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Real Analog Chapter 2: Circuit Reduction

2 Introduction and Chapter Objectives

In Chapter 1, we presented Kirchhoff's laws (which govern the interaction between circuit elements) and Ohm's law (which governs the voltage-current relationships for resistors). These analytical tools provide us with the ability to analyze any circuit containing only resistors and ideal power supplies. However, we also saw in Chapter 1 that a circuit analysis, which relies strictly on a brute-force application of these tools can become complex rapidly - we essentially must use as our unknowns the voltage differences across all resistors and the currents through all resistors. This generally results in a large number of unknowns and a correspondingly large number of equations, which must be written and solved in order to analyze any but the simplest circuit.

In the next few chapters, we will still apply Kirchhoff's laws and Ohm's law in our circuit analysis, but we will focus on improving the efficiency of our analyses. Typically, this improvement in efficiency is achieved by reducing the number of unknowns in the circuit, which reduces the number of equations, which must be written to describe the circuit's operation.

In this chapter, we introduce analysis methods based on *circuit reduction*. Circuit reduction consists of combining resistances in a circuit to a smaller number of resistors, which are (in some sense) equivalent to the original resistive network. Reducing the number of resistors, of course, reduces the number of unknowns in a circuit.

We begin our discussion of circuit reduction techniques by presenting two specific, but very useful, concepts: *Series* and *parallel* resistors. These concepts will lead us to *voltage* and *current divider* formulas. We then consider reduction of more general circuits, which typically corresponds to identifying multiple sets of series and parallel resistances in a complex resistive network. This chapter then concludes with two important examples of the application of circuit reduction techniques: the analysis of *non-ideal power sources* and *non-ideal measurement* devices; without an understanding of these devices, it is impossible to build practical circuits or understand the consequences of a voltage or current measurement.

After Completing this Chapter, You Should be Able to:

- Identify series and parallel combinations of circuit elements
- Determine the equivalent resistance of series resistor combinations
- Determine the equivalent resistance of parallel resistor combinations
- State voltage and current divider relationships from memory
- Determine the equivalent resistance of electrical circuits consisting of series and parallel combinations of resistors
- Sketch equivalent circuits for non-ideal voltage and current meters
- Analyze circuits containing non-ideal voltage or current sources
- Determine the effect of non-ideal meters on the parameter being measured

2.1 Series Circuit Elements and Voltage Division

There are a number of common circuit element combinations that are quite easily analyzed. These “special cases” are worth noting since many complicated circuits contain these circuit combinations as sub-circuits. Recognizing these sub-circuits and analyzing them appropriately can significantly simplify the analysis of a circuit.

This chapter emphasizes two important circuit element combinations: elements in series and elements in parallel. Also discussed is the use of these circuit element combinations to reduce the complexity of a circuit’s analysis.

2.1.1 Series Connections

Circuit elements are said to be connected in *series* if all of the elements carry the same current. An example of two circuit elements connected in series is shown in Fig. 2.1. Applying KCL at node a and taking currents out of the node as positive we see that:

$$-i_1 + i_2 = 0$$

Or

$$i_1 = i_2 \qquad \text{Eq. 2.1}$$

Equation (2.1) is a direct outcome of the fact that the (single) node a in Fig. 2.1 interconnects only two elements - there are no other elements connected to this node through which current can be diverted. This observation is so apparent (in many cases[1]) that equation (2.1) is generally written by inspection for series elements such as those shown in Fig. 2.1 without explicitly writing KCL.

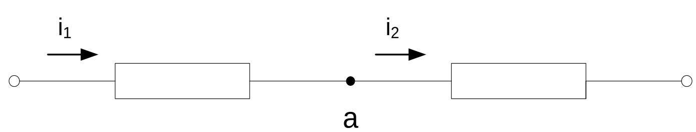

Figure 2.1. Circuit elements connected in series.

When resistors are connected in series, a simplification of the circuit is possible. Consider the resistive circuit shown in Fig. 2.2(a). Since the resistors are in series, they both carry the same current. Ohm's law gives:

$$v_1 = R_1 i$$

$$v_2 = R_2 i \qquad \text{Eq. 2.2}$$

Applying KVL around the loop:

$$-v + v_1 + v_2 = 0 \Rightarrow v = v_1 + v_2 \qquad \text{Eq. 2.3}$$

Substituting equations (2.2) into equation (2.3) and solving for the current *i* results in:

$$i = \frac{v}{R_1 + R_2} \qquad \text{Eq. 2.4}$$

Now consider the circuit of Fig. 2.2(b). Application of Ohm's law to this circuit and solution for the current *i* gives:

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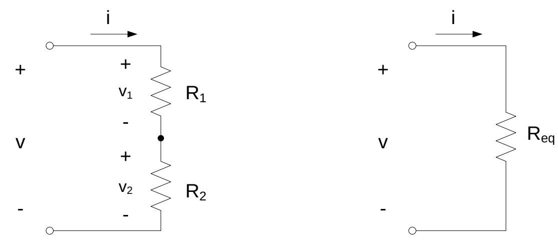
[1] If there is any doubt whether the elements are in series, apply KCL! Assuming elements are in series which are not in series can have disastrous consequences.

$$i = \frac{v}{R_{eq}} \qquad \text{Eq. 2.5}$$



(a) Series resistors *(b) Equivalent Circuit*

Figure 2.2. Series resistors and equivalent circuit.

