## Real Analog Chapter 6: Lab Projects

### 6.4.1: Inductor Voltage-current Relations

In this assignment, we will measure the relationship between the voltage difference across a capacitor and the current passing through it. We will apply several types of time-varying signals to a series combination of a resistor and a capacitor. The voltage difference across the resistor, in conjunction with Ohm's law, will provide an estimate of the current through the capacitor. This current can be related to the voltage difference across the capacitor.

The results of our voltage-current measurements will be compared to analytical expectations.

#### Before beginning this lab, you should be able to:

- State voltage-current relationships for inductors in both differential and integral form
- Apply the inductor voltage-current relations to calculate a inductor's voltage from its current and vice-versa
- Use the Analog Discovery's arbitrary waveform generator and oscilloscope to apply and measure time-varying waveforms (Lab 6.2.1)

#### After completing this lab, you should be able to:

- Use the Analog Discovery oscilloscope's math function to calculate the current through a known resistor from the measured voltage difference.
- Export data acquired by the Analog Discovery to files for post-processing by other programs
- Verify a inductor's voltage-current relations using measured data

#### This lab exercise requires:

- Analog Discovery module
- Digiilent Analog Parts Kit

#### Symbol Key:

|                 |   |
|-----------------|---|
| <b>DEMO</b>     | Demonstrate circuit operation to teaching assistant; teaching assistant should initial lab notebook and grade sheet, indicating that circuit operation is acceptable. |
| <b>ANALYSIS</b> | Analysis; include principle results of analysis in laboratory report.   |
| <b>SIM</b>      | Numerical simulation (using PSPICE or MATLAB as indicated); include results of MATLAB numerical analysis and/or simulation in laboratory report.                      |
| <b>DATA</b>     | Record data in your lab notebook.   |

#### General Discussion:

We will use the circuit of Figure 1 in this lab assignment. Both the voltage difference across the inductor and the resistor ( $v_L(t)$  and  $v_R(t)$ ) will be measured. From this data, we can readily compare the voltage across the inductor with the current through the inductor. Since the voltage across the resistor is measured, we can readily infer the current through the resistor via Ohm's law:

$$i_R(t) = \frac{v_R(t)}{R} \quad \text{Eq. 1}$$

The resistor and inductor are in series, so the current through the inductor is the same as the current through the resistor, so:

$$i_L(t) = \frac{v_R(t)}{R} \quad \text{Eq. 2}$$



Since we are also measuring the voltage difference across the inductor,  $v_L(t)$ , we can readily compare these parameters with our expectations based on our mathematical models of the capacitor voltage-current relationships.

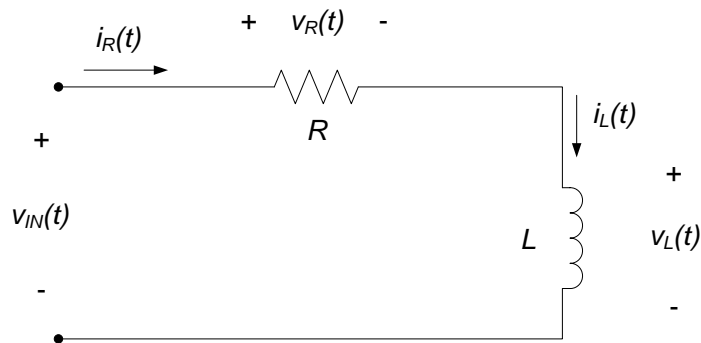


Figure 1. Series RL circuit.

### Pre-lab:

In this lab, we will apply sinusoidal signals to the inductor of Fig. 1. Mathematically, the form of the inductor current will be:

$$i_L(t) = A \cos(2\pi f t) \quad \text{Eq. 3}$$

#### ANALYSIS

Where  $A$  is the amplitude of the sinusoid (in volts) and  $f$  is the frequency (in Hz). The waveform is shown graphically in Fig. 2. For the circuit of Fig. 1, use the inductor voltage-current relations to calculate the inductor voltage resulting from application of the voltage of equation (3). Your results may be dependent up on the parameters  $A$ ,  $f$ , and  $L$ .

