

Real Analog Chapter 5: Operational Amplifiers

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5 Introduction and Chapter Objectives

Operational amplifiers (commonly abbreviated as *op-amps*) are extremely useful electronic devices. Some argue, in fact, that operational amplifiers are the single most useful integrated circuit in analog circuit design. Operational amplifier-based circuits are commonly used for *signal conditioning*, performing *mathematical operations*, and *buffering*. These topics are discussed briefly below.

Signal conditioning is the process of manipulating a given signal (such as a voltage) to improve its properties or usefulness. Examples of common signal conditioning processes are:

- **Level adjustment:** the overall level of a signal may be too small to be usable. For example, the voltage output from a thermocouple (an electrical component used to measure temperature) may be only a few thousandths of a volt. It is often desirable to *amplify* the signal to increase the output voltage – this is often done using circuits containing operational amplifiers.
- **Noise reduction:** electrical signals are susceptible to noise; an undesirable component of a signal. (For example, static on a radio signal.) Operational amplifier circuits can be used to remove, or filter out undesirable components of a voltage signal.
- **Signal manipulation:** Electrical signals are often used to transmit information. For example, the voltage output of a thermocouple changes as the temperature of the thermocouple changes. The sensitivity of the thermocouple output to temperature changes may be changed by an operational amplifier circuit to provide a more readily usable output voltage-to-temperature relationship.

A common use of electrical circuits is to perform mathematical operations. So far, we have focused on developing mathematical models of existing circuits – we have been performing analysis tasks. The design process, conversely, can be considered to consist of implementing an electrical circuit that will perform a desired mathematical operation. (Of course, a large part of the design process consists of determining what mathematical operation is to be performed by the circuit.) Operational amplifier circuits are readily developed to perform a wide range of mathematical operations, including addition, subtraction, multiplication, differentiation, and integration.

Buffers allow us to electrically isolate one section of an electrical circuit from another. For example, using an electrical circuit to supply power to a second electrical circuit may result in undesirable loading effects, in which the power requirements of the second circuit exceed the power that the first circuit can provide. In this case, a buffer can be used to isolate the two circuits and thus simplifying design problems associated with integrating the two circuits. Operational amplifier circuits are commonly used for this purpose.

Operational amplifiers (or *op-amps*) are active devices. This differs from *passive* devices, such as resistors, in that an external power source must be provided to the operational amplifier in order to make it function properly. Op-amps are rather complex devices, consisting of a number of interconnected transistors and resistors. We will not be interested at this point in a detailed description of the internal operation of operational amplifiers – instead, we

will use an op-amp model which provides us with relatively simple input-output relations for the overall circuit. In fact, our op-amp model will most often take the form of a dependent source¹. In their most basic form, operational amplifiers are most readily modeled as voltage controlled voltage sources, but it can be used within other circuits to create devices which act as other types of dependent sources. This simplified model will be adequate for many analysis and design purposes.

The operational amplifier symbol which we will most often use is shown in Fig. 5.1. Operational amplifiers are essentially three-terminal devices (ignoring the power supply connections previously mentioned for the moment), having two input terminals and one output terminal. The inputs are called the inverting terminal (indicated by the – sign) and the non-inverting terminal (indicated by the + sign). We will use v_n and i_n to denote the voltage and current at the inverting terminal, and v_p and i_p to denote the voltage and current at the non-inverting terminal. The voltage and current at the output terminal are denoted as v_{OUT} and i_{OUT} . The voltages v_p , v_n , and v_{OUT} are all measured relative to some common reference voltage level, such as ground.

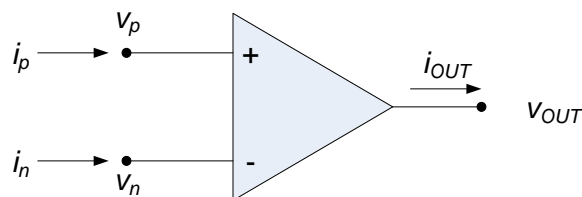


Figure 5.1. Operational amplifier symbol.

After Completing this Chapter, You Should be Able to:

- State ideal operational amplifier modeling rules
- State constraints on the operational amplifier output voltage
- Represent operational amplifiers as dependent voltage sources
- Be able to identify standard operational amplifier pin connections
- Analyze electrical circuits containing ideal operational amplifiers and resistors
- Sketch op-amp based circuits which perform the following operations:
 - Inverting voltage amplification
 - Non-inverting voltage amplification
 - Summation (addition)
 - Differencing (subtraction)
 - Buffering
- Describe the operation of a comparator
- Briefly describe the effect of the following non-ideal op-amp parameters, relative to ideal op-amp performance:
 - Finite input resistance
 - Finite output resistance
 - Finite op-amp gain

5.1 Ideal Operational Amplifier Model

We will begin by summarizing the rules governing ideal operational amplifiers. In the following section, we will provide some background material relative to these rules and some additional criteria which the operational

¹ Op-amp circuits will be our first exposure to physical devices which act as dependent sources.

amplifier must satisfy. It should be emphasized that these rules govern ideal operational amplifiers; modeling of non-ideal operational amplifiers will most likely be presented in later electronics courses.

Ideal Op-amp Modeling Rules

1. No current flows into the input terminals: $i_n = i_p = 0$
2. The voltages at the input terminals are the same: $v_n = v_p$ (when sufficient negative feedback is applied).

No requirements are placed on the output voltage and current. One may not conclude that $i_{OUT} = 0$ simply because the input currents are zero. It may appear, from the input-output relations governing the op-amp, that the op-amp violates Kirchhoff's current law – this is because we are not examining the details of the internal operation of the op-amp. Since the op-amp is an active device with its own power supply, it can provide an output current with no input current. Operational amplifiers, unlike passive devices, are capable of adding power to a signal. The presence of the external power supplies raises some additional constraints relative to op-amp operation; we address these issues next.

A more complete schematic symbol for an operational amplifier, including the op-amp's external power supplies, is shown in Fig. 5.2. Figure 5.2 shows two additional op-amp terminals. One is connected to a voltage source V^+ and the other is connected to a voltage source V^- . These terminals are sometimes called the *positive* and *negative power supply terminals*. We must set the external voltage supplies so that the positive power supply voltage is greater than the negative power supply voltage: $V^+ > V^-$. In our discussions, it will be assumed that the power supply voltages are relative to the same reference voltage as all other voltages on the schematic.

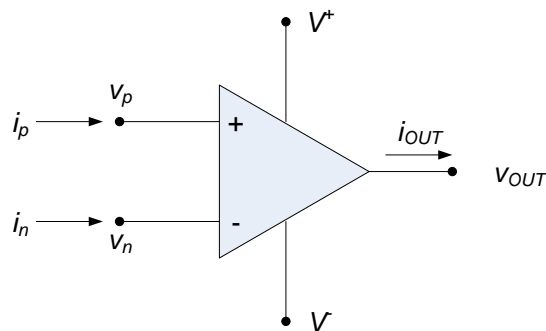


Figure 5.2. Operational amplifier schematic, including external power supplies.

The power supply voltages provide a constraint on the range of allowable output voltages, as provided below:

Output Voltage Constraint:

The output voltage is constrained to be between the positive and negative power supply voltages: $V^- < v_{OUT} < V^+$

The above constraint is based on pure inequalities – in general, the output voltage range will be somewhat less than the range specified by V^- and V^+ . The margin between the output and the supply voltages will vary depending on the specific op-amp. Any attempt to drive the output voltage beyond the range specified by the supply voltages will cause the output to *saturate* at the appropriate supply voltage. Similarly, it makes sense that the power supply voltages will constrain the range of allowable input voltages, as provided below:

Input Voltage Constraint:

- The input voltages, v_p and v_n , are constrained to be between the positive and negative power supply voltages: $V^- < v_p, v_n < V^+$.

The above constraint is based on pure inequalities – in general, the input voltage range will be somewhat less than the range specified by V_- and V_+ . The margin between the inputs and the supply voltages will vary depending on the specific op-amp. Any attempt to drive the input voltages beyond the range specified by the supply voltages will cause the op-amp to no longer operate as we describe in this simple ideal model.

It is important to keep in mind, when analyzing operational amplifier circuits, that all of the terminal voltages shown in Fig. 5.2 should be taken as having the same reference voltage². Figure 5.3 provides an explicit illustration of what is implied by this statement.

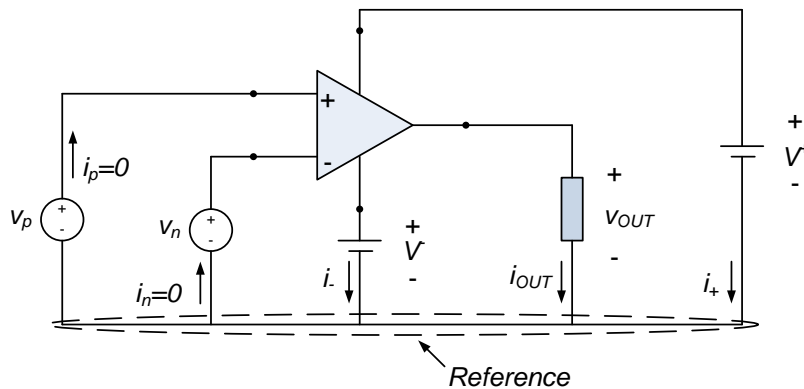


Figure 5.3. Op-amp voltages with reference node defined.

All voltages in Fig. 5.3, including the power supply voltages V_+ and V_- , have the same reference. It is obvious from Fig. 5.3 that KCL at the reference node provides:

$$i_p + i_n = i_- + i_{OUT} + i_+ = 0$$

So that the positive and negative power supplies provide the current to the output. However, it is common to leave the power supply terminals off of the op-amp diagram (as in Fig. 5.1). If one interprets these types of diagrams literally, the figure corresponding to Fig. 5.3 will be as shown in Fig. 5.4.

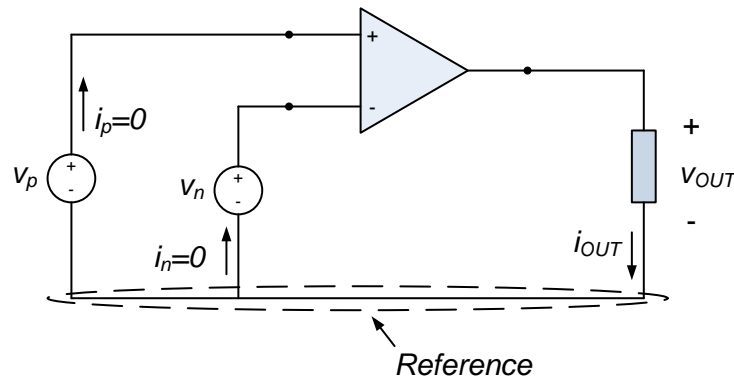


Figure 5.4. Op-amp voltages with reference node defined, but without supply voltages explicitly noted.

Figure 5.4. Op-amp voltages with reference node defined, but without supply voltages explicitly noted.

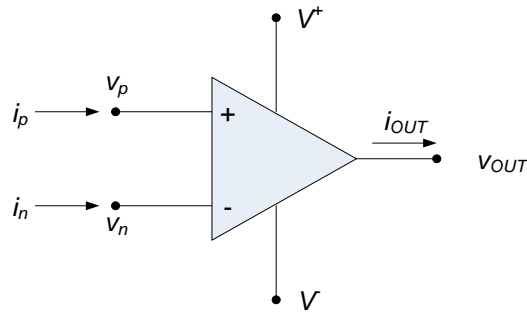
$$i_p + i_n = i_{OUT} = 0$$

² This can be difficult at times, since circuit schematics containing operational amplifiers often do not emphasize this point. It is common to assume that the person reading the schematic understands op-amp operation.

And it is tempting to infer that the current out of the op-amp must be zero. This is not true; it is a misconception based upon an attempt to literally interpret a somewhat incomplete schematic.

Section Summary

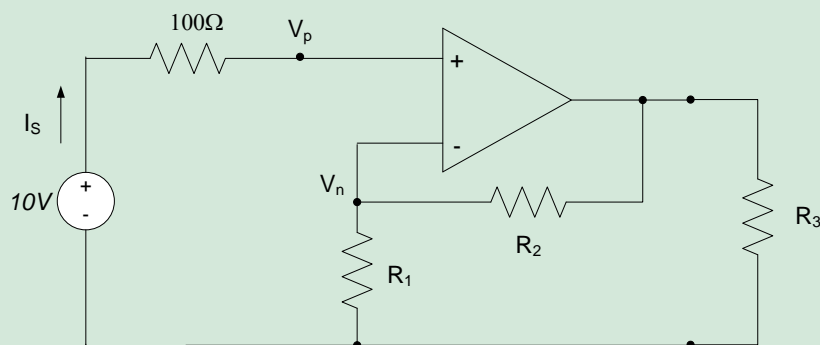
- The operational amplifier symbol is:



- The operation of ideal operational amplifiers follows the rules below:
 - No current flows into the input terminals: $i_n = i_p = 0$
 - The voltages at the input terminals are the same: $v_n = v_p$
 - The output voltage is constrained to be between the positive and negative power supply voltages: $V^- < v_{OUT} < V^+$
 - Nothing is known about the current out of the op-amp, i_{OUT}
- All voltages on the above diagram are relative to the same reference.

5.1 Exercises

- The op-amp in the circuit with negative feedback below is ideal. Find:
 - The current I_s
 - The voltage V_p
 - The voltage V_n



5.2 Operational Amplifier Model Background

The rules provided in section 5.1 governing our ideal operational amplifier model can be applied directly to operational amplifier circuits, but some background information will allow more insight into the basis for these rules. We will still treat the operational amplifier as a single circuit element with some input-output relationship, but our more complete description will model the op-amp as a dependent source. Certain assumptions relative to this dependent source allow us to recover the op-amp rules presented in section 5.2, but our more complete model will allow us to later introduce some basic non-ideal operational amplifier effects.

An operational amplifier operates as a differential amplifier with a very high gain. That is, the output of the amplifier is the difference between the input voltages, multiplied by a large gain factor, K . Figure 5.5 shows the operation of the op-amp, from a systems-level standpoint:

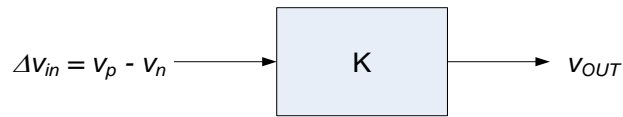


Figure 5.5. Block diagram of op-amp operation.

Thus, the input-output relation for an operational amplifier is:

$$V_{OUT} = K(v_p - v_n) = K \cdot \Delta v_{in} \quad \text{Eq. 5.1}$$

Where in Δv_{in} is the difference between the voltages at the input terminals and K is a very large number. (Values of K for typical commercially available operational amplifiers can be on the order of 10^6 or higher.) Since the output voltage is constrained to be less than the supply voltages,

$$V^- < K \cdot \Delta v_{in} < V^+$$

So

$$\frac{V^-}{K} < \Delta v_{in} < \frac{V^+}{K} \quad \text{Eq. 5.2}$$

If the voltage supplies are finite and K is very large, the difference in the input voltages must be very small. Thus,

$$\Delta v_{in} \approx 0$$

And $\Delta v_p \approx v_n$. This is of course only true when $V^- < v_{OUT} < V^+$.