Comparing equation (2.4) with equation (2.5), we can see that the circuits of Figs. 2.2(a) and 2.2(b) are indistinguishable if we select:

$$R_{eq} = R_1 + R_2 \qquad \text{Eq. 2.6}$$

Figures 2.2(a) and 2.2(b) are called *equivalent circuits* if the equivalent resistance of Fig. 2.2(b) is chosen as shown in equation (2.6). R_{eq} of equation (2.6) is called the equivalent resistance of the series combination of resistors R_1 and R_2.

This result can be generalized to a series combination of N resistances as follows:

A series combination of N resistors $R_1, R_{2,} \cdots, R_N$ can be replaced with a single equivalent resistance $R_{eq} = R_1 + R_2 + \cdots + R_N$. The equivalent circuit can be analyzed to determine the current through the series combination of resistors.

2.1.2 Voltage Division

Combining equations (2.2) with equation (2.4) results in the following expressions for V_i and v_2:

$$v_1 = \frac{R_1}{R_1+R_2} v \qquad \text{Eq. 2.7}$$

$$v_2 = \frac{R_2}{R_1+R_2} v \qquad \text{Eq. 2.8}$$

These results are commonly called *voltage divider* relationships, because they state that the total voltage drop across a series combination of resistors is divided among the individual resistors in the combination. The ratio of each individual resistor's voltage drop to the overall voltage drop is the same as the ratio of the individual resistance to the total resistance.

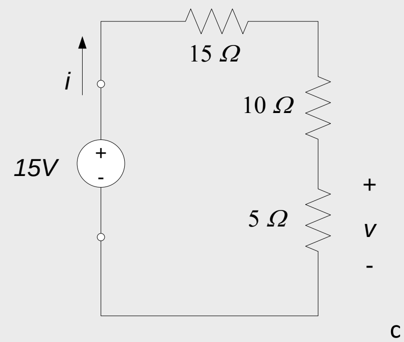
The above results can be generalized for a series combination of N resistance as follows:

The voltage drop across any resistor in a series combination of N resistances is proportional to the total voltage drop across the combination of resistors. The constant of proportionality is the same as the ratio of the individual resistor value to the total resistance of the series combination. For example, the voltage drop of the k^{th} resistance in a series combination of resistors given by:

$$v_k = \frac{R_k}{R_1+R_2+\cdots+R_N} v \qquad \text{Eq. 2.9}$$

Example 2.1

For the circuit below, determine the voltage across the 5Ω resistor, *v*, the current supplied by the source, *i*, and the power supplied by the source.



The voltage across the 5Ω resistor can be determined from our voltage divider relationship:

$$v = \left[\frac{5\Omega}{5\Omega + 15\Omega + 10\Omega}\right] \cdot 15V = \frac{5}{30} \cdot 15V = 2.5V$$

The current supplied by the source can be determined by dividing the total voltage by the equivalent resistance:

$$i = \frac{15V}{R_{eq}} = \frac{15V}{5\Omega + 15\Omega + 10\Omega} = \frac{15V}{30\Omega} = 0.5A$$

The power supplied by the source is the product of the source voltage and the source current:

$$P = iv = (0.5A)(15V) = 7.5W$$

We can double-check the consistency between the voltage *V* and the current *i* with Ohm's law. Applying Ohm's law to the 5Ω resistor, with a 0.5A current, results in $v = (5\Omega)(0.5A) = 2.5V$, which agrees with the result obtained using the voltage divider relationship.

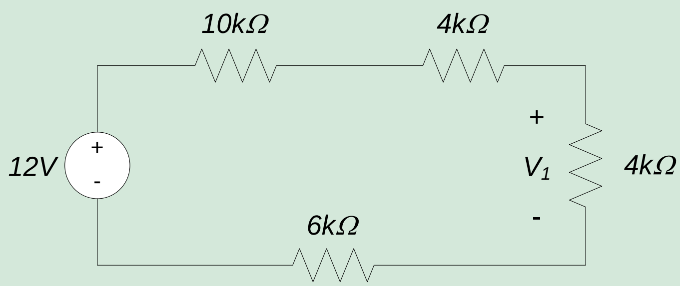
Section Summary:

- If only two elements connect at a single node, the two elements are in *series*. A more general definition, however, is that circuit elements in series all share the same current - this definition allows us to determine series combinations that contain more than two elements. Identification of series circuit elements allows us to simplify our analysis, since there is a reduction in the number of unknowns: there is only a single unknown current for all series elements.
- A series combination of resistors can be replaced by a single *equivalent resistance*, if desired. The equivalent resistance is simply the sum of the individual resistances in the series combination. Therefore, a series combination of *N* resistors $R_1, R_2, \cdots, R_N$ can be replaced with a single equivalent resistance $R_{eq} = R_1 + R_2 + \cdots + R_N$.
- If the total voltage difference across a set of series is known, the voltage differences across any individual resistor can be determined by the concept of *voltage division*. The term voltage division comes from the fact that the voltage drop across a series combination of resistors is divide among the individual resistors. The ratio between the voltage difference across a particular resistor and the total voltage difference is the same as the ratio between the resistance of that resistor and the total resistance of the combination. If V_k is the voltage across the k_{th} resistor, and R_{TOT} is the total resistance of the series combination, the mathematical statement of this concept is:

$$\frac{v_k}{v_{TOT}} = \frac{R_k}{R_{TOT}}$$

2.1 Exercises

1. Determine the voltage V_1 in the circuit below.



2.2 Parallel Circuit Elements and Current Division

Circuit elements are said to be connected in *parallel* if all of the elements share the same pair of nodes. An example of two circuit elements connected in parallel is shown in Fig. 2.3. Applying KVL around the loop of Fig. 2.3 results in:

$$v_1 = v_2 \qquad \text{Eq. 2.10}$$

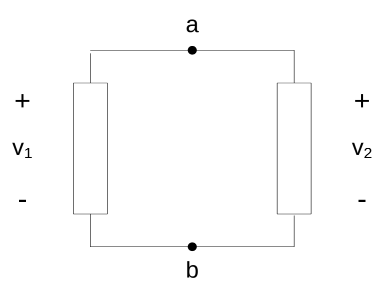


Figure 2.3. Parallel connection of circuit elements.

We can simplify circuits, which consist of resistors connected in parallel. Consider the resistive circuit shown in Fig. 2.4(a). The resistors are connected in parallel, so both resistors have a voltage difference of v. Ohm's law applied to each resistor results in:

$$i_1 = \frac{v}{R_1}$$

$$i_2 = \frac{v}{R_2} \qquad \text{Eq. 2.11}$$

Applying KCL at node a:

$$i = i_1 + i_2 \qquad \text{Eq. 2.12}$$

Substituting equations (2.11) into equation (2.12):

$$i = \left[\frac{1}{R_1} + \frac{1}{R_2}\right] v \qquad \text{Eq. 2.13}$$

Or

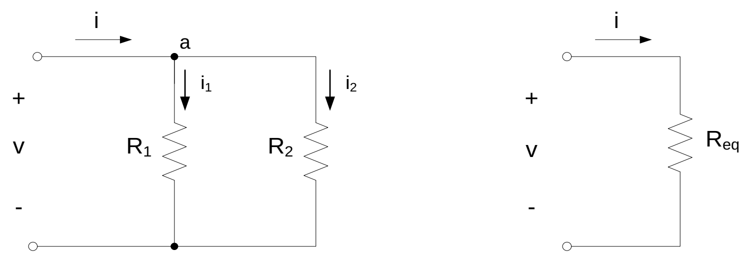
$$v = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}} \cdot i \qquad \text{Eq. 2.14}$$

If we set $R_{eq} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}$, we can draw Fig. 2.4(b) as being equivalent to Fig. 2.4(b).

We can generalize this result for *N* parallel resistances:

A parallel combination of *N* resistors $R_1, R_2, \cdots, R_N$ can be replaced with a single equivalent resistance:

$$R_{eq} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\cdots\frac{1}{R_N}} \qquad \text{Eq. 2.15}$$



(a) Parallel resistance combination *(b) Equivalent circuit*

Figure 2.4. Parallel resistances and equivalent circuit.

For the special case of two parallel resistances, R_1 and R_2, the equivalent resistance is commonly written as:

$$R_{eq} \frac{R_1 R_2}{R_1+R_2} \qquad \text{Eq. 2.16}$$

This alternative way to calculate R_{eq} can be also used to calculate R_{eq} for larger numbers of parallel resistors since any number of resistors could be combined two at a time.

2.2.1 Current Division

Substituting equation (2.14) into equations (2.11) results in:

$$i_1 = \frac{1}{R_1} \cdot \frac{i}{\frac{1}{R_1}+\frac{1}{R_2}} \qquad \text{Eq. 2.17}$$

Simplifying:

$$i_1 = \frac{R_2}{R_1+R_2} \qquad \text{Eq. 2.18}$$

Likewise, for the current i_2:

$$i_2 = \frac{R_1}{R_1+R_2} \qquad \text{Eq. 2.19}$$

Equations (2.18) and (2.19) are the current *divider relationships* for two parallel resistances, so called because the current into the parallel resistance combination is divided between the two resistors. The ratio of one resistor's current to the overall current in the same as the ratio of the other resistance to the total resistance.