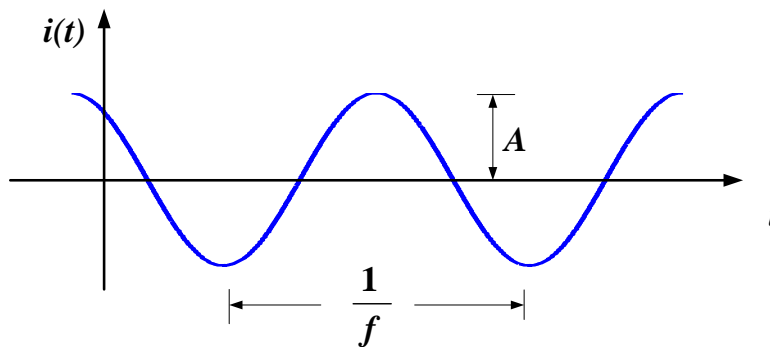


Figure 2. Basic waveform used in this lab.

### Lab Procedures:

Construct the circuit of Fig. 1 with  $L = 1\text{mH}$  and  $R = 100\Omega$ . Use channel 1 of your oscilloscope to measure the resistor voltage difference, and channel 2 of your oscilloscope to measure the inductor voltage difference. Use channel 1 of your waveform generator (W1) to apply the voltage  $v_{in}(t)$  in Fig. 1. Set up a math channel to calculate the current through the inductor per equation (2) in the pre-

lab<sup>9</sup>. Set the oscilloscope measurements to provide at least the amplitude of each of the three displayed waveforms.

**DATA**

1. Apply a sinusoidal input voltage with frequency = 1kHz, amplitude = 2V, and offset = 0V to the circuit of Fig. 1. Use your oscilloscope to display the data listed above (waveforms corresponding to C1, C2, and M1; measurement window displaying amplitudes of C1, C2, and M1). Export the data in the oscilloscope time window to a .csv file for later processing.

**DATA**

2. Apply a sinusoidal input voltage with frequency = 2 kHz, amplitude = 2V, and offset = 0V to the circuit of Fig. 1. Use your oscilloscope to display the data listed above (waveforms corresponding to C1, C2, and M1; measurement window displaying amplitudes of C1, C2, and M1). Export the data in the oscilloscope time window to a .csv file for later processing.

**DEMO**

3. Demonstrate operation of your circuit to the Teaching Assistant. Have the TA initial the appropriate page(s) of your lab notebook and the lab checklist.

### Post-lab Exercises:

**ANALYSIS**

Import the data acquired in the lab procedures into your favorite numerical analysis software (e.g. Excel, Matlab, Octave, etc.). Use the software and the results of your pre-lab analysis to calculate the expected inductor voltage waveforms corresponding to the inductor current waveforms you measured in the lab procedures. Use the software to overlay plots of the expected and measured inductor voltages for each of the cases tested in the lab procedures. Comment briefly on the agreement between the measured and expected inductor voltages for each of the cases. In your comments, be sure to include a quantitative comparison (including percent difference) between the expected and measured amplitudes of the inductor voltages.