The second operational amplifier modeling rule is a result of the high input resistance of operational amplifiers. We assume that any difference in the input terminal voltages is due to the operational amplifier's input resistance, R_{in} , times the current at the input terminals. This is illustrated conceptually in Fig. 5.6.

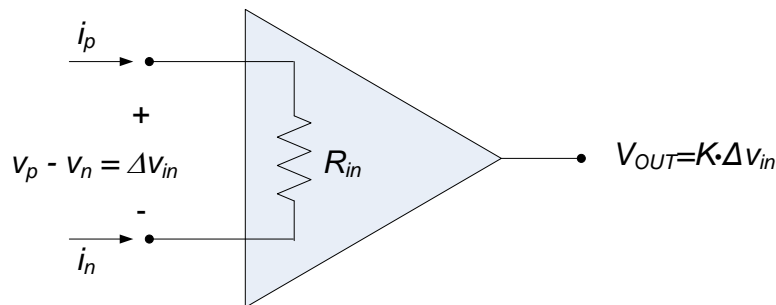


Figure 5.6. Operational amplifier symbol with "input resistance" indicated.

From Fig. 5.6, we see that the voltage difference between the input terminals can be considered to result from an input current passing through this input resistance.

$$v_p - v_n = R_{in} \cdot i_p$$

We will also assume that KCL applies across the input terminals of Fig. 5.6, so that:

$$i_p = -i_n$$

The above equations can be combined to give:

$$i_p = -i_n = \frac{v_p - v_n}{R_{in}} \tag{Eq. 5.3}$$

Since the input resistance of operational amplifiers is very large (commercial operational amplifiers have input resistances of several mega-ohms or higher) and the voltage difference across the input terminals is very small,

$$i_p = -i_n \approx 0$$

The above results suggest that an operational amplifier operates as a voltage-controlled-voltage source as shown in Fig. 5.7. Typically, commercially available operational amplifiers have very high gains, K , very high input resistances, R_{in} , and very low output resistances, R_{OUT} .

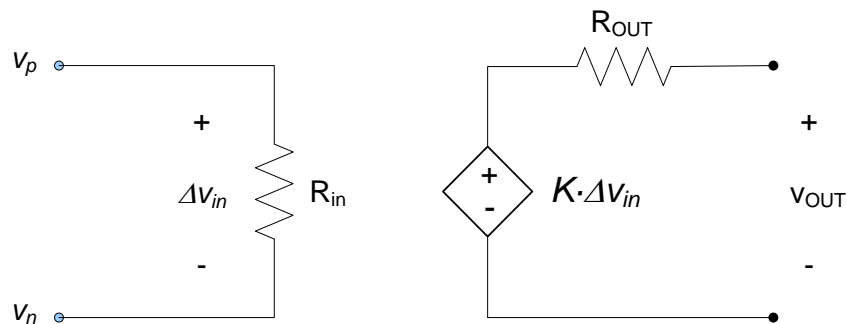


Figure 5.7. Equivalent circuit for operational amplifier model.

Combining the criteria provided by equations (5.1) and (5.2) results in the input-output relationship shown graphically in Fig. 5.8 below. The circuit operates linearly only when the output is between the supply voltages. When the output attempts to go outside this range, the circuit saturates and the output remains at the appropriate supply voltage. Notice that the negative supply voltage in Fig. 5.8 is indicated as a negative number; this is fairly typical, though not a requirement.

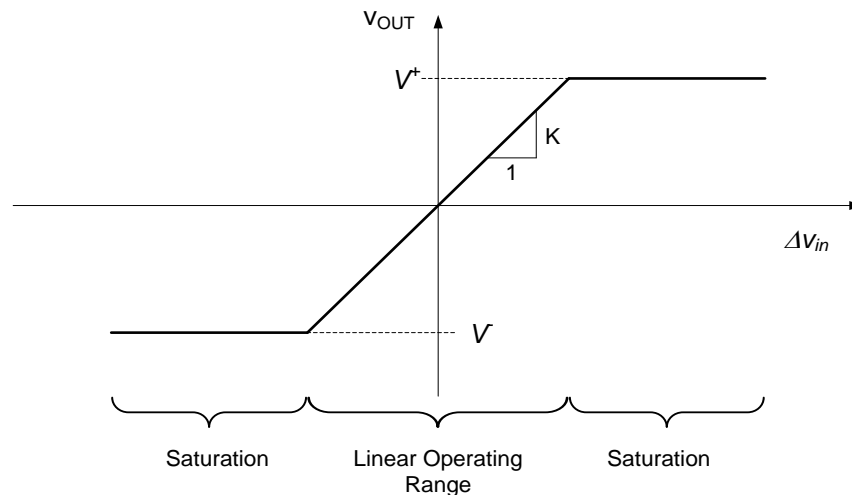


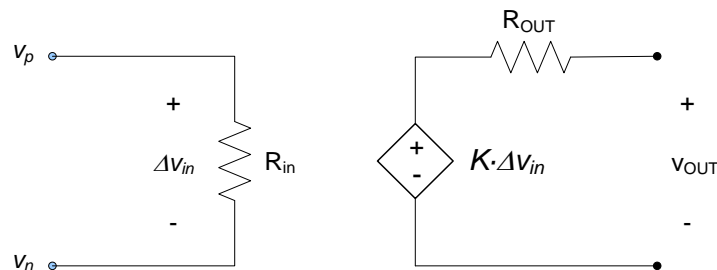
Figure 5.8. Op-amp input-output relationship.

Our ideal operational amplifier model rules are based on the above, more general, operational amplifier relationships. The assumptions relative to ideal operational amplifier operation, along with their associated conclusions, are provided below:

- The output voltage is bounded by the power supply voltages: $V^- < v_{OUT} < V^+$
- $K \rightarrow \infty$. This, in conjunction with equation (5.2) implies that $\Delta v_{in} = 0$ and $v_p = v_n$.
- $R_{in} \rightarrow \infty$. This, in conjunction with equation (5.3) implies that $i_p = -i_n = 0$.
- $R_{OUT} = 0$.

Section Summary

- A circuit modeling the behavior of an operational amplifier is:



- For ideal operational amplifiers, the parameters in the circuit above are:
 - $K \rightarrow \infty$. This implies that $\Delta v_{in} = 0$ and $v_p = v_n$.
 - $R_{in} \rightarrow \infty$. This implies that $i_p = -i_n = 0$.
 - $R_{OUT} = 0$. This implies that the operational amplifier can provide infinite power as its output.
- In the operational amplifier model above, it is still assumed that $V^- < v_{OUT} < V^+$.

5.2 Exercises

1. An operational amplifier has a gain $K = 10,000$. The voltage supplies are $V^+ = 20V$ and $V^- = -10V$. Determine the output voltage if the voltage difference between the input terminals ($v_p - v_n$) is:
 - a. 1mV
 - b. 2mV
 - c. 4mV
 - d. -0.2mV
 - e. -2mV

5.3 Commercially Available Operational Amplifiers

Operational amplifiers are available commercially as integrated circuits (ICs). They are generally implemented as *dual in-line packages (DIPs)*, so called because the terminals (pins) on the package are in pairs and line-up with one another. A typical DIP is shown in Fig. 5.9. The pins on DIPs are numbered; in order to correctly connect the DIP, pin 1 must be correctly oriented. Pin 1 is commonly located by looking for a notch at one end of the IC – pin 1 will be to the immediate left of this notch, if you are looking at the IC from the top. Alternate methods of indicating pin 1 are also used: sometimes the corner of the IC nearest pin 1 is shaved off or a small indentation or dot is located at the corner of the IC nearest pin 1.

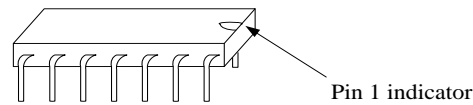


Figure 5.9. Dual in-line transistor package.

One common op-amp device is the 741 op-amp. The 741 is an eight-lead DIP; a top view of the package, with the leads labeled, is shown in Fig. 5.10. Key features of the package are as follows:

- Orientation of the pins is determined by the location of a semicircular notch on the package, as shown in Fig. 5.10. (Recall that Fig. 5.10 is a top view of the device.) Alternately, some packages place a circular indentation near pin 1 in order to provide the orientation of the pins.
- Inverting and non-inverting inputs are pins 2 and 3, respectively in Fig. 5.10
- The output terminal is pin 6 on the package.
- The positive and negative power supplies are labeled as V_{CC+} and V_{CC-} in Fig. 5.10. They are pins 7 and 4, respectively. V_{CC+} should be less than +15 volts and V_{CC-} should be more than -15 volts. A larger range of power supply voltages may destroy the device.
- The pins labeled OFFSET NULL 1, OFFSET NULL 2, and NC (pins 1, 5, and 8) will not be used for this class. The offset null pins are used to improve the op-amp's performance. The NC pin is never used. (NC stands for "not connected").

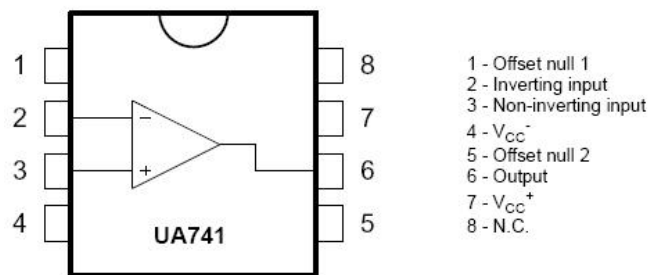


Figure 5.10. 741-type operational amplifier pin connections.

Most commercially available operational amplifiers will conform to a relatively standard pin connection layout. However, there will tend to be variations to one extent or another. For example, pin connections for an OP27 operational amplifier are shown in Fig. 5.11(a). Some operational amplifier chips will also contain more than one operational amplifier on the chip. For example, Fig. 5.11(b) provides pin connections for an OP282 package, in which two operational amplifiers are included. (A chip with two operational amplifiers is commonly called a "dual package". Chips with four operational amplifiers are also common; they are often called "quad packages".) In Fig. 5.11, $V+$ and $V-$ are the positive and negative power supplies, respectively. (Both operational amplifiers on the OP282 chip share the same power supplies.) $+IN$ and $-IN$ are the positive and negative input terminals, and OUT is the output terminal. In the OP27 amplifier, the VOS TRIM terminals perform the same purpose as the Offset Null pins on the 741-type operational amplifier. The OP282 chip contains two amplifiers, "A" and "B". In Fig. 5.11(b), the inputs and outputs for the two amplifiers are identified as being associated with the "A" or "B" amplifier by appending the appropriate letter.



Figure 5.11. Additional operational amplifier pin connection examples.

Note: Always check the manufacturer's data sheet for the specific operational amplifier you are using. This can eliminate irritating and time-consuming errors when wiring your circuits!

5.3 Exercises:

- Go to the Analog Devices website, <http://www.analog.com>, and look up the pin connections for the OP482 operational amplifier package. Sketch the package and label the pin connections. Briefly describe your interpretation of the various pin connections.

5.4 Analysis of Op-amp Circuits

Operational amplifiers can be used as either linear or nonlinear circuit elements. When used as nonlinear circuit elements, the op-amp is deliberately operated so that the output voltage from the op-amp is driven to the power supply voltages. In this mode of operation, the output of the op-amp is said to be saturated and does not necessarily change as the input to the system changes. When the operational amplifier is used as a linear circuit element, the output is maintained within the range of the power supply voltages and the output voltage is a linear function of some input voltage or voltages.

In this chapter, we will be concerned with the use of operational amplifiers as linear circuit elements. When used in this mode, the overall circuit is generally constructed to provide *negative feedback* around the operational amplifier itself. When operated in a negative feedback mode, the output of the operational amplifier is connected to the inverting input terminal, generally through some other circuit elements. Negative feedback tends to make the overall circuit less sensitive to the specific value of the op-amp gain and reduces the likelihood of saturation at the op-amp output. We will not be concerned here with the details of why this is true, beyond noting that these devices will generally not operate linearly without feedback. (Later electronics courses will discuss why this is true.)

Nodal analysis is often the most efficient way to approach the analysis of an operational amplifier-based circuit. When applying nodal analysis to a circuit containing an ideal operational amplifier, the first step should be to apply the basic op-amp rules to the overall circuit. These were presented in the previous chapter, and are repeated here for convenience:

- The voltages at the input terminals of the operational amplifier are the same.
- The currents into the input terminals of the operational amplifier are zero.

It should be emphasized that application of rule 1 above does not imply that both of the op-amp input terminals can be treated as being part of the same node. The op-amp input terminals should be treated as being two separate nodes, with the same voltage potential. After applying the basic op-amp rules, it is generally appropriate to apply Kirchhoff's current law at the input terminals of the operational amplifier. Additional nodes in the circuit may necessitate application of KCL at other points, but the above approach is generally an extremely good starting point.

Important Tip: Applying KCL at the output node of an operational amplifier is often not productive. Since no information is available about the current out of an operational amplifier (due to the active nature of the device, as noted in the previous section) application of KCL at the output node generally provides an additional equation, at the expense of introducing an additional unknown. Application of KCL at an op-amp output node is generally only productive if one must determine the current output of the op-amp.

When analyzing an operational amplifier as a linear circuit element, the external power supply voltages will generally be ignored. We will assume that the output voltage is within the voltage range specified by the external power supplies. If the output voltage is not within this range, the circuit will not behave linearly, and our analysis will be invalid. The final step of any analysis of an operational amplifier circuit is to determine whether the output voltage is within the external power supply voltage range; meeting this constraint often results on a constraint on the input voltages applied to the circuit.

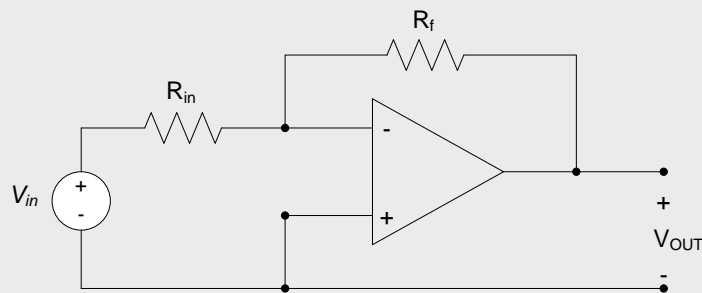
Suggested Analysis Approach:

1. Apply ideal operational amplifier rules to circuit. (Voltage potentials at op-amp input terminals are the same; no current enters the op-amp input terminals.)
2. Apply KCL at op-amp input terminals.
3. Apply KCL at other circuit nodes, if necessary.
4. Check to ensure that output voltage remains within range specified by op-amp power supply voltages.