The above results can be generalized for a series combination of *N* resistances. By Ohm's law, $v = R_{eq} i$. Substituting our previous result for the equivalent resistance for a parallel combination of *N* resistors results in:

$$v = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\cdots\frac{1}{R_N}} \cdot i$$ Eq. 2.20

Since the voltage difference across all resistors is the same, the current through the k^th resistor is, by Ohm's law:

$$i_k \frac{v}{R_k}$$ Eq. 2.21

Where R_k is the resistance of the k^th resistor. Combining equations (2.20) and (2.21) gives:

$$i_k = \frac{\frac{1}{R_k}}{\frac{1}{R_1}+\frac{1}{R_2}+\cdots\frac{1}{R_N}} \cdot i$$ Eq. 2.22

It is often more convenient to provide the generalized result of equation (2.20) in terms of the conductance of the individual resistors. Recall that the conductance is the reciprocal of the resistance, $G = \frac{1}{R}$. Thus, equation (2.22) can be re-expressed as follows:

The Current through any resistor in a parallel combination of *N* resistances is proportional to the total current into the combination of resistors. The constant of proportionality is the same as the ratio of the conductance of the individual resistor value to the total conductance of the parallel combination. For example, the current through the k^th resistance in a parallel combination of resistors is given by:

$$i_k = \frac{G_k}{G_1+G_2+\cdots+G_N} i$$ Eq. 2.23

Where *i* is the total current through the parallel combination of resistors.

One final comment about notation: two parallel bars are commonly used as shorthand notation to indicate that two circuit elements are in parallel. For example, the notation $R_1 \parallel R_2$ indicates that the resistors R_1 and R_2 are in parallel. The notation $R_1 \parallel R_2$ is often used as shorthand notation for the equivalent resistance of the parallel resistance combination, in lieu of equation (2.16).

Double-checking results for parallel resistances:

- The equivalent resistance for a parallel combination of *N* resistors will always be less than the smallest resistance in the combination. In fact, the equivalent resistance will always obey the following inequalities:

$$\frac{R_{min}}{N} \leq R_{eq} \leq R_{min}$$

- Where R_{min} is the smallest resistance value in the parallel combination.
- In a parallel combination of resistances, the resistor with the smallest resistance will have the largest current and the resistor with the largest resistance will have the smallest current.

Section Summary

- If several elements interconnect the same two nodes, the two elements are in *parallel*. A more general definition, however, is that circuit elements in parallel all share the same voltage difference. As with series circuit elements, identification of parallel circuit elements allows us to simplify our analysis, since there is a reduction in the number of unknowns: there is only a single unknown voltage difference for all of the parallel elements.
- A parallel combination of resistors can be replaced by a single *equivalent resistance*, if desired. The conductance of the parallel combination is simply the sum of the individual conductance of the parallel

resistors. Therefore, a parallel combination of N resistors $R_1, R_2, \cdots, R_N$ can be replaced with a single equivalent resistance:

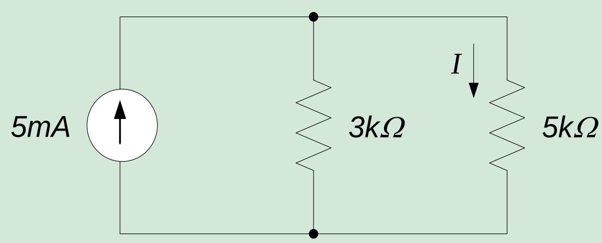
$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots \frac{1}{R_N}}$$

- If the total current through a set of parallel resistors is known, the current through any individual resistor can be determined by the concept of *current division*. The term current division comes form the fact that the current through a parallel combination of resistors is divided among the individual resistors. The ratio between the current through a particular resistor and the total current is the same as the ratio between the conductance of that resistor and the total conductance of the combination. If i_k is the voltage across the k[th] resistor, i_{TOT}, is the total current through the parallel combination, G_k is the conductance of the k[th] resistor, and G_{TOT} is the total conductance of the parallel combination, the mathematical statement of this concept is:

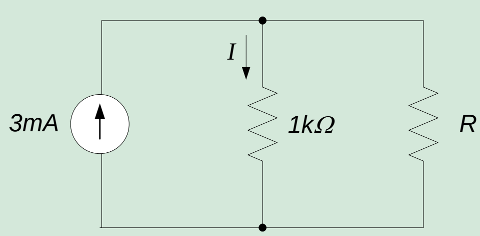
$$\frac{v_k}{i_{TOT}} = \frac{G_k}{G_{TOT}}$$

1.2 Exercises

1. Determine the value of *I* in the circuit below.



2. Determine the value of *R* in the circuit below which makes I=2mA.



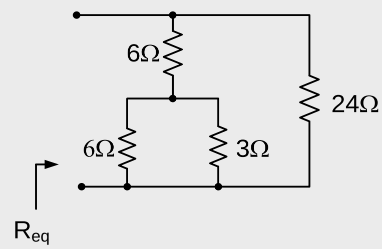
2.3 Circuit Reduction and Analysis

The previous results give us an ability to potentially simplify the analysis of some circuits. This simplification results if we can use *circuit reduction* techniques to convert a complicated circuit to a simpler, but equivalent, circuit which we can use to perform the necessary analysis. Circuit reduction is not always possible, but when it is applicable it can significantly simplify the analysis of a circuit.