**ANALYSIS**

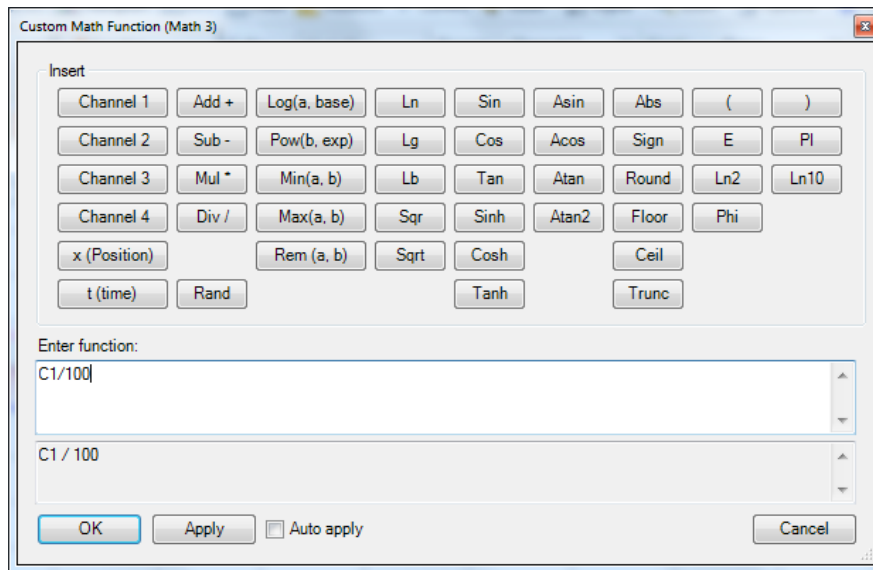
## Appendix A: Math channel to calculate current from resistor's voltage

The analog discovery provides capabilities for performing mathematical operations on the displayed waveforms and displaying the result. Essentially, there are two basic “types” of mathematical operations which can be performed: “Simple” and “Custom”. The simple math operations consist of addition, subtraction, or multiplication of the two channels. The custom operations are much more wide-ranging. In order to determine the resistor current, we want to divide the resistor voltage by a constant (the resistance value), so we will create a custom math channel. To do this, follow the steps below:

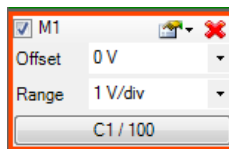
1. Click on “Add Channel”. Select “Add Mathematic Channel” from the resulting drop-down menu and choose “Custom”.
2. A custom math function window will open, as shown below. Type the desired math function (typically a function of the scope channels, C1 and C2) in the text box in this window or use the buttons in the window to create the function. We are using channel 1 (C1) to measure the resistor voltage. The current through the resistor is simply the resistor voltage divided by the resistance value (100Ω), so our function is: C1/100, also shown in the Figure below. Click “OK” to display the function in the main window.

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<sup>9</sup> Detailed instructions for doing this are provided in Appendix A.



- The properties of the math channel display can be adjusted using the channel's control box, just as any with any other channel displayed by the scope. A typical control box is shown below:



- The units of our math channel are amperes. It is nice to have the displayed units agree with the actual units of the measurement. To change the units, click on the Units icon on the control box and select "Units" from the resulting drop-down menu. Volts, will typically be the default unit; if you want the vertical axis in amps, click the drop-down arrow icon and select **A** from the resulting menu<sup>10</sup>.

<sup>10</sup> Choices of units are volts (V), amps (A), and watts (W).

## Real Analog Chapter 6: Lab Worksheets

### 6.4.1: Inductor Voltage-current Relations (40 points total)

1. In the space below, provide the mathematical expression for the inductor voltage resulting from the provided current waveform. (6 pts)
2. Attach to this worksheet an image of the oscilloscope window, showing the inductor voltage, the resistor voltage, and the inductor current and the measured amplitudes of the waveforms for a 1kHz sinusoidal input. (8 pts)
3. Attach to this worksheet an image of the oscilloscope window, showing the inductor voltage, the resistor voltage, and the inductor current and the measured amplitudes of the waveforms for a 2kHz sinusoidal input. (8 pts)
4. **DEMO:** Have a teaching assistant initial this sheet, indicating that they have observed your circuit's operation for the sinusoidal input. (6 pts)