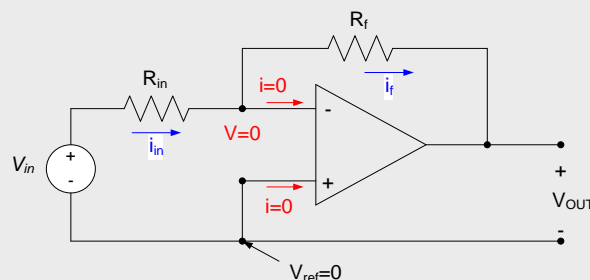
We illustrate the above analysis approach with several examples. The example circuits provided below illustrate the use of operational amplifier circuits to perform the mathematical operations of scaling (multiplication by a constant), addition, and subtraction. We also provide an example circuit which performs a buffering operation – this circuit can be useful for isolating different parts of a circuit from one another.

Example 5.1

Determine V_{OUT} as a function of V_{IN} for the circuit shown below.



Choosing the non-inverting terminal voltage as our reference voltage and applying the ideal operational amplifier rules allows us to label the voltages and currents shown in red below.



Applying KCL at the non-inverting input terminal provides no information (we know the current and voltage at the non-inverting input). Applying KCL at the inverting input terminal results in:

$$i_{in} = i_f$$

Using Ohm's law to write these currents in terms of node voltages and taking advantage of the fact that the voltage at the inverting terminal of the op-amp is zero (because there is no voltage difference across the input terminals of the op-amp and we have chosen the non-inverting terminal voltage as our reference) results in:

$$\frac{V_{in} - 0}{R_{in}} = \frac{0 - V_{OUT}}{R_f}$$

Solving for V_{OUT} results in:

$$V_{OUT} = -\left(\frac{R_f}{R_{in}}\right)V_{in}$$

Comments:

- This circuit is called an *inverting voltage amplifier*. The output voltage is a scaled version of the input voltage, hence the term "voltage amplifier". The change in sign between the output and input voltage makes the amplifier "invert".
- The output voltage must be between the op-amp power supply voltages. Depending on the values of R_f and R_{in} , this sets limits on the magnitude of the input voltage to avoid saturation.

It is worthwhile at this point to make a few comments relative to some concepts presented sections 5.1 and 5.2, in the context of the op-amp circuit of Example 5.1.

- The input-output relationship governing the circuit of Example 5.1 can be represented conceptually as a dependent source-based circuit. The input-output relationship for the circuit, as determined in Example 5.1, is:

$$V_{OUT} = -\left(\frac{R_f}{R_{in}}\right)V_{in}$$

While the current provided by the source to the circuit is:

$$i_{in} = \frac{V_{in} - 0}{R_{in}}$$

These two relationships are satisfied by the voltage controlled voltage source (VCVS) shown in Fig. 5.12 below. The input resistance of this circuit is the resistance R_{in} ; the input resistance governs the relationship between the voltage applied by the source and the source current necessary to maintain that voltage. Thus, increasing R_{in} reduces the power which the source must provide to maintain the output voltage V_{OUT} .

Note: The input resistance of the circuit of Example 5.1 is not the same as the input resistance of the operational amplifier itself, as shown in Figs. 5.6 and 5.7.

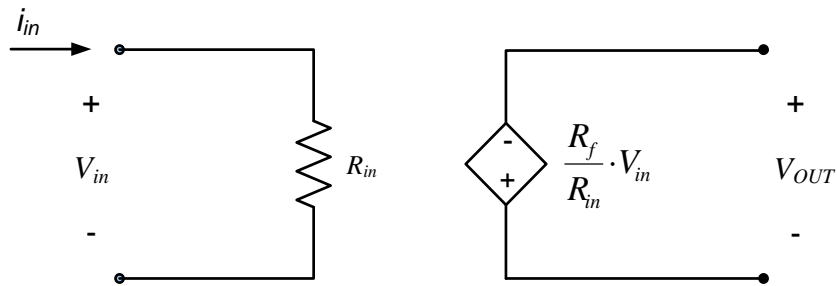


Figure 5.12. VCVS to model circuit of Example 5.1.

- The representation given in Fig. 5.12 of the circuit of Example 5.1 is not unique. For example, the current controlled voltage source (CCVS) model shown in Fig. 5.13 is also a valid model for the circuit.

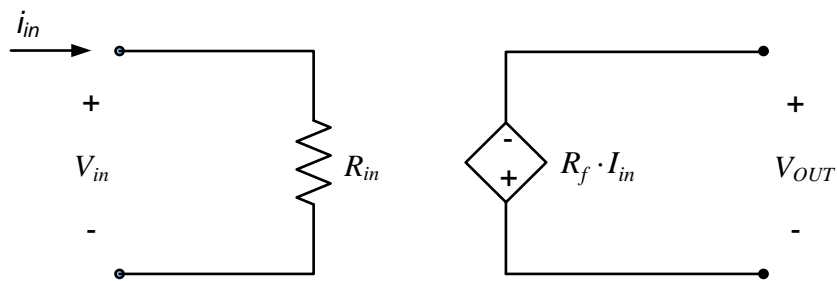


Figure 5.13. CCVS model for circuit of Example 5.1.

- The circuit of Example 5.1 has an open-circuit at the op-amp output terminal. Thus, no current is provided to the output. However, this does not imply that the current at the output terminal of the op-amp is zero! KCL at the op-amp output terminal indicates that the current through the feedback resistor goes into the op-amp output terminal, where (from our diagram) it seems to disappear! Recall, however, from our discussion in section 5.1 (Fig. 5.3 in particular) that this current will pass through the supply voltages and then to the reference node. The supply voltages and their associated path to the reference node are not shown on the circuit diagram in Example 5.1, but they do exist.
- If we apply a load resistor R_L to the output terminal of the operational amplifier in Exercise 5.1, we obtain the circuit shown in Fig. 5.14. The analysis of the circuit proceeds exactly as in Exercise 5.1, and we again obtain:

$$V_{OUT} = -\left(\frac{R_f}{R_{in}}\right)V_{in}$$

This result does not depend on the current through the load, i_L ! This is because we made no assumptions relative to the current out of the operational amplifier output; the operational amplifier adjusts its output current as necessary to provide the current required to maintain the output voltage as in the above expression.

Note: Our ideal operational amplifier will draw power from the supply voltages to provide whatever output current is necessary to satisfy the rules governing operational amplifier provided in section 5.1. Real operational amplifiers, of course, have current limitations. We will discuss these limitations in section 5.6.

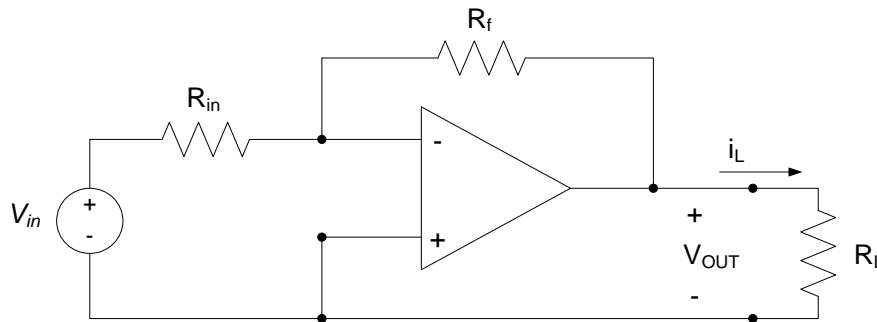
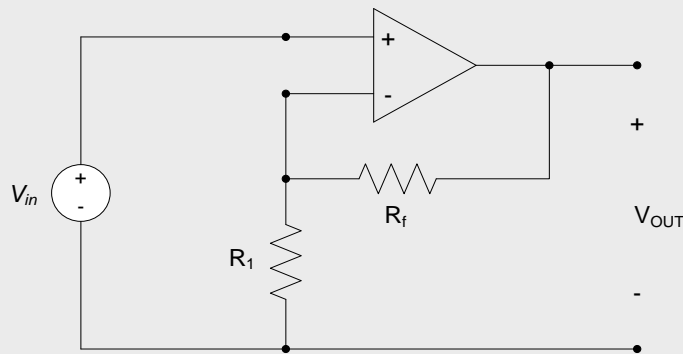


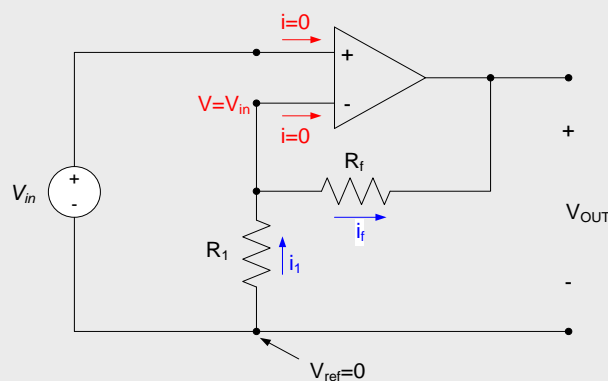
Figure 5.14. Inverting voltage amplifier with load resistor.

Example 5.2

Determine V_{OUT} as a function of V_{IN} for the circuit shown below.



Choosing our reference voltage at the negative terminal of both V_{in} and V_{OUT} and applying the ideal operational amplifier rules allows us to label the voltages and currents shown in red below. (Note that since the input voltage sets the voltage of the non-inverting op-amp terminal, it also indirectly sets the voltage at the inverting terminal of the op-amp.)



Applying KCL at the inverting terminal of the op-amp results in $i_1 = i_f$. Using Ohm's law to write these in terms of voltages provides:

$$\frac{0 - V_{in}}{R_1} = \frac{V_{in} - V_{OUT}}{R_f}$$

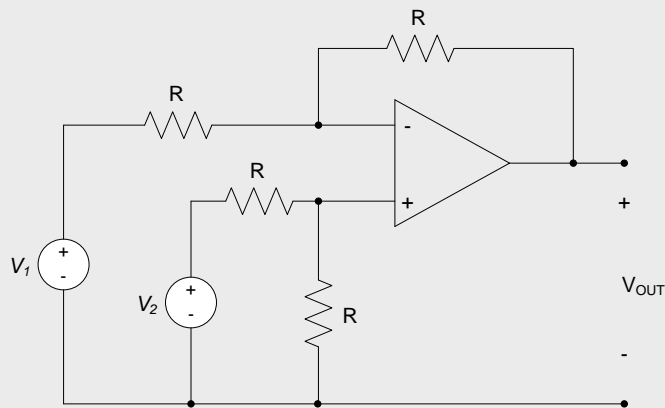
Solving this for V_{OUT} gives $V_{OUT} = \left(1 + \frac{R_f}{R_1}\right) V_{in}$

Comments:

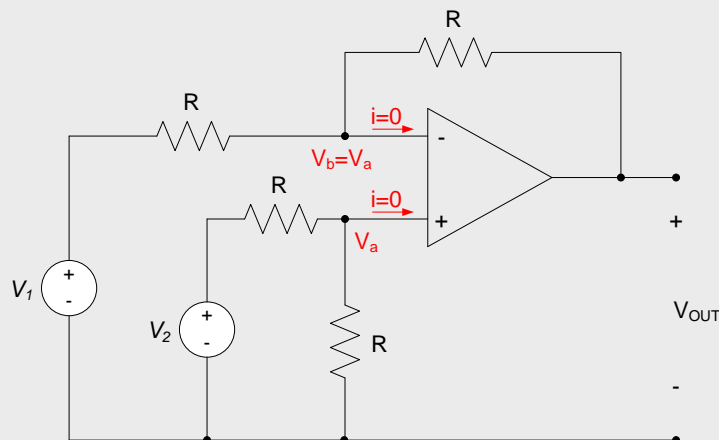
- The output voltage can be expressed as a gain (a multiplicative factor) times the input voltage; the circuit is a “voltage amplifier”. Since there is no sign change between the input and output voltage, the circuit is a *non-inverting voltage amplifier*.
- The output voltage must be between the op-amp power supply voltages. Depending on the values of R_f and R_1 , this sets limits on the magnitude of the input voltage to avoid saturation.

Example 5.3

Determine V_{OUT} as a function of V_1 and V_2 for the circuit shown below.



Denoting the non-inverting terminal of the op-amp as node a and the inverting terminal as node b, and applying the ideal op-amp rules results in the figure below:



The voltage V_a can be determined from a voltage divider relation (or by applying KCL at node a) as $V_a = \frac{V_2}{2}$.

Thus, the voltage at the inverting terminal is $V_b = V_a = \frac{V_2}{2}$. Applying KCL at node b results in:

$$\frac{V_1 - V_b}{R} = \frac{V_b - V_{OUT}}{R} \Rightarrow \frac{V_1 - \frac{V_2}{2}}{R} = \frac{\frac{V_2}{2} - V_{OUT}}{R}$$

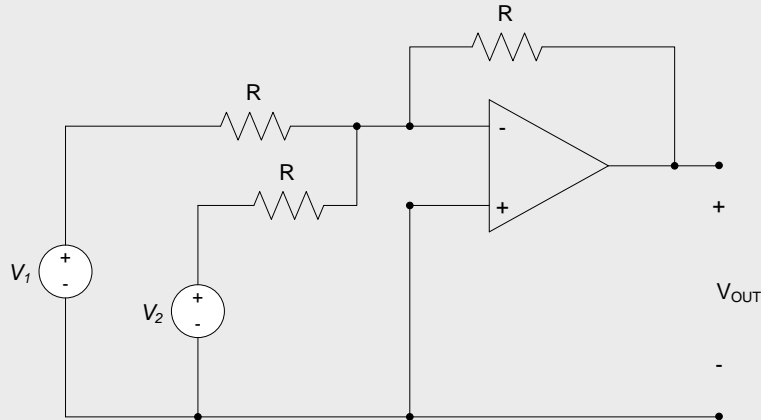
Simplification of the above results in $V_{OUT} = V_2 - V_1$.

Comments:

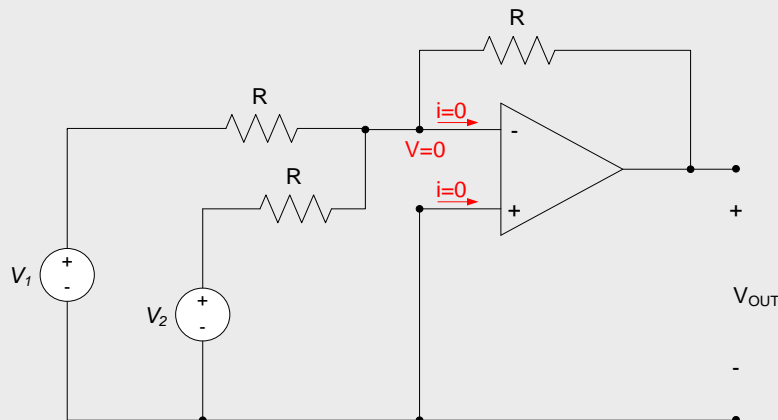
- The above circuit performs a subtraction operation. The voltage V_1 is subtracted from the voltage V_2 .
- The inverting and non-inverting terminals of the op-amp are treated as separate nodes in this analysis, even though the op-amp constrains the voltages at these nodes to be the same. Thus, we apply KCL at each input terminal of the op-amp.