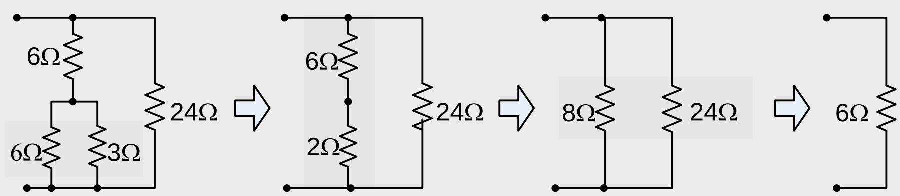
Circuit reduction relies upon identification of parallel and series combinations of circuit elements. The parallel and series elements are then combined into equivalent elements and the resulting *reduced* circuit is analyzed. The principles of circuit reduction are illustrated below in a series of examples.

Example 2.2

Determine the equivalent resistance seen by the terminals of the resistive network shown below.

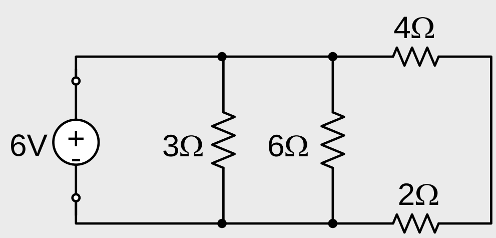


The sequence of operations performed is illustrated below. The 6Ω and 3Ω resistances are combined in parallel to obtain an equivalent 2Ω resistance. This 2Ω resistance and the remaining 6Ω resistance are in series, these are combined into an equivalent 8Ω resistance. Finally, this 8Ω resistor and the 24Ω resistor are combined in parallel to obtain an equivalent 6Ω resistance. Thus, the equivalent resistance of the overall network is 6Ω.



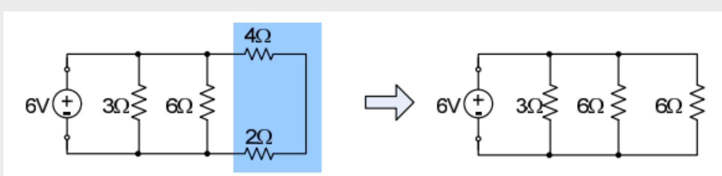
Example 2.3

In the circuit below, determine the power delivered by the source.

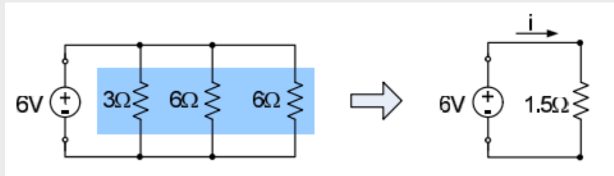


In order to determine power delivery, we need to determine the total current provided by the source to the rest of the circuit. We can determine current easily if we convert the resistor network to a single, equivalent resistance. A set of step for doing this are outlined below.

Step 1: The 4-ohm and 2-ohm resistors, highlighted in the figure to the left in blue, are in series. Series resistances add directly, so these can be replaced with a single 6-ohm resistor, as shown on the figure to the right below.



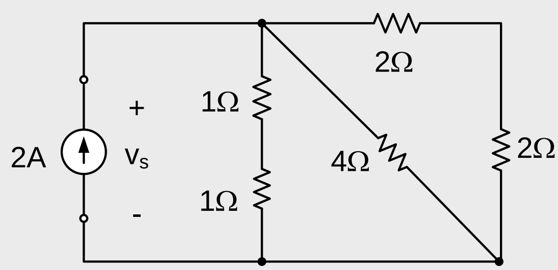
Step 2: The 3-ohm resistor and the two 6-ohm resistors are now all in parallel, as indicated on the figure to the left below. These resistances can be combined into a single equivalent resistor $R_{eq} = \frac{1}{\frac{1}{3}+\frac{1}{6}+\frac{1}{6}} = 1.5\Omega$. The resulting equivalent circuit is shown to the right below.



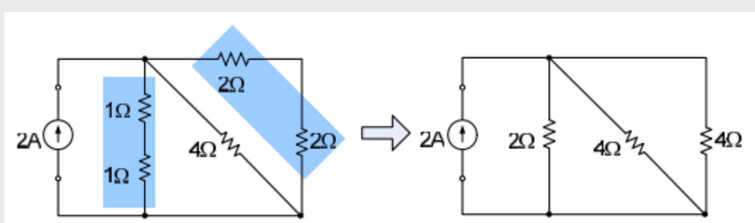
The current out of the source can now be readily determined from the figure to the right above. The voltage drop across the 1.5Ω resistor is 6V, so Ohm's law gives $i = \frac{6V}{1.5\Omega} = 4A$. Thus, the power delivered by the source is $P = (4A)(6V) = 24W$. Since the sign of the current relative to the current does not agree with the passive sign convention, the power is generated by the source.