TA Initials: \_\_\_\_\_

5. Attach to this worksheet plots of:
- Expected (per your pre-lab analysis) and measured inductor voltages for a 1kHz sinusoidal input (6 pts)
  - Expected (per your pre-lab analysis) and measured inductor voltages for a 2kHz sinusoidal input (6 pts)

## Real Analog Chapter 6: Lab Projects

### 6.4.2: Non-Ideal Inductor Effects

Voltage-current relationships for ideal inductors do not always adequately explain measured inductor voltages and currents. In this assignment, we will measure inductor voltages and currents which do not agree well with idealized inductor relationships we have employed earlier. After recognizing the differences between the measured data and our mathematical models, we will use a non-ideal inductor model to better approximate the physical behavior of the inductor.

**Before beginning this lab, you should be able to:**

- Apply the inductor voltage-current relations to calculate a inductor's voltage from its current and vice-versa
- Use the Analog Discovery's arbitrary waveform generator and oscilloscope to apply and measure time-varying waveforms (Lab 6.2.1)
- Use a math channel on the Analog Discovery to determine the current through a resistor from the voltage drop across the resistor (Labs 6.3.1, 6.4.1)





**After completing this lab, you should be able to:**

- Model an non-ideal inductor as an ideal inductor in series with a resistor
- Identify some effects of non-ideal inductors from measured data

**This lab exercise requires:**

- Analog Discovery module
- Digilent Analog Parts Kit
- Digital multimeter (DMM)

### Symbol Key:

|   |   |
|---|---|
|  | Demonstrate circuit operation to teaching assistant; teaching assistant should initial lab notebook and grade sheet, indicating that circuit operation is acceptable. |
|  | Analysis; include principle results of analysis in laboratory report.   |
|  | Numerical simulation (using PSPICE or MATLAB as indicated); include results of MATLAB numerical analysis and/or simulation in laboratory report.                      |
|  | Record data in your lab notebook.   |

### General Discussion:

The basic circuit we will use in this assignment is the same series resistor-inductor circuit used in lab 6.4.1. This circuit is repeated in Fig. 1 below, for convenience. As in lab 6.4.1, the inductor voltage can be inferred from the voltage across the resistor and the resistance, as shown below:

$$i_L(t) = \frac{v_R(t)}{R} \quad \text{Eq. 1}$$

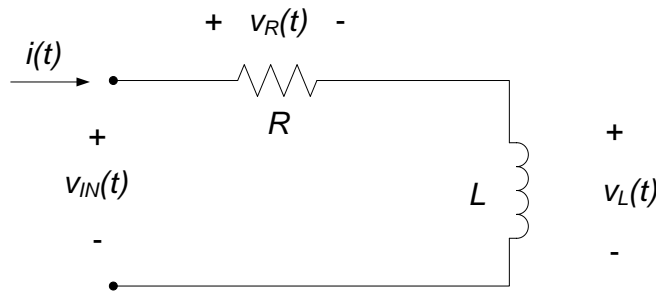


Figure 1. Series RL circuit.

In this assignment, we will apply triangular and square waveforms to the circuit of Fig. 1 above. The measured inductor voltage and current resulting from these waveforms will not agree well with the idealized inductor voltage-current relationships we have used to date. In order to interpret the data we acquire in this assignment, we will employ a non-idealized model of the inductor consisting of a series combination of an ideal inductor and a resistor as shown in Fig. 2.

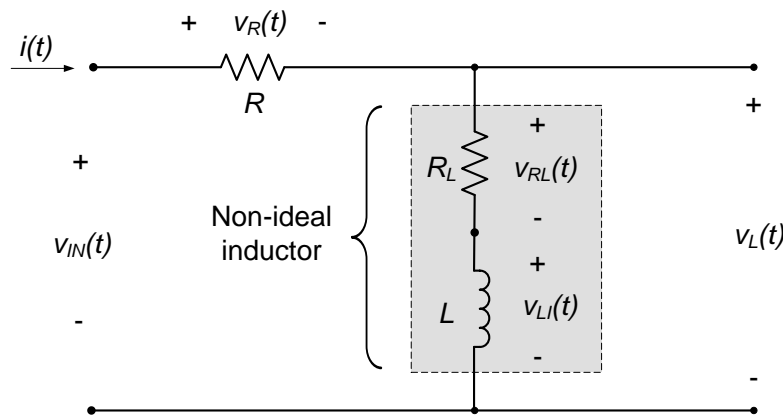


Figure 2. Circuit of Figure 1, with non-ideal model of inductor.