Example 5.4

Determine V_{OUT} as a function of V_1 and V_2 for the circuit shown below.



Choosing the non-inverting terminal voltage as our reference voltage and applying the ideal operational amplifier rules allows us to label the voltages and currents shown in red below:



Applying KCL at the inverting terminal of the op-amp results in:

$$\frac{V_1 - 0}{R} + \frac{V_2 - 0}{R} = \frac{0 - V_{OUT}}{R}$$

Or

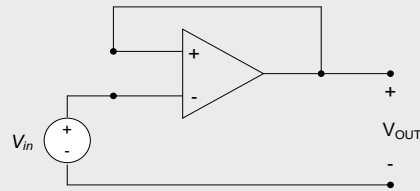
$$V_{OUT} = -(V_1 + V_2)$$

Comments:

- The circuit inverts the sum of the inputs. One can use an inverting amplifier with a gain of one in conjunction with the above circuit to obtain a non-inverted sum of the inputs.
- An arbitrary number of inputs can be summed, by simply increasing the number of input signals and resistors applied at the inverting terminal of the op-amp, which is often referred to as the “summing node”.

Example 5.5

Determine V_{OUT} as a function of V_{IN} for the circuit shown below.

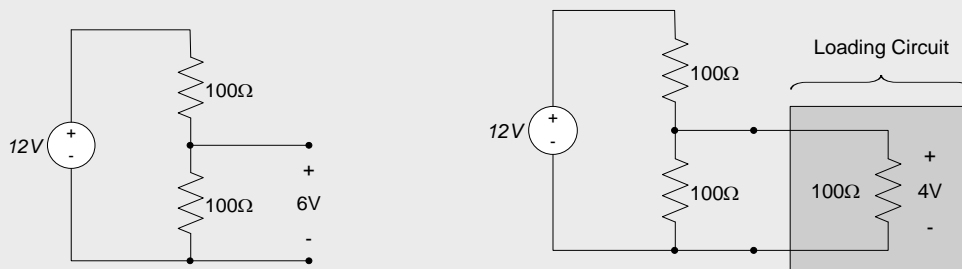


Since there is no circuit element in the feedback loop, the inverting terminal voltage is identical to the output voltage, V_{OUT} . The ideal op-amp rules require that the inverting and non-inverting terminal voltages are the same, so:

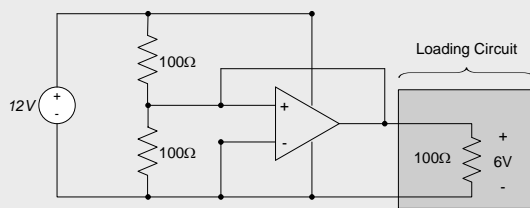
$$V_{OUT} = V_{in}$$

The circuit is called a *voltage follower*, since the output voltage simply “follows” the input voltage. This circuit, though it appears to do nothing, is actually extremely useful. Since the input voltage is applied directly to an op-amp input terminal, the input resistance to the circuit is infinite and no current is drawn from the source. Thus, the source provides no power in order to generate the output voltage - all power provided to the load comes from the op-amp power supplies. This can be extremely useful in isolating different portions of a circuit from one another.

Consider, as an example, the following case. We have a loading circuit with an equivalent resistance of 100Ω . We wish to apply 6V to the circuit, but only have access to a 12V source. It is decided that we will use a voltage divider containing two 100Ω resistors in series to reduce the supply voltage to the desired 6V level as shown in the circuit to the left below. However, adding the loading circuit to the voltage divider changes the voltage provided to the load, as shown to the right below.



Addition of a voltage follower to the circuit isolates the voltage divider from the load, as shown below. Power to the op-amp can be provided by connecting the 12V source to $V+$ and grounding $V-$, as shown, since the desired op-amp output is between 0V and 12V.



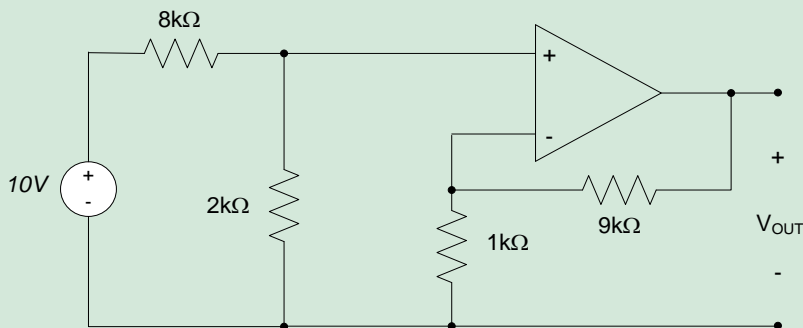
Section Summary

- Analysis of linear operational amplifier circuits typically consists of the following components:
 - Assume that the voltage difference across the input terminals is zero.
 - Assume that the currents into the input terminals is zero.
 - Apply KCL at op-amp input terminals.
 - Apply KCL at other circuit nodes, if necessary.
 - Check to ensure that output voltage remains within range specified by op-amp power supply voltages.
- Op-amp circuits which perform the following functions are presented in this section:
 - Inverting voltage amplification
 - Non-inverting voltage amplification
 - Summation (addition)
 - Differencing (subtraction)
 - Buffering

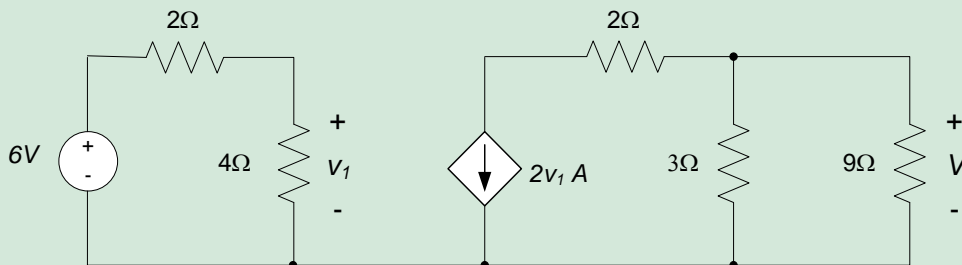
The reader should be able to sketch circuits, which perform the functions above.

5.4 Exercises

1. Represent the circuit of Example 5.2 as a voltage controlled voltage source.
2. Represent the circuit of Example 5.2 as a voltage controlled current source.
3. Find V_{out} for the circuit below.



4. Find V in the circuit below.



5.5 Comparators

Operational amplifiers are intended to be incorporated into circuits which *feeds back* the op-amp output to one or both of the input terminals. That is, the output voltage is connected in some way to the op-amp inputs. Typically, for stable operation, the output is fed back to the inverting input terminal for stable operation (as in all of our circuit examples in section 5.4). If the output is not fed back to the input of the op-amp, the op-amp may not function as expected.

Comparators are operational amplifier –like devices which are intended to be operated without feedback from the output to the input. The circuit symbol for a comparator looks like an op-amp symbol, reflecting their similarities. Figure 5.15 provides a typical comparator symbol, with applicable voltages labeled.

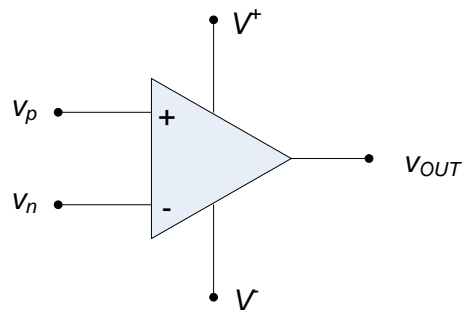


Figure 5.15. Comparator circuit symbol.

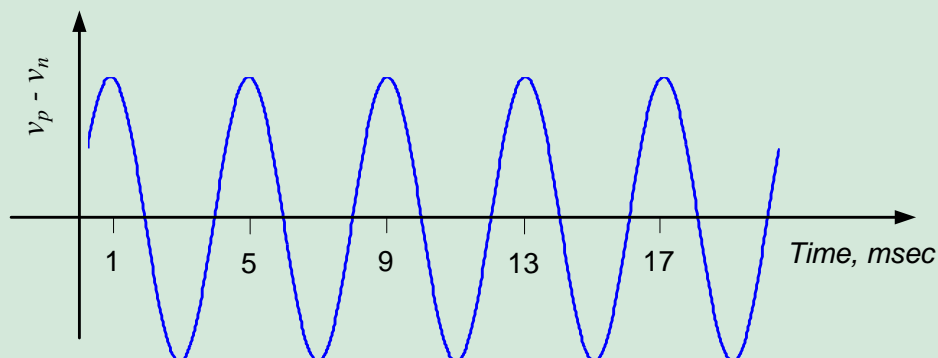
Operation of the comparator is simple: if v_p is greater than v_n , the output goes to the high supply voltage, V^+ . If v_p is less than v_n , the output goes to the low supply voltage, V^- . The comparator is essentially checking the sign between the voltage at the inverting and non-inverting inputs, and adjusting the output voltage accordingly.

Mathematically, the operation of a comparator can be expressed as:

$$V_{OUT} = \begin{cases} V^+, & v_p - v_n > 0 \\ V^-, & v_p - v_n < 0 \end{cases}$$

5.5 Exercises

1. A comparator like that shown in Fig. 5.14 has the sinusoidal signal below applied across the input terminals. (E.g. the plot below is $v_p - v_n$ vs. time.) Sketch the output voltage $v_{out}(t)$.



5.6 A Few Non-ideal Effects

In section 5.2, we indicated that operational amplifiers are designed to have high input resistances, low output resistances and high gains between the input voltage difference and the output voltage. Figure 5.7 of section 5.2 provided a model of an operational amplifier as a dependent source, including input and output resistances. This model is repeated below as Fig. 5.16 for convenience.

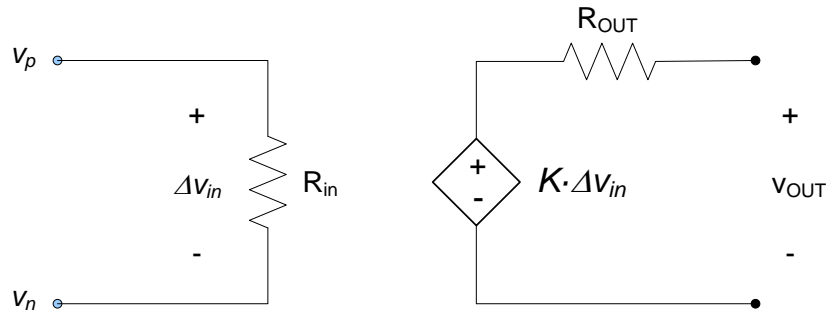


Figure 5.16. Operational Amplifier model.

In section 5.2, we also provided the assumptions applicable to ideal operational amplifier operation, along with their associated conclusions, and are provided below:

- The output voltage is bounded by the power supply voltages: $V^- < V_{OUT} < V^+$
- $K \rightarrow \infty$. Thus, $\Delta v_{in} = 0$ and $v_p = v_n$.
- $R_{in} \rightarrow \infty$. Thus, $i_p = -i_n = 0$, and the operational amplifier draws no power at its input.
- $R_{OUT} = 0$. Thus, there is no limit on the output current (or power) which can be provided by the op-amp.

Practical operational amplifiers have finite gains (K for most amplifiers is in the range $10^5 - 10^7$), finite input resistances (typical values are on the order of a few mega-ohms to hundreds or thousands of mega-ohms) and non-zero output resistances (generally on the order of 10 to 100 ohms). In this section, we will very briefly discuss a few of the ramifications of these non-ideal parameters.

5.6.1 Input Resistance Effects

The high input resistance of the operational amplifier means that circuits connected to the op-amp input do not have to provide much power to the op-amp circuit. This is the op-amp property that is employed in buffer amplifiers and instrumentation amplifiers. Instrumentation systems, for example, have very limited power output capabilities; these limitations are typically modeled as high output resistances in the instrumentation systems. Thermocouples, for example, provide low voltage levels, and very small power output – they can be modeled as a voltage source with a fairly high output resistance. When a system of this type is connected to the input terminals of an op-amp, the situation is as shown in Fig. 5.17.

It is apparent from Fig. 5.17 that the output resistance of the system, if it is large enough, can have an effect on the voltage difference across the op-amp input terminals, since:

$$\Delta v_{in} = V_s \left(\frac{R_{in}}{R_{in} + R_s} \right)$$

Since we generally want to amplify V_s directly, any difference between V_s and Δv_{in} will degrade our output voltage from its desired value.

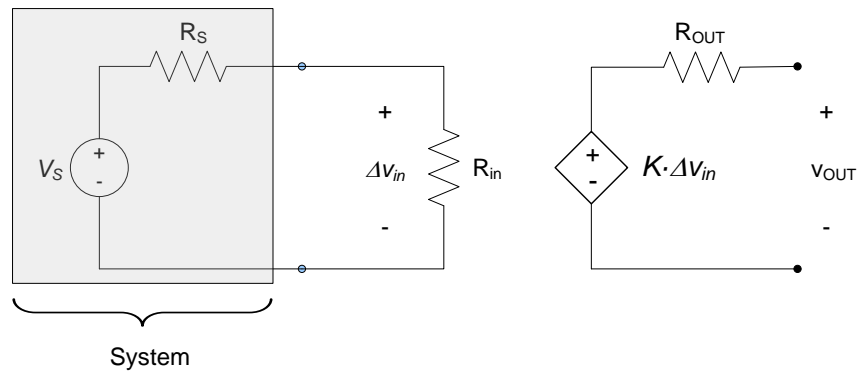


Figure 5.17. Effect of input resistance on output.

5.6.2 Output Resistance Effects

The op-amp output resistance essentially limits the amount of power the op-amp can provide at its output terminal. This can become a problem if we want to connect very low resistance loads to the output of an operational amplifier. For example, audio speakers commonly have an 8Ω resistance. Figure 5.18 shows an 8Ω speaker connected to the output of an operational amplifier which has an 80Ω output resistance. In this case, we expect the maximum output voltage to be:

$$v_{OUT} = K \cdot V_S \left(\frac{8\Omega}{8\Omega + 80\Omega} \right) = 0.09KV_S$$

If the maximum output voltage of the op-amp is low, we may not have nearly enough power to operate the speaker.