Example 2.4

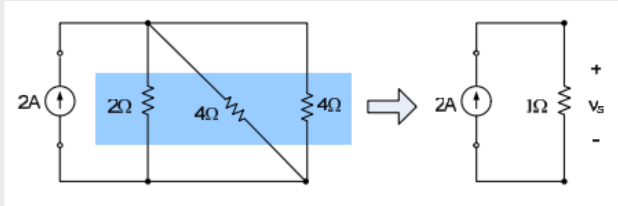
For the circuit shown below, determine the voltage, v_s, across the 2A source.



The two 1Ω resistors and the two 2Ω resistors are in series with one another, as indicated on the figure to the left below. These can be combined by simply adding the series resistances, leading to the equivalent circuit shown to the right below.



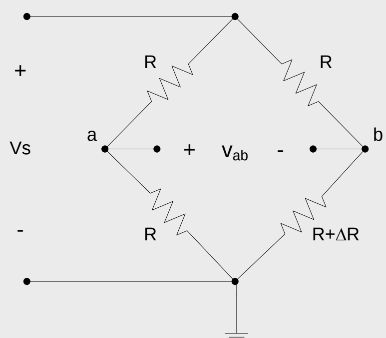
The three remaining resistors are all in parallel (they all share the same nodes) so they can be combined using the relation $R_{eq} = \frac{1}{\frac{1}{2}+\frac{1}{4}+\frac{1}{4}}$. Note that it is not necessary to combine all three simultaneously, the same result is obtained by successive combinations of two resistances. For example, the two 4Ω resistors can be combined using equation (2.16) to obtain: $R_{eq1} = \frac{4 \cdot 4}{4+4} = 2\Omega$. The total equivalent resistance can then be determined by a parallel combination of R_{eq1} and the 2Ω resistor: $R_{eq} = \frac{2 \cdot 2}{2+2} = 1\Omega$.

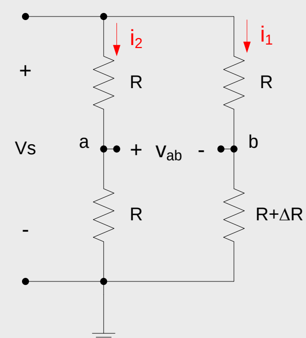


The voltage across the source can now be determined from Ohm's law: $v_s = (1\Omega)(2A) = 2V$. The assumed polarity of the source voltage is correct.

Example 2.5: Wheatstone Bridge

A Wheatstone bridge circuit is shown below. The bridge is generally presented as shown in the figure to the left; we will generally use the equivalent circuit shown to the right. A Wheatstone bridge is commonly used to convert a variation in resistance to a variation in voltage. A constant supply voltage V_s is applies to the circuit. The resistors in the circuit all have a nominal resistance of R; the variable resistor has a variation ΔR from this nominal value. The output voltage v_{ab} indicates the variation ΔR in the variable resistor. The variable resistor in the network is often a transducer whose resistance varies dependent upon some external variable such as temperature.





By voltage division, the voltages v_b and v_a (relative to ground) are:

$$v_b = \frac{(R + \Delta R)}{2R + \Delta R} V_s$$

And

$$v_a = R i_2 = \frac{V_s \cdot R}{2R} = \frac{V_s}{2}$$

The voltage v_{ab} is then:

$$v_{ab} = v_a - v_b = \left(\frac{1}{2} - \frac{R + \Delta R}{2R + \Delta R}\right) V_s = \left(\frac{(2R + \Delta R) - 2(R + \Delta R)}{2(2R + \Delta R)}\right) V_s = -\frac{\Delta R}{2(2R + \Delta R)} \cdot V_s$$

For the case in which $\Delta R << 2R$, this simplifies to:

$$v_{ab} \approx -\frac{V_s}{4R} \Delta R$$

And the output voltage is proportional to the change in resistance of the variable resistor.

Practical applications:

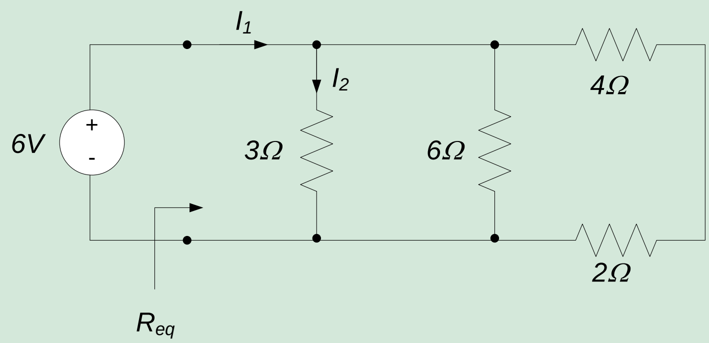
A number of common sensors result in a resistance variation resulting from some external influence. *Thermistors* change resistance as a result of temperature changes; *strain gages* change resistance as a result of deformation, generally due to application of a load to the part to which the gage is bonded; *photoconductive transducers*, or *photoresistors*, change resistance as a result of changes in light intensity. Wheatstone bridges are commonly used in conjunction with these types of sensors.