### Pre-lab:

In this lab, we will apply triangular and square signals to the circuit of Fig. 1<sup>11</sup>. The waveforms we will use are shown graphically in Fig. 3. In both cases,  $A$  is the amplitude of the signal (in amperes) and  $f$  is the frequency of the signal (in Hz).

<sup>11</sup> Please keep in mind that Figs. 1 and 2 are actually schematics of the same physical circuit. The difference between the two circuits is simply in the way the inductor is modeled mathematically – in Fig. 1 the inductor is modeled as being ideal, while in Fig. 2, the inductor is modeled as including a resistance. In Fig. 2, we are simply accounting for the fact that any physical inductor will dissipate energy.

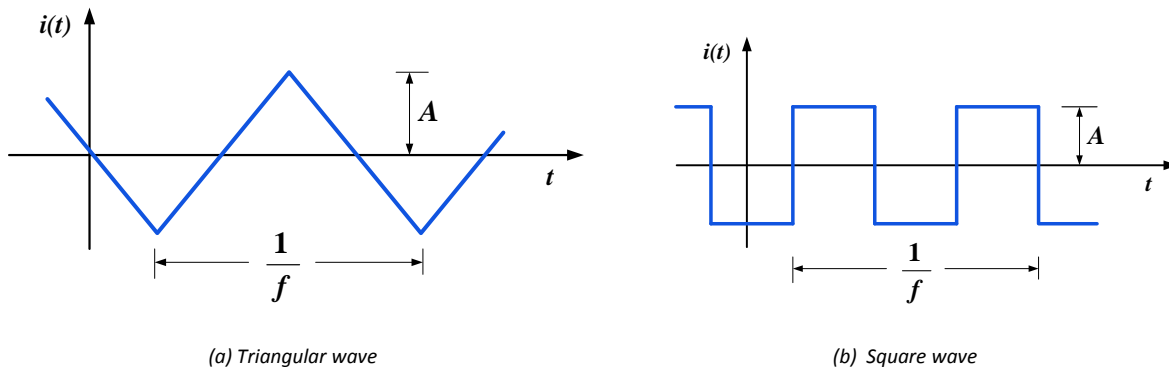


Figure 3. Basic waveforms used in this lab.

For the circuit of Figure 1, use the inductor voltage-current relations to overlay sketches of the inductor voltage and the inductor current ( $v_L(t)$  and  $i(t)$ ) if the inductor current is:

**ANALYSIS**

- a. A triangular wave,  $i(t)$ , with frequency ( $f$ ) and amplitude ( $A$ ) as shown in Fig. 2(a)

**ANALYSIS**

- b. A square wave,  $i(t)$ , with frequency ( $f$ ) and amplitude ( $A$ ) as shown in Fig. 2(b).

Label your sketch to show the amplitude and period of the inductor current for both of the above cases. Your results may be dependent on the parameters  $A$ ,  $f$ , and  $L$ . Be sure that your sketches of voltage and current share the same time axis!

**Lab Procedures:**

1. **Triangular waveform:** Construct the circuit of Fig. 1 with  $L = 1\text{mH}$  and  $R = 100\Omega$ . Use channel 1 of your oscilloscope to measure the resistor voltage difference, and channel 2 of your oscilloscope to measure the voltage across the inductor. Use channel 1 of your waveform generator (W1) to apply the voltage  $v_{in}(t)$  in Fig. 1. Set up a math channel to calculate the current through the capacitor per equation (2) in the pre-lab. Set the oscilloscope measurements to provide at least the amplitude of each of the three displayed waveforms.