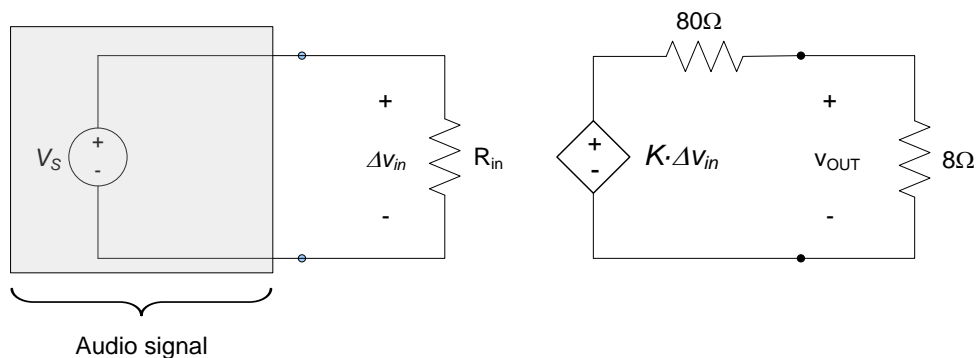


Figure 5.18. Audio amplifier.

5.6.3 Finite Gain Effects

As an example of the effects of a finite voltage gain, let us assume that an operational amplifier has a gain of $K = 10,000$ and supply voltages $V^+ = 10V$ and $V^- = -10V$. From equation (5.2), the linear operating range of the operational amplifier is over the range of input voltage differences:

$$\frac{V^-}{K} \leq \Delta \leq \frac{V^+}{K}$$

The non-ideal operational amplifier of interest can then allow input terminal voltage differences of up to $-1mV \leq \Delta v_{in} \leq 1mV$. Although voltage differences of a millivolt will be considered to be essentially zero for any of the voltage levels we will deal with in this class, these voltages are definitely not zero for some applications.

Section Summary

- The effect of a finite input resistance on an operational amplifier's operation is that the current into the input terminals will not be identically zero. Thus, a real operational amplifier with finite input resistance will always draw some power from a circuit connected to it. Whether this has a significant effect on the overall circuit's operation is generally a function of the output resistance of the circuit to which the amplifier is connected.
- The effect of a non-zero output resistance on an operational amplifier's operation is that the power output of the amplifier is limited. Thus, a realistic operational amplifier will not be able to provide any arbitrary current to a load. Whether this has a significant effect on the overall circuit's operation is primarily dependent upon the value of the load resistance.
- The effect of a finite op-amp gain is that the voltage difference across the input terminals may not be identically zero.

Real Analog Chapter 5: Lab Projects

5.4.1: Inverting Voltage Amplifier

In this assignment, we implement a simple operational amplifier-based circuit. Since operational amplifiers are used commonly in circuits used to implement mathematical operations, we implement the processes of multiplication by a negative constant.

Before beginning this lab, you should be able to:

- Analyze operational amplifier-based circuits

After completing this lab, you should be able to:

- Design and build an operational amplifier-based inverting voltage amplifier

This lab exercise requires:

- Analog Discovery 2 module
- Digilent Analog Parts Kit
- Digital multimeter (optional)

Symbol Key:


- DEMO** Demonstrate circuit operation to teaching assistant; teaching assistant should initial lab notebook and grade sheet, indicating that circuit operation is acceptable.
- ANALYSIS** Analysis; include principle results of analysis in laboratory report.
- SIM** Numerical simulation (using PSPICE or MATLAB as indicated); include results of MATLAB numerical analysis and/or simulation in laboratory report.
- DATA** Record data in your lab notebook.

General Discussion:

The circuit shown in Fig. 1 is called an inverting amplifier. Appropriate pin numbers for the OP27 operational amplifier are provided on Fig. 1. v_{in} is the applied (input) voltage to the circuit. v_{out} is the output voltage from the circuit. The relationship between v_{in} and v_{out} for this circuit is:

$$v_{out} = -\frac{R_2}{R_1} v_{in}$$

Thus, the output voltage is an *inverted* (due to the sign change) and *amplified* or *scaled* (due to the multiplicative factor $\frac{R_2}{R_1}$) version of the input voltage. The scaling factor $\frac{R_2}{R_1}$ is sometimes called the *gain* of the amplifier. The

ground symbol, , is used to denote the reference voltage from which all other voltages are measured. Note that if R_1 and R_2 are the same, the output voltage is simply the negative of the input voltage.

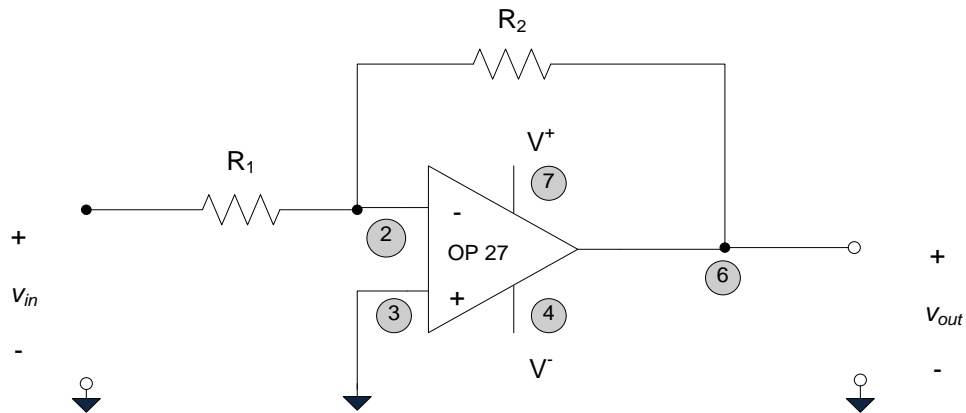


Figure 1. Inverting amplifier circuit.

Pre-lab:

ANALYSIS

Design an inverting amplifier which provides a gain of approximately 2 and an input resistance, R_1 , of approximately $2\text{k}\Omega$. (The input resistance is defined as the input voltage divided by the input current. Since pin 2 provides a “virtual” ground, the input resistance is simply R_1 .)

Lab Procedures:

DATA

1. Implement the amplifier design you generated in the pre-lab. Create a schematic of the circuit in your lab notebook, record actual resistance values, and label supply voltages on your schematic. Recommended connections are as follows:
 - Use V_+ as positive supply rail to the op-amp and V_- as the negative supply rail to the op-amp.
 - Use one of the waveform generator channels on your Analog Discovery 2 to provide the input voltage v_{in} to your circuit.
 - Measure both the input voltage, v_{in} and the output voltage, v_{out} , using your DMM and/or the scope channels on your Analog Discovery 2.

DATA

2. Test your design with input voltages of approximately -3V to $+4\text{V}$ by step sizes of $.5\text{V}$. Tabulate your results (v_{in} and v_{out}) in your lab notebook. Also in your lab notebook, create a plot of v_{in} vs. v_{out} and comment on your results (make sure that you calculate a circuit gain – the rate of change of output to input – and compare it to your expectations based on your pre-lab). Note in your lab notebook the range of output voltages over which the circuit response is linear.

DEMO

3. Demonstrate operation of your circuit to the Teaching Assistant. Have the TA initial the appropriate page(s) of your lab notebook and the lab checklist.

Real Analog Chapter 5: Lab Projects

5.4.2: Summing Amplifier

In this assignment, we implement a simple operational amplifier-based circuit. Since operational amplifiers are used commonly in circuits used to implement mathematical operations, we implement the processes of summing two voltages.

Before beginning this lab, you should be able to:

- Analyze operational amplifier-based circuits

After completing this lab, you should be able to:

- Design and build an operational amplifier-based inverting voltage amplifier

This lab exercise requires:

- Analog Discovery 2 module
- Digilent Analog Parts Kit
- Digital multimeter (optional)

Symbol Key:

- DEMO** Demonstrate circuit operation to teaching assistant; teaching assistant should initial lab notebook and grade sheet, indicating that circuit operation is acceptable.
- ANALYSIS** Analysis; include principle results of analysis in laboratory report.
- SIM** Numerical simulation (using PSPICE or MATLAB as indicated); include results of MATLAB numerical analysis and/or simulation in laboratory report.
- DATA** Record data in your lab notebook.

General Discussion:

The circuit shown in Fig. 1 is a summing amplifier circuit. Appropriate pin numbers for the OP27 operational amplifier are provided on Fig. 1. The output voltage, v_{out} , is an inverted and scaled version of the sum of the input voltages, v_a and v_b . If $R_1 = R_2$, the input voltages are not individually scaled and the output voltage is:

$$v_{out} = -\frac{R_3}{R_1}(v_a + v_b)$$

Note that if, in addition, $R_3 = R_1$, the output voltage is simply the sum of the two input voltages.

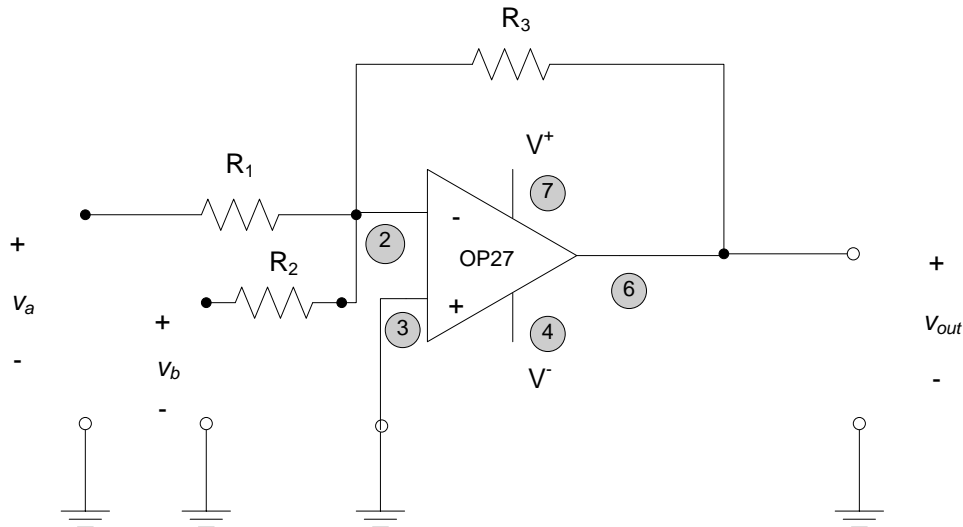


Figure 1. Summing amplifier circuit.

Pre-lab:

ANALYSIS

Design an inverting summing circuit which performs an addition of two signals. The input resistance seen by the two voltage sources (v_a and v_b) should be at least $1\text{k}\Omega$. (The input resistance of a circuit is defined as the input voltage divided by the input current. Since the inverting input terminal of the operational amplifier in Fig. 1 is a “virtual” ground, the input resistance seen by v_a and v_b are R_1 and R_2 , respectively.)

Lab Procedures:

DATA

1. Implement the design you generated in the pre-lab. Create a schematic of the circuit in your lab notebook, record actual resistance values, and label supply voltages on your schematic.

Recommended connections are as follows:

- Use V- on the Analog Discovery 2 to set the negative op-amp supply V^- to -5V ; set the positive op-amp supply V^+ to 5V using V+ on the Analog Discovery 2.
- Use one waveform generator channel to provide the input voltage v_a to your circuit, and use the other waveform generator channel to set $v_b = 1\text{V}$.
- Measure the output voltage, v_{out} .

DATA

1. Test your design with $v_a = -4\text{V}, -2\text{V}, -1\text{V}, 0\text{V}, 1\text{V}, 2\text{V}, 3\text{V},$ and 5V . Tabulate your results (v_a , v_b and v_{out}) in your lab notebook (you may assume that the values you set for v_a and v_b are correct – you do not need to measure these). Also in your lab notebook, comment on your results (make sure that you compare your results with your expectations based on your pre-lab).

DEMO

3. Demonstrate operation of your circuit to the Teaching Assistant. Have the TA initial the appropriate page(s) of your lab notebook and the lab checklist.

Real Analog Chapter 5: Lab Projects

5.4.3: Non-inverting Voltage Amplifier

In this assignment, we implement a simple operational amplifier-based circuit. Since operational amplifiers are used commonly in circuits used to implement mathematical operations, we implement the process of multiplication by a positive constant.

Before beginning this lab, you should be able to:

- Analyze operational amplifier-based circuits

After completing this lab, you should be able to:

- Design and build an operational amplifier-based non-inverting voltage amplifier

This lab exercise requires:

- Analog Discovery 2 module
- Digilent Analog Parts Kit
- Digital multimeter (optional)

Symbol Key:

- DEMO** Demonstrate circuit operation to teaching assistant; teaching assistant should initial lab notebook and grade sheet, indicating that circuit operation is acceptable.
- ANALYSIS** Analysis; include principle results of analysis in laboratory report.
- SIM** Numerical simulation (using PSPICE or MATLAB as indicated); include results of MATLAB numerical analysis and/or simulation in laboratory report.
- DATA** Record data in your lab notebook.

General Discussion:

The circuit shown in Fig. 1 is called a non-inverting amplifier. Appropriate pin numbers for the OP27 operational amplifier are provided on Fig. 1. v_{in} is the applied (input) voltage to the circuit. v_{out} is the output voltage from the circuit.

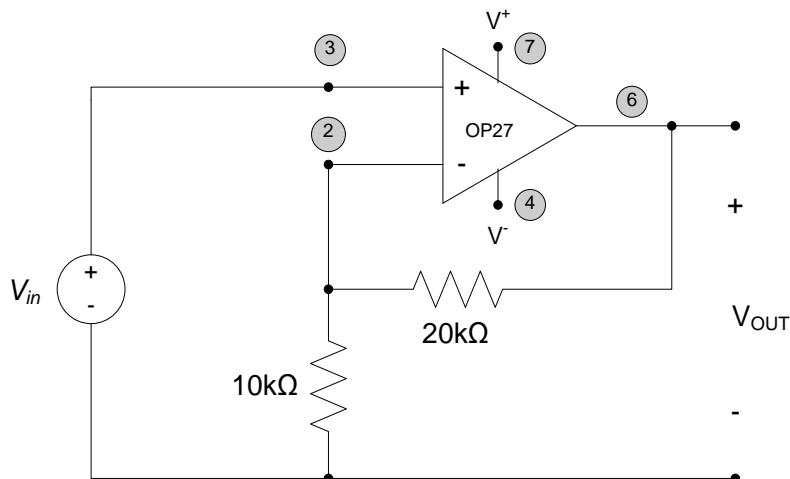


Figure 1. Inverting amplifier circuit.

Pre-lab:**ANALYSIS**

Determine the relationship between V_{in} and V_{out} for the circuit shown in Fig. 1. Why is the circuit called a non-inverting voltage amplifier?

Lab Procedures:**DATA**

1. Implement the amplifier shown in Fig. 1. Create a schematic of the circuit in your lab notebook, record actual resistance values, and label supply voltages on your schematic. Recommended connections are as follows:
 - Use V- on the Analog Discovery 2 to set the negative op-amp supply V^- to -5V; use V+ on the Analog Discovery 2 to set the positive op-amp supply V^+ to +5V.
 - Use W1 on your Analog Discovery 2 to provide the input voltage v_{in} to your circuit.
 - Measure V_{OUT} .