Section Summary

- In a circuit, which contains obvious series and/or parallel combinations of resistors, analysis can be simplified by combining these resistances into equivalent resistances. The reduction in the overall number of resistances reduces the number of unknowns in the circuit, with a corresponding reduction in the number of governing equations. Reducing the number of equations and unknowns typically simplifies the analysis of the circuit.
- Not all circuits are reducible.

2.3 Exercises

1. For the circuit shown, determine:
   a. R_{eq} (the equivalent resistance seen by the source)
   b. The currents I_1 and I_2



2.4 Non-ideal Power Supplies

In section 1.2, we discussed ideal power sources. In that section, an ideal voltage supply was characterized as providing a specified voltage regardless of the current requirements made upon the device. Likewise, an ideal current source was defined as providing a specified voltage regardless of the voltage potential difference across the source. These models are not realistic - since an ideal voltage source can provide infinite current with non-zero voltage difference and an ideal current source can provide infinite voltage difference with non-zero current, either device is capable of delivering infinite power. In many cases, the ideal voltage and current source models will be adequate, but in cases where we need to more accurately replicate the operation of realistic power supplies, we will need to modify our models of these devices.

In this section, we present simple models for voltage and current sources which incorporate more realistic assumptions as to the behavior of these devices.

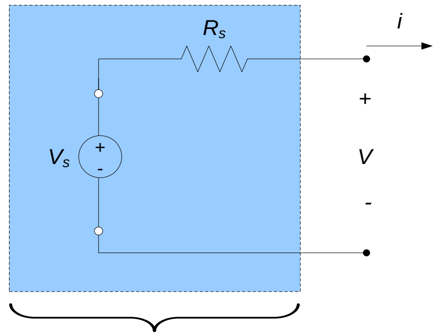
2.4.1 Non-ideal Voltage Sources

An ideal voltage source was defined in section 1.2 as providing a specified voltage, regardless of the current flow out of the device. For example, an ideal 12V battery will provide 12V across its terminals, regardless of the load connected to the terminals. A real 12V battery, however, provides 12V across its terminals only when its terminals are open-circuited. As we draw current from the terminals, the battery will provide less than 12V - the voltage will decrease as more and more current is drawn from the battery. The real battery thus appears to have an internal voltage drop which increases with increased current.

We will model a real or *practical* voltage source as a series connection of an ideal voltage source and an *internal resistance*. This model is depicted schematically in Fig. 2.5, in which the non-ideal voltage source contains an ideal voltage source providing voltage V_s and an internal resistance R_s . The non-ideal voltage source delivers a voltage V and a current *i*, where:

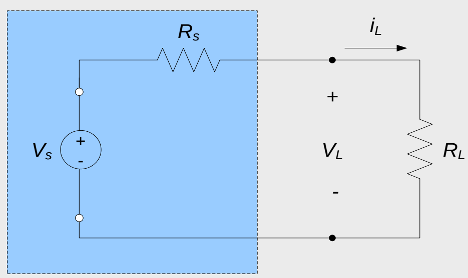
$$V = V_s - i \cdot R_s \quad \text{Eq. 2.24}$$

Equation (2.24) indicates that the voltage delivered by our non-ideal voltage source model decreases as the current out of the voltage source increases, which agrees with expectations.



Example 2.6

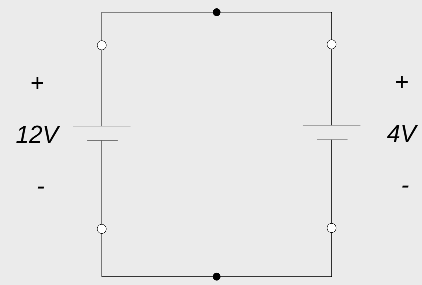
Consider the case in which we connect a resistive load to the non-ideal voltage source. The figure below provides a schematic of the overall system; R_L is the load resistance, V_L is the voltage delivered to the load, and i_L is the current delivered to the load.



In the case above, the current delivered to the load is $i = \frac{V_S}{R_S+R_L}$ and the load voltage is $V_L = V_s \frac{R_L}{R_S+R_L}$. Thus, if the load resistance is infinite (the load is an open circuit), $V_L = V_S$, but the power supply delivers no current and hence no power to the load. If the load resistance is zero (the load is a short circuit), $V_L = 0$ and the power supply delivers current $i_L = \frac{V_S}{R_S}$ to the load; the power delivered to the load, however, is still zero.