**DATA**

- i. Apply a triangular input voltage with frequency = 1kHz, amplitude = 1V, and offset = 0V to the circuit of Figure 1. Use your oscilloscope to display the data listed above (waveforms corresponding to C1, C2, and M1; measurement window displaying amplitudes of C1, C2, and M1). Export the image of the oscilloscope window, showing the waveforms and their measured amplitudes.

**DEMO**

- ii. Demonstrate operation of your circuit to the Teaching Assistant. Have the TA initial the appropriate page(s) of your lab notebook and the lab checklist.
- iii. If the experiment goes according to plan, the measured voltage across the inductor will look something like the waveform shown in Fig. 4. This is not (hopefully) what your sketch of your expected response of the inductor voltage that you created in part (a) of the pre-lab looks like (since you were assuming that the inductor was ideal, per the assumptions of Fig. 1). We need to revisit our expectations as to how inductors actually behave!

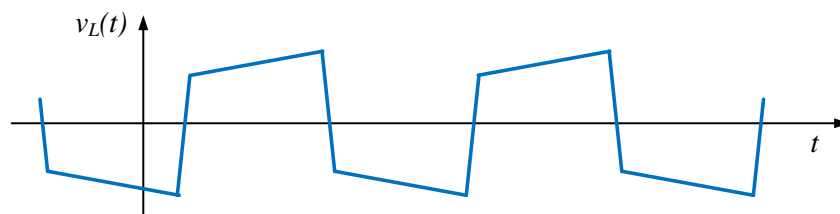
**DATA**

Figure 4. Likely shape of the measured inductor voltage waveform.

- iv. Now we will examine our measured inductor voltage response in the light of the non-ideal inductor model shown in Fig. 2. In Fig. 2, the measured inductor voltage is the sum of the “ideal” inductor voltage,  $v_{LI}(t)$ , and the voltage difference induced by the inductor’s internal resistance,  $v_{RL}(t)$ . We can still assume that the measured current ( $i(t)$ , in Figs. 1 and 2) is the current through both elements of our non-ideal inductor. Using your digital multimeter as an ohmmeter, measure the resistance of your inductor. Based on your measured waveform of the inductor current from part (b), sketch the voltages internal to the non-ideal inductor,  $v_{LI}(t)$  and  $v_{RL}(t)$ . Sketch the sum of these contributions and compare the result to the measured inductor voltage from part (b) above. Briefly discuss the ideal, non-ideal, and measured inductor voltages.

**DATA****ANALYSIS**

2. Apply a square wave input voltage with frequency = 100 Hz, amplitude = 2V, and offset = 0V to the circuit of Fig. 1. Repeat the process of part 1 of the experimental procedures to generate:

**DATA**

- v. The measured waveforms and amplitudes (as an image of the oscilloscope window).
- vi. A sketch of the expected inductor voltage using an ideal inductor model from the pre-lab. (Since the current is a piecewise constant value, the expected inductor voltage should be a constant, except when the current transitions from one value to another – during this transition, the inductor voltage should go to infinity.)

**ANALYSIS**

- vii. A sketch of your expected inductor voltage, based on the non-ideal inductor model of Fig. 3 and your measured inductor current. Include a brief discussion comparing the ideal, non-ideal, and measured inductor voltage differences.