DATA

2. Test your design with input voltages of approximately -3V to +3V with steps of .5V. Tabulate your results (v_{in} and v_{out}) in your lab notebook. Also in your lab notebook, create a plot of V_{in} vs. V_{out} and comment on your results (make sure that you calculate a circuit gain – the rate of change of output to input – and compare it to your expectations based on your pre-lab.

DEMO

3. Demonstrate operation of your circuit to the Teaching Assistant Have the TA initial the appropriate page(s) of your lab notebook and the lab checklist.

Real Analog Chapter 5: Lab Projects

5.4.3: Difference Amplifier

In this assignment, we implement a simple operational amplifier-based circuit. Since operational amplifiers are used commonly in circuits used to implement mathematical operations, we implement the process of taking the difference between two voltages.

Before beginning this lab, you should be able to:

- Analyze operational amplifier-based circuits

After completing this lab, you should be able to:

- Design and build an operational amplifier-based difference amplifier

This lab exercise requires:

- Analog Discovery 2 module
- Digilent Analog Parts Kit
- Digital multimeter (optional)

Symbol Key:

- DEMO** Demonstrate circuit operation to teaching assistant; teaching assistant should initial lab notebook and grade sheet, indicating that circuit operation is acceptable.
- ANALYSIS** Analysis; include principle results of analysis in laboratory report.
- SIM** Numerical simulation (using PSPICE or MATLAB as indicated); include results of MATLAB numerical analysis and/or simulation in laboratory report.
- DATA** Record data in your lab notebook.

General Discussion:

The circuit shown in Fig. 1 is called a difference amplifier. v_a and v_b are the applied (input) voltages to the circuit. v_{out} is the output voltage from the circuit.

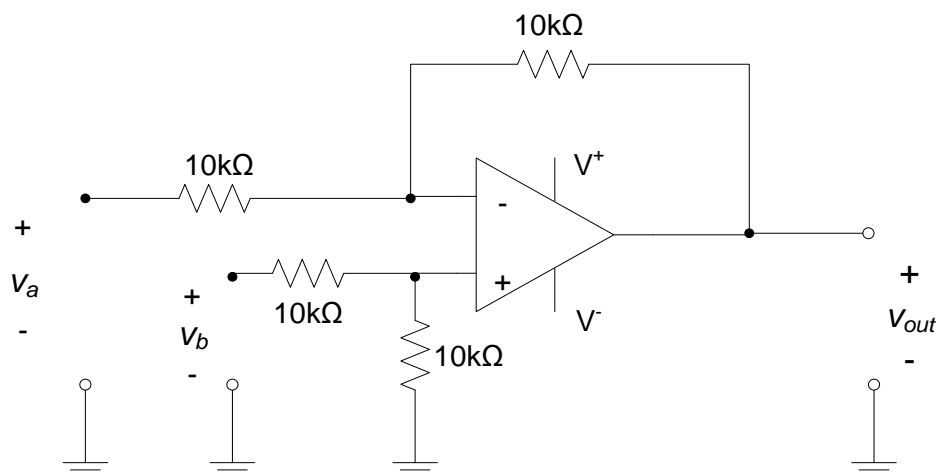


Figure 1. Difference amplifier circuit.

Pre-lab:**ANALYSIS**

Determine the relationship between v_a , v_b and V_{out} for the circuit shown in Fig. 1. Why is the circuit called a difference amplifier?

Lab Procedures:**DATA**

1. Implement the amplifier shown in Fig. 1, using an OP27 operational amplifier. Create a schematic of the circuit in your lab notebook, record actual resistance values, and label supply voltages on your schematic. Recommended connections are as follows:

- Use V- on the Analog Discovery 2 to set the negative op-amp supply V^- to -5V; use V+ on the Analog Discovery 2 to set the positive op-amp supply V^+ to +5V.
- Use AWG1 on your Analog Discovery 2 to provide the input voltage v_a to your circuit. Use AWG2 on your Analog Discovery 2 to provide the input voltage v_b to your circuit.
- Measure V_{OUT} .

DATA

2. Set $v_b = 1V$. Test your design with $v_a = -4V, -2V, -1V, 0V, 1V, 3V$, and $5V$. Tabulate your results (v_a , v_b and v_{out}) in your lab notebook. You may assume that the set values for v_a and v_b are correct, and that you do not need to measure them. Also in your lab notebook, comment on your results (including a comparison between your test results and your expectations based on your pre-lab).

DATA

3. Repeat the above tests with $v_b = -1V$. Tabulate your results (v_a , v_b and v_{out}) in your lab notebook. Also in your lab notebook, comment on your results (including a comparison between your test results and your expectations based on your pre-lab).

DEMO

4. Demonstrate operation of your circuit to the Teaching Assistant. Have the TA initial the appropriate page(s) of your lab notebook and the lab checklist.

Real Analog Chapter 5: Lab Projects

5.4.5: Temperature Measurement System Design

In this lab, we will design a simple temperature measurement system which outputs a DC voltage which indicates temperature. Our system will use a *thermistor* to indicate the temperature; the electrical resistance of the thermistor changes as the temperature changes. We will use a *Wheatstone bridge* circuit to convert this resistance change to a voltage change. The voltage output of the Wheatstone bridge circuit will be small relative to the amount of temperature change (the measurement is said to have low *sensitivity*), so we will use a *difference amplifier* to increase the overall sensitivity of the temperature measurement system.

Before beginning this lab, you should be able to:

- State rules governing ideal op-amps
- Analyze electrical circuits which include ideal op-amps
- Describe the operation of a thermistor (Background information for Lab assignments 1.4.4, 2.1.1)





After completing this lab, you should be able to:

- Design and balance a Wheatstone bridge circuit
- Design and implement an operational amplifier-based difference amplifier
- Integrate the above subsystems to create an overall temperature measurement system

This lab exercise requires:

- Analog Discovery 2 module
- Digilent Analog Parts Kit
- Digital multimeter (optional)

Symbol Key:

- | | |
|---|---|
|  | Demonstrate circuit operation to teaching assistant; teaching assistant should initial lab notebook and grade sheet, indicating that circuit operation is acceptable. |
|  | Analysis; include principle results of analysis in laboratory report. |
|  | Numerical simulation (using PSPICE or MATLAB as indicated); include results of MATLAB numerical analysis and/or simulation in laboratory report. |
|  | Record data in your lab notebook. |

General Discussion:

In this lab assignment, we will design and implement a measurement system which outputs a voltage which is indicative of temperature. A thermistor will be used to measure temperature. The resistance of the thermistor changes with temperature.

The design requirements for the system are as follows:

1. The output voltage from the system is $0V \pm 20mV$ at room temperature (approximately $25^{\circ}C$).
2. Output voltage is positive for temperatures above room temperature, negative for temperatures below room temperature.
3. Output voltage increases by a minimum of 2V over a temperature range of $25^{\circ}C$ to $37^{\circ}C$. (These temperatures correspond approximately to room temperature and body temperature, respectively.)

A common approach to this problem (and the one we will implement) is to use a Wheatstone bridge circuit in conjunction with a difference amplifier circuit to achieve the necessary sensitivity between temperature and output voltage. A block diagram of the overall system is shown in Fig. 1. The input to the overall system is the temperature of the thermistor. The thermistor converts this temperature into an output resistance. This resistance change is used in a Wheatstone bridge circuit, which converts this resistance change to a voltage change. The voltage difference output by the Wheatstone bridge is generally smaller than desired, so an amplifier is used to increase the amplitude of the overall output voltage from the system.

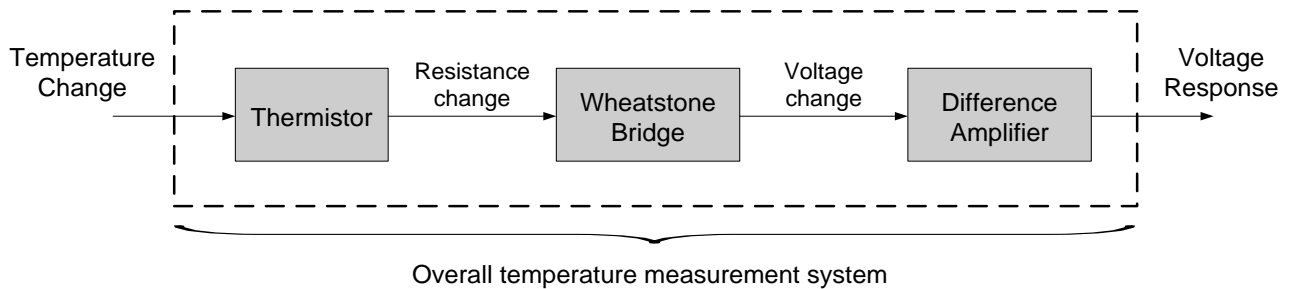


Figure 1. Overall temperature measurement system block diagram.

Pre-lab:

ANALYSIS

Read the information relative to Wheatstone bridge circuits provided in Appendix A of this lab assignment. It is not necessary to exhaustively follow all derivations provided in Appendix A, but you should be able to summarize, in a few sentences, the overall approach toward setting up and balancing a Wheatstone bridge circuit prior to coming to lab. Pay particular attention to equation (A9), which can be used to choose nominal resistances for the bridge circuit, and the practical note on adjusting the nominal values to balance the circuit. Please be sure to note that Appendix A recommends using a potentiometer to balance your Wheatstone bridge circuit.

Read the information relative to difference amplifiers provided in Appendix B of this lab assignment.

Lab Procedures:

Design approaches tend to vary from individual to individual, however, the recommended lab procedures for this assignment consist of the three discrete steps provided below. Feel free to modify these steps if you wish, but be prepared to explain your design approach to a teaching assistant.

1. Thermistor Characterization:

DATA

Measure the nominal resistance of the thermistor (when the thermistor is at room temperature) and the resistance variation from this value when the thermistor is approximately at body temperature (37°C). Apply the 37°C temperature to the thermistor by firmly grasping the thermistor between two fingers. Record these values in your lab notebook.

2. Wheatstone bridge design and balancing:

ANALYSIS

Design and build a Wheatstone bridge circuit which converts the resistance variation of the thermistor to a voltage variation. The output of this circuit should be (approximately) zero volts when the thermistor is at room temperature. Provide a schematic of your Wheatstone bridge circuit in your lab notebook, along with the desired and actual resistance values used in the circuit. Also in your lab notebook, record the voltage variation provided by the Wheatstone bridge circuit, resulting from the full range of temperature change.

DATA

Design Hint:

You may not wish to spend a lot of time balancing the bridge at this stage. It is probably more productive to roughly balance the bridge at this point, and then do a final balance after the amplification stage described in step (c) has been implemented.

DEMO

Demonstrate your Wheatstone bridge operation to a teaching assistant and have them initial your lab notebook and the lab checklist.

ANALYSIS

3. Difference amplifier design and implementation:

Design a difference amplifier which amplifies the output voltage difference from the Wheatstone bridge to the levels specified in the design requirements. The circuit schematic and governing equations for a difference amplifier are provided in Appendix B. Implement the circuit. Provide a schematic of your circuit, along with desired and actual resistance values used in your circuit in your lab notebook.

DATA

DATA

Connect your circuit to the thermistor/Wheatstone bridge assembly and measure the output voltage resulting from the temperature range provided in the specifications (approximately 25°C to 37°C). Verify that voltage increases for as temperature increases and decreases as temperature decreases. Record the range of voltages in your lab notebook, corresponding to the full range of temperature change provided in the design requirements. Compare your measured voltage response to the original design requirements. You do not need to re-design your system if the design requirements are not met, but you should provide comments in your lab notebook as to why you think the circuit behaves differently than expected.

DEMO

Demonstrate your Wheatstone bridge operation to a teaching assistant and have them initial your lab notebook and the lab checklist.

Design Hint:

1. Your amplifier circuit in step (c) may “load” your Wheatstone bridge circuit in a different manner than was done in step (b) above. (In step (b), you measured the output voltage from the Wheatstone bridge with a DMM; your difference amplifier may require the Wheatstone bridge to provide different power levels than the DMM does. These effects can be mitigated by using relatively high resistance values in your difference amplifier. (Recall the discussion of input resistance provided in Lab 2. The DMM has a very high input resistance; an alternate circuit may have lower input resistance, and thus make different power demands on the Wheatstone bridge. The change in power requirements may affect the operation of the Wheatstone bridge.
2. After implementing your amplifier circuit in step (c), you may wish to re-balance the Wheatstone bridge circuit. The added sensitivity of the overall system will make the balancing process simpler.
3. The amplifier circuit of Lab 4 (the inverting voltage amplifier) is not appropriate for the implementation of step (c) of this lab assignment. The inverting voltage amplifier of Lab 4 amplifies a voltage which is relative to the ground of the amplifier. In our current application, we have two voltages which are both measured relative to ground; we need to amplify the difference between these voltages.

Appendix A: Wheatstone Bridge Circuits

Wheatstone bridge circuits are most often used to convert variations in resistance to variations in voltage. Wheatstone bridges are commonly used in measurement systems, as a number of common sensors provide a resistance variation in response to some external influence. For example, *thermistors* change resistance in response to temperature changes, *strain gages* change resistance in response to deformations, and *photoconductive transducers* change resistance in response to changes in light intensity. Wheatstone bridges are generally used in conjunction with these sensors in order to convert these resistance changes to voltage changes since voltages are generally easier to record and transmit than resistances.

Wheatstone bridge sensitivity to resistance variations:

A Wheatstone bridge circuit is shown in Fig. A1. The bridge is generally presented as shown in the figure to the left; we will use the equivalent circuit shown to the right in our analysis. A Wheatstone bridge is commonly used to convert a variation in resistance to a variation in voltage. A constant supply voltage V_S is applied to the circuit. The resistors in the circuit all have a nominal resistance of R ; the variable resistor has a variation ΔR from this nominal value. The output voltage v_{ab} indicates the variation ΔR in the variable resistor. The variable resistor in the network is often a transducer whose resistance varies dependent upon some external variable such as temperature.

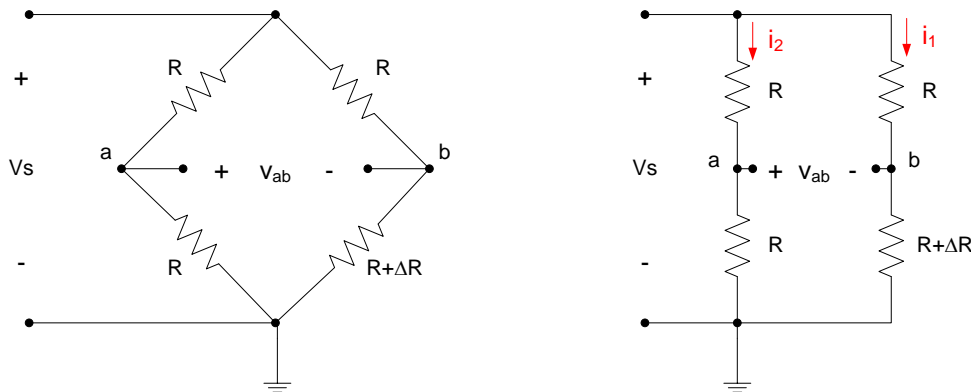


Figure A1. Wheatstone bridge circuit.

By voltage division, the voltages v_b and v_a (relative to ground) are

$$v_b = \frac{(R+\Delta R)}{2R+\Delta R} V_S \text{ and } v_a = R i_2 = \frac{V_S \cdot R}{2R} = \frac{V_S}{2} \quad \text{Eq. A1}$$

The voltage v_{ab} is then

$$v_{ab} = v_a - v_b = \left(\frac{1}{2} - \frac{R+\Delta R}{2R+\Delta R} \right) V_S = \left(\frac{(2R+\Delta R) - 2(R+\Delta R)}{2(2R+\Delta R)} \right) V_S = -\frac{\Delta R}{2(2R+\Delta R)} \cdot V_S \quad \text{Eq. A2}$$

For the case in which $\Delta R \ll 2R$, this simplifies to:

$$v_{ab} \approx -\frac{V_S}{4R} \Delta R \quad \text{Eq. A3}$$

and the output voltage is proportional to the change in resistance of the variable resistor.

Balancing the Wheatstone Bridge Circuit:

In Fig. A1, it is assumed that all four resistances in the Wheatstone bridge have identical nominal values. In the case when the Wheatstone bridge output voltage is to result from a varying resistance sensor, this requires one to obtain three resistors with resistance exactly equal to the sensor's nominal resistance. In general, this is not possible. In this chapter, we present an approach for balancing a Wheatstone bridge so that the output voltage is zero when the variable resistance is at its nominal value (i.e. ΔR in Fig. A1 is zero), even if the other three resistances in the bridge are not identical.

A schematic of a Wheatstone bridge with non-equivalent resistances is shown in Fig. A2. R_{Nom} is the nominal value of a variable resistance; it is desired that the output voltage from the bridge circuit is zero for this resistance value. R_2 and R_3 are fixed resistors; R_1 is a variable resistor which will be used to *balance* the bridge circuit.

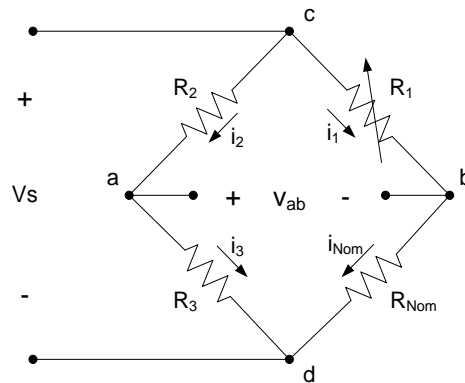


Figure A2. Wheatstone bridge.

Governing equations for balanced circuit:

If the Wheatstone bridge of Fig. A2 is balanced, $v_{ab} = 0$, and

$$i_2 R_2 = i_1 R_1 \quad \text{Eq. A4}$$

KCL at nodes a and b tells us that $i_2 = i_3$ and $i_1 = i_{Nom}$. Using this along with Ohm's law gives:

$$V_S = i_2 (R_2 + R_3) \quad \text{Eq. A5}$$

And

$$V_S = i_1 (R_1 + R_{Nom}) \quad \text{Eq. A6}$$

Equating (A4) and (A5) and taking advantage of equation (A3) provides

$$i_3 R_3 = i_1 R_{Nom} \quad \text{Eq. A7}$$

Or

$$\frac{i_1}{i_2} = \frac{R_3}{R_{Nom}} \quad \text{Eq. A8}$$

From equation (A3), $\frac{i_1}{i_2} = \frac{R_2}{R_1}$, so equation (A7) becomes

$$\frac{R_2}{R_1} = \frac{R_3}{R_{Nom}} \quad \text{Eq. A9}$$

and the variable resistance can be set according to:

$$R_1 = \frac{R_2 R_{Nom}}{R_3} \quad \text{Eq. A10}$$

Where R_{Nom} is the variable resistance for which the circuit is balanced.

Practical Note:

The value of the resistance R_1 must be set very accurately, so it is common to use a variable-resistance potentiometer to set the resistance R_1 . Specifically, a relatively large-resistance potentiometer can be placed in parallel with a fixed resistor with a resistance slightly higher than the value specified by equation (9) in order to provide the ability to provide very fine adjustments to the value of the resistor R_1 .

Balancing the bridge circuit is commonly performed by setting the variable resistance to its nominal value R_{Nom} , and monitoring the voltage v_{ab} while adjusting the resistance R_1 . The resistance R_1 is at its desired value when the voltage v_{ab} is zero. (The actual value of R_1 required to balance the circuit is generally not important – setting the output voltage to zero at the nominal variable resistance is the ultimate goal.)

Appendix B: Difference Amplifier

The circuit shown in Fig. B1 is called a difference amplifier. The output of the circuit (v_{out} in Fig. B1) is proportional to the difference between the two inputs, v_a and v_b . With the four independent resistances (R_1 , R_2 , R_3 , and R_4) shown in Fig. B1, the input voltages can be scaled independently. In order to apply the same scaling factor to both inputs, we can apply a requirement to the choice of the resistances. If we choose

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

then the expression for the output voltage becomes:

$$v_{out} = \frac{R_2}{R_1} (v_b - v_a)$$

The output voltage is, then, a scaled version of the difference between the two input voltages. Note that if v_a is zero, the circuit is a *non-inverting* amplifier. (The output is a scaled version of the single input v_b , with no sign change.) Also note that if $R_1 = R_2 = R_3 = R_4$, the circuit simply subtracts the voltage v_a from the voltage v_b .

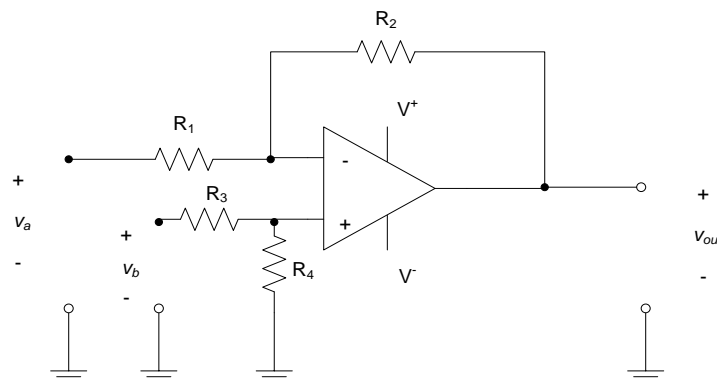


Figure B1. Difference amplifier circuit.

Real Analog Chapter 5: Lab Worksheets

5.4.5: Temperature Measurement System (100 points total)

1. Thermistor Characterization (10 pts total)
 - a. In the space below, provide the measured sensor resistance variation over the specified temperature range. (10 pts)

2. Wheatstone Bridge Design (45 pts total)
 - a. Sketch below the circuit to convert resistance variation to voltage variation. Include desired and actual values for circuit components (e.g. resistor values). (15 pts)

 - b. Provide below the measured output voltage for nominal (approximately 25°C) temperature. (5 pts)

 - c. Provide below the measured range of output voltage over specified temperature range (25°C to 37°C). (10 pts)

 - d. **DEMO:** Have a teaching assistant initial this sheet, indicating that they have observed your circuits' operation. (15 pts)

TA Initials: _____

3. Difference Amplifier Design (45 pts total)
 - a. Provide a sketch of difference amplifier circuit, including desired and actual values for circuit components. (10 pts)

- b. Verify that output voltage increases as temperature increases and decreases as temperature decreases. Provide below the measured range of output voltage over specified temperature range (25°C to 37°C) (10 pts)

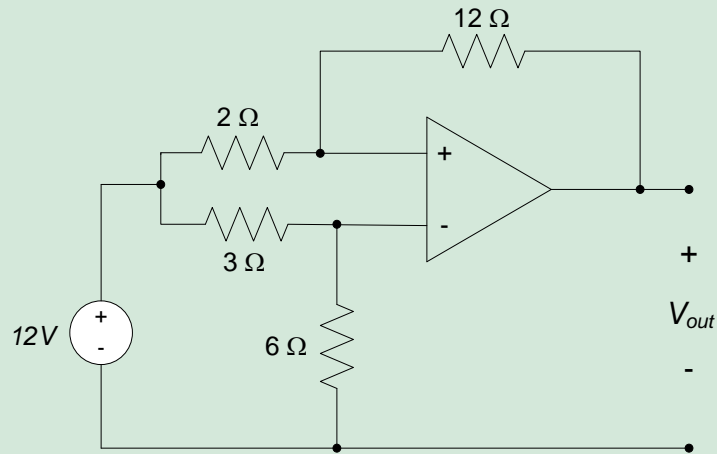
- c. **DEMO:** Have a teaching assistant initial this sheet, indicating that they have observed your circuits' operation. (15 pts)

TA Initials: _____

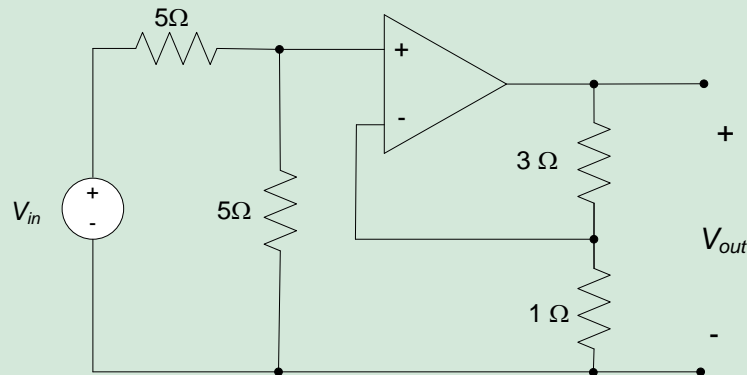
- d. Discuss below the performance of your temperature measurement system performance and comparison with design specifications. (10 pts)

Real Analog Chapter 5: Homework

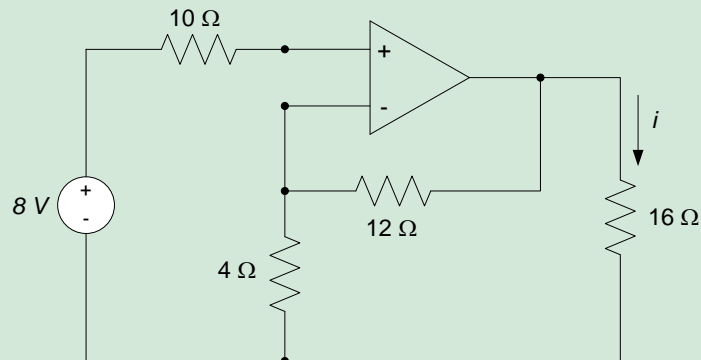
5.1 For the circuit shown, find V_{out} .



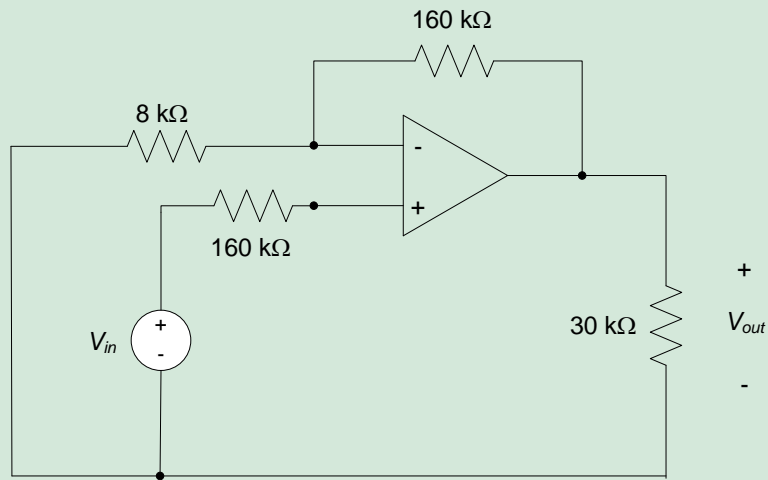
5.2 For the circuit shown, find the relationship between V_{out} and V_{in} .



5.3 For the circuit below find i , the current through the $16\ \Omega$ resistor.

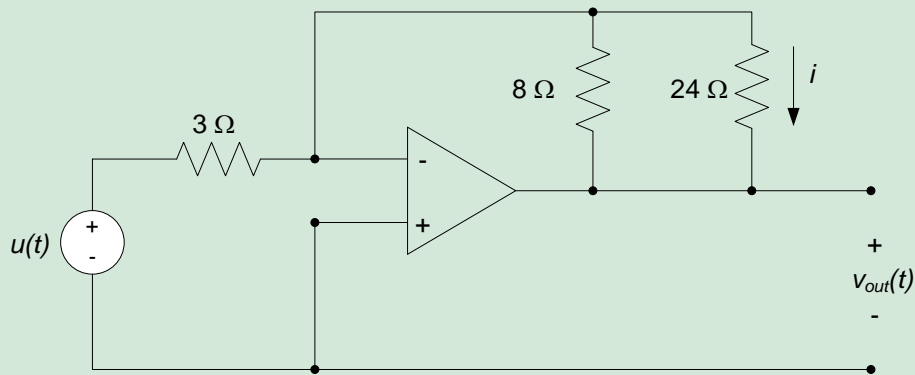


5.4 For the circuit shown, find V_{out} , the voltage across the $30\text{k}\Omega$ resistor.

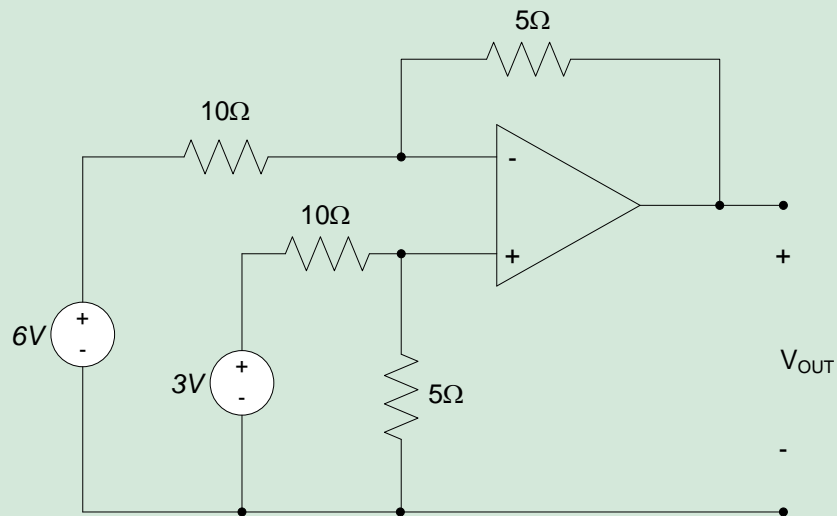


5.5 For the circuit below, determine:

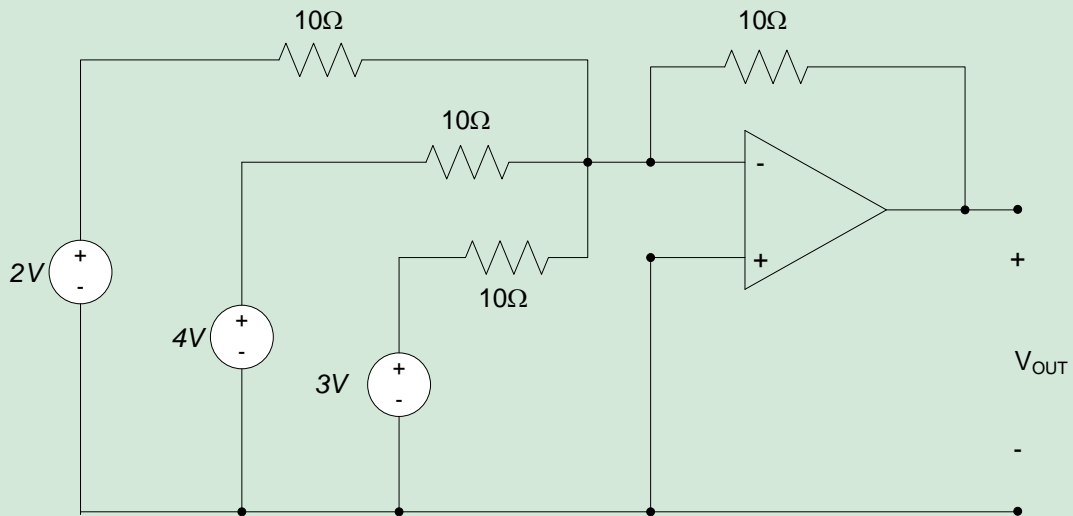
- a. The current, i , and voltage v_{out} if $u(t) = 4V$
- b. The current, i , and voltage v_{out} if $u(t) = 2\cos(5t)$