## Real Analog Chapter 6: Lab Worksheets

### 6.4.2: Non-ideal Inductor Effects (45 points total)

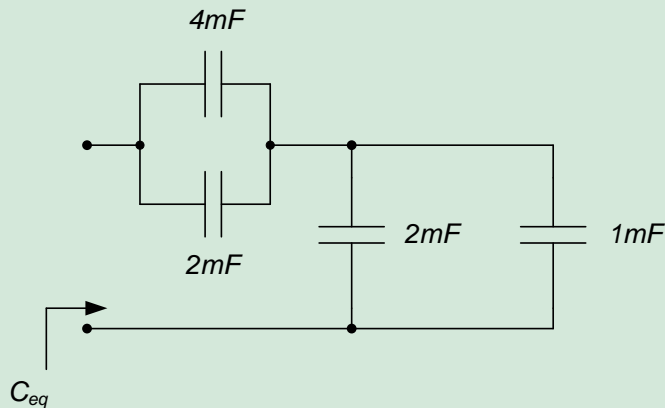
1. Attach to this worksheet your sketches of the inductor voltage and current resulting from the triangular and square waves of Fig. 3. (5 pts)
2. Attach to this worksheet an image of the oscilloscope window, showing the resistor voltage, the inductor voltage, and the inductor current waveforms and the measured amplitudes of these waveforms resulting from the 1kHz triangular input. (8 pts)
3. **DEMO:** Have a teaching assistant initial this sheet, indicating that they have observed your circuit's operation for sinusoidal inputs. (5 pts)

**TA Initials:** \_\_\_\_\_

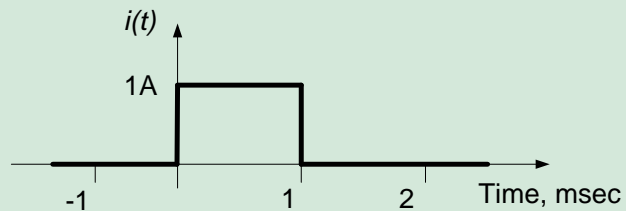
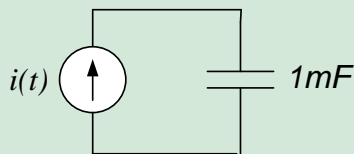
4. In the space below, provide the measured resistance of the inductor. (2 pts)
5. In the space below, sketch the expected voltages  $v_{LI}(t)$  and  $v_{RL}(t)$  as indicated in Fig. 2 (based on your resistance measurement of 4 above). Also sketch the sum of these contributions. Briefly discuss the relationships between this latter sketch and your measured inductor voltage waveform. (5 pts)
6. Attach to this worksheet an image of the oscilloscope window, showing the resistor voltage, the inductor voltage, and the inductor current waveforms and the measured amplitudes of these waveforms resulting from the 100 Hz square wave input. (8 pts)
7. In the space below, sketch the expected inductor voltage based on the non-ideal inductor model and your measured inductor current. (7 pts)
8. In the space below, briefly compare the ideal, non-ideal, and measured inductor voltages. (5 pts)

## Real Analog Chapter 6: Homework

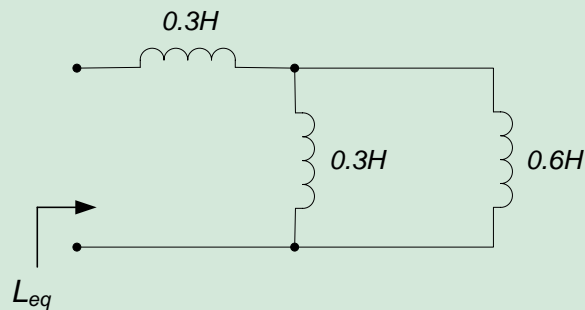
- 6.1 Determine the equivalent capacitance of the circuit below.



- 6.2 The capacitor in the circuit to the left below is initially uncharged. A 1A current is applied to the capacitor for 1msec, as shown to the right below. Sketch the voltage across the capacitor.



- 6.3 Determine the equivalent inductance of the circuit below.



- 6.4 The voltage across a 1mH inductor is as shown below. Sketch the current through the inductor. Assume that the inductor is relaxed at time  $t = 0$ .