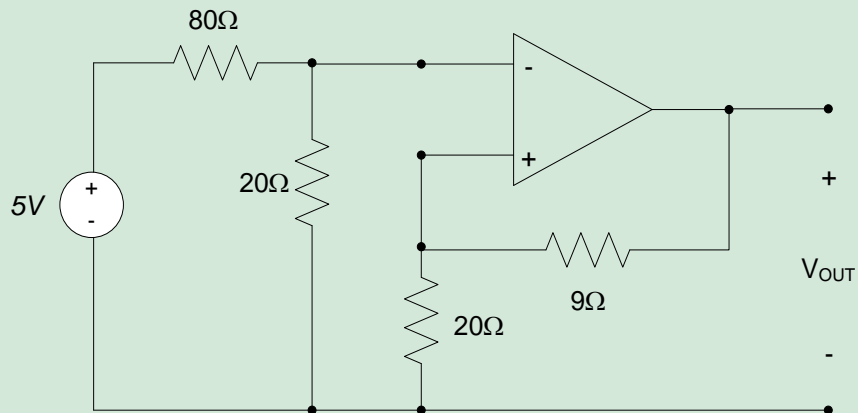
5.6 For the circuit shown, find V_{out} .



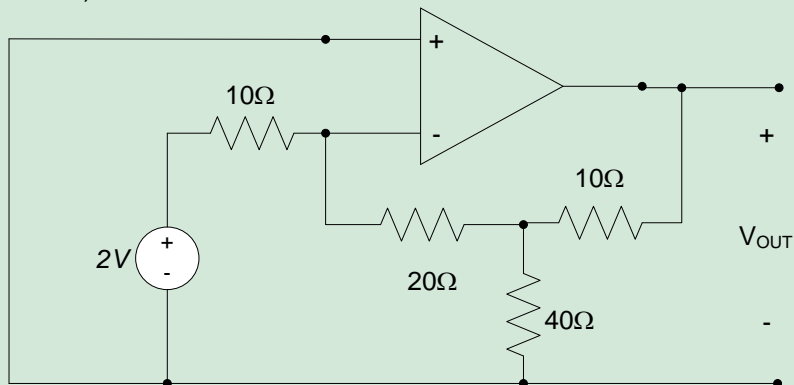
5.7 For the circuit shown, find V_{out} .



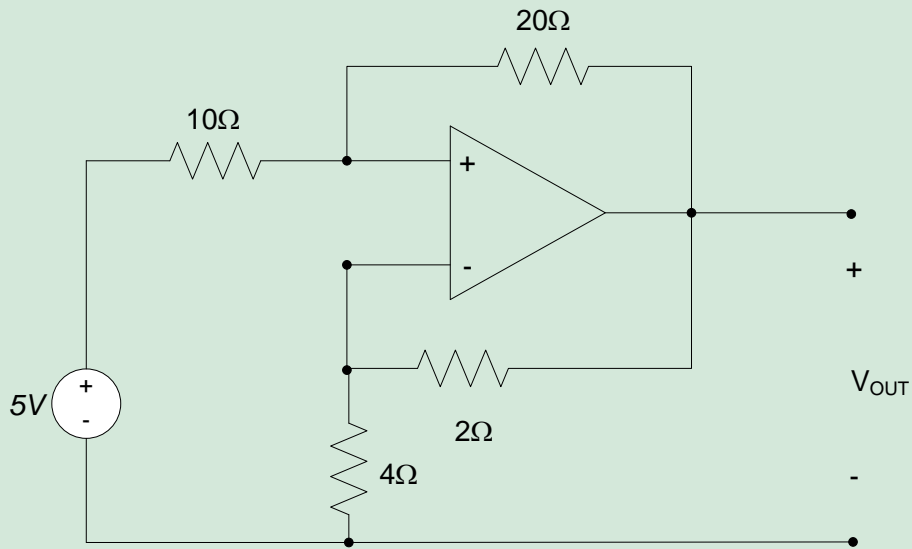
5.8 For the circuit shown, find V_{out} .



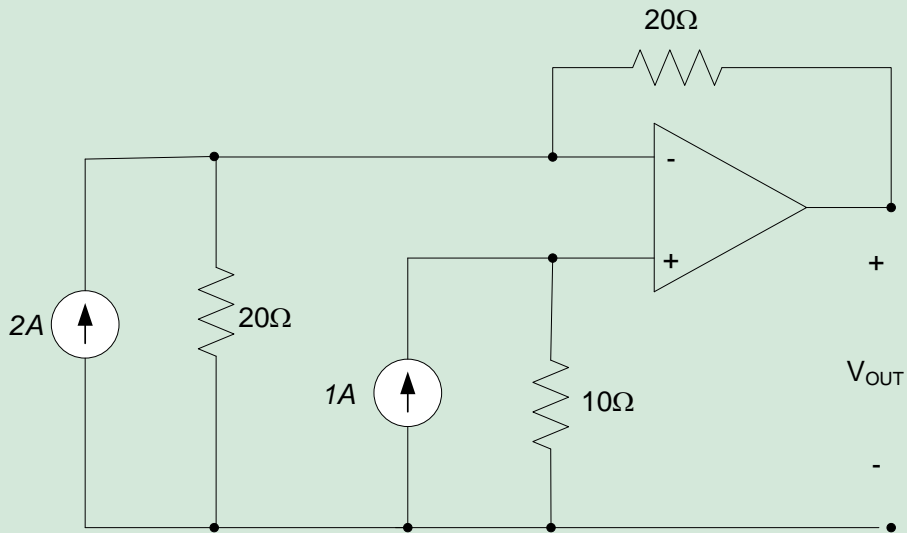
5.9 For the circuit shown, find V_{out} .



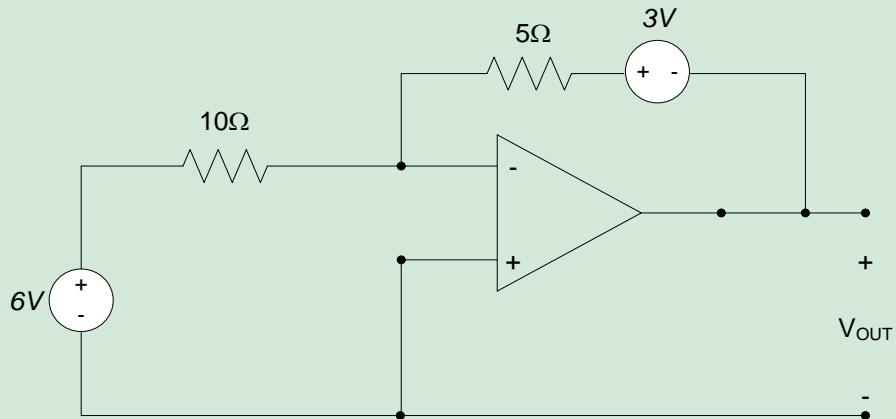
5.10 For the circuit shown, find V_{out} .



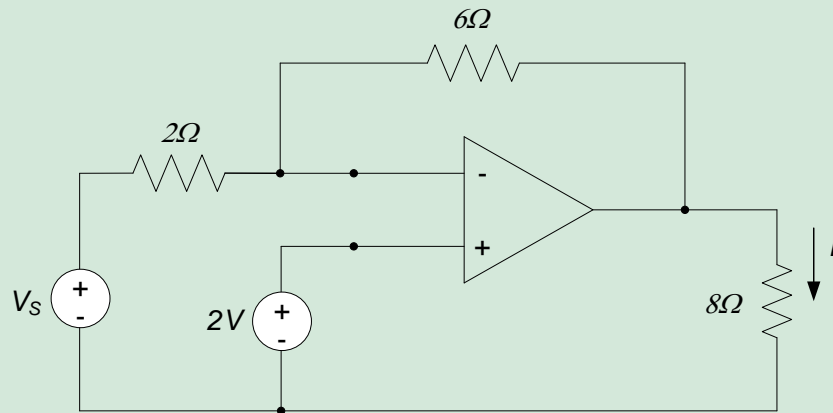
5.11 For the circuit shown, find V_{out} .



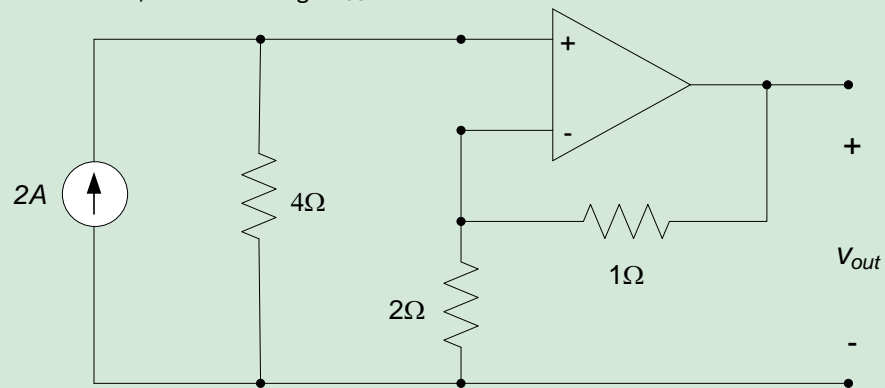
5.12 For the circuit shown, find V_{out} .



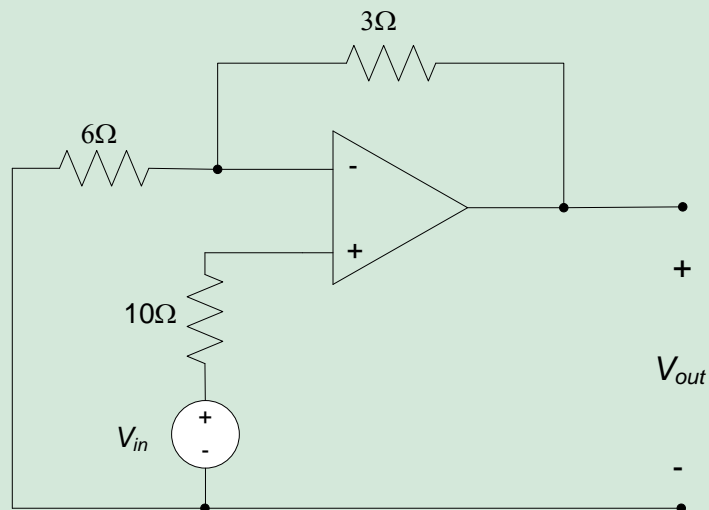
5.13 Find the current I in the circuit below. Your answer may be a function of the voltage supply V_s . Clearly show all your work.



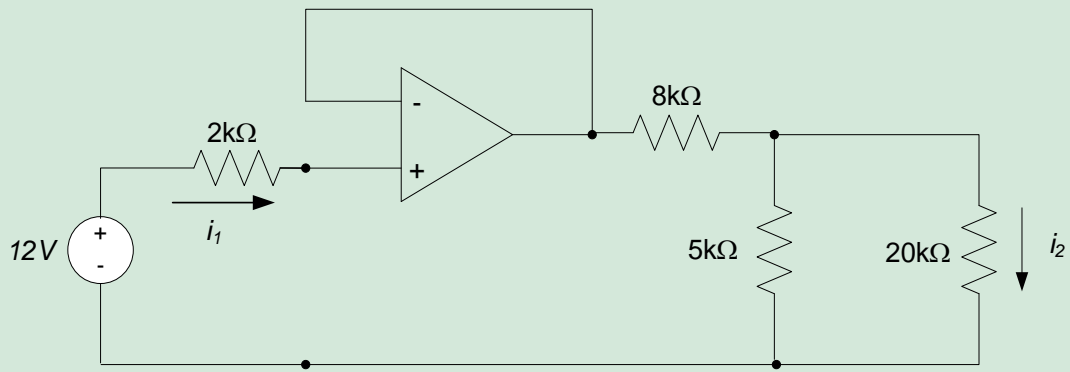
5.14 For the circuit shown, find the voltage V_{out} .



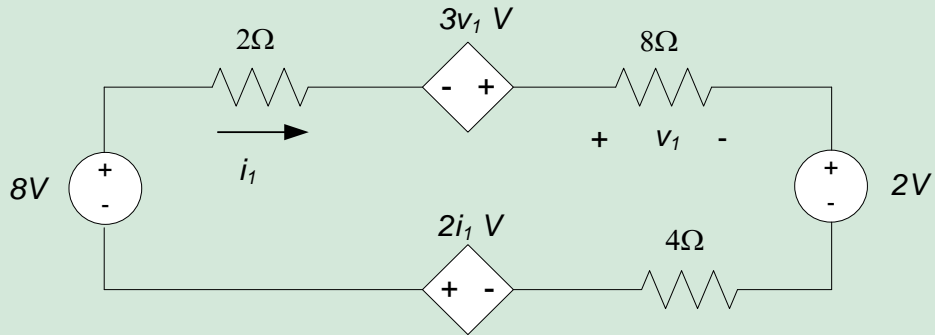
5.15 For the circuit shown, find V_{out} as a function of V_{in} .



5.16 Determine the currents i_1 and i_2 in the circuit below.



5.17 Determine the voltage v_1 in the circuit below.



5.18 Determine the current i_2 in the circuit below.