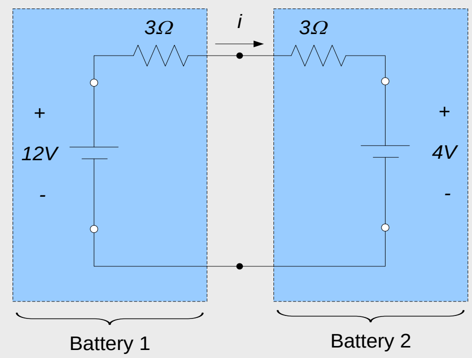
Example 2.7: Charging a Battery

We have a “dead” car battery which is providing only 4V across its terminals. We want to charge the battery using a spare battery which is providing 12V across its terminals. To do this, we connect the two batteries as shown below:



If we attempt to analyze this circuit by applying KVL around the loop, we obtain 12V=4V. This is obviously incorrect and we cannot proceed with our analysis - our model disagrees with reality!

To resolve this issue, we will include the internal resistance of the batteries. Assuming a 3Ω internal resistance in each battery, we obtain the following model for the system:



Applying KVL around the loop, and using Ohm's law to write the voltages across the battery internal resistances in terms of the current between the batteries results in:

$$-12V + (3\Omega)i + (3\Omega)i + 4V = 0$$

Which can be solved for the current *i* to obtain:

$$i = \frac{12V - 4V}{6\Omega} = 1.33A$$

Notice that as the voltage of the "dead" battery increases during the charging process, the current delivered to the "dead" battery decreases.

2.4.2 Non-ideal Current Sources

An ideal current source was defined in section 1.2 as providing a specified current, regardless of the voltage difference across the device. This model suffers from the same basic drawback as our ideal voltage source model - the model can deliver infinite power, which is inconsistent with the capabilities of a real current source.

We will use the circuit shown schematically in Fig. 2.6 to model a non-ideal current source. The non-ideal model consists of an ideal current source, i_s, placed in parallel with an internal resistance, R_s. The source delivers a voltage *V* and current *i*. The output current is given by:

$$i = i_S - \frac{V}{R_S} \qquad \text{Eq. 2.25}$$

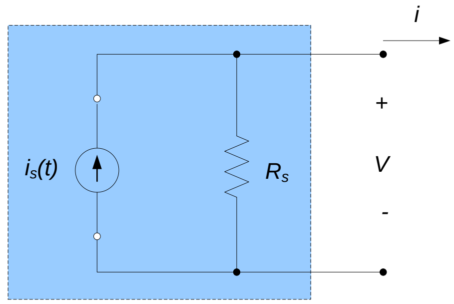
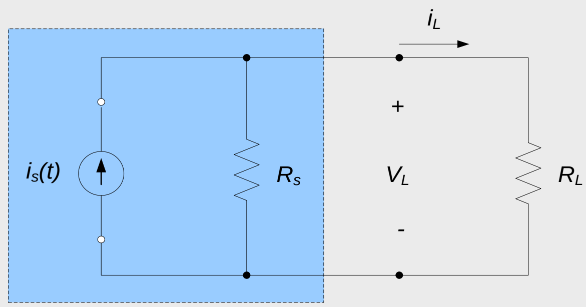


Figure 2.6. Non-ideal current source model.

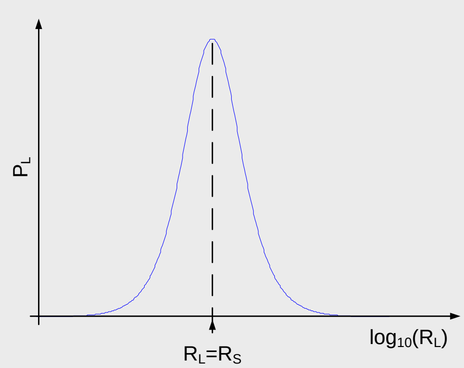
Example 2.8

Consider the case in which we connect a resistive load to the non-ideal current source. The figure below provides a schematic of the overall system; R_L is the load resistance, V_L is the voltage delivered to the load, and i_L is the current delivered to the load.



In the case above, the current delivered to the load can be determined from a current divider relation as $i_L = i_s \cdot \frac{R_S}{R_S+R_L}$ and the load voltage, by Ohm's law, is $V_L = i_L R_L = i_S \frac{R_S R_L}{R_S+R_L}$. If the load resistance is zero (the load is a short circuit), $i_L = i_S$, but the power supply delivers no voltage and hence no power to the load. In the case of infinite load resistance (the load is open circuit), $i_L = 0$. In this case, we can neglect R_S in the denominator of the load voltage equation to obtain $V_L \approx i_S \frac{R_S R_L}{R_L}$ so that $V_L \approx i_S R_S$. Since the current is zero, however, the power delivered to the load is still zero.

If we explicitly calculate the power delivered to the load, we obtain $V_L = i_S^2 \frac{R_S R_L}{R_S+R_L} \cdot \frac{R_S}{R_S+R_L}$. A plot of the power delivered to the load as a function of the load resistance is shown below; a logarithmic scale is used on the horizontal axis to make the plot more readable. As expected, the power is zero for high and low load resistances. The peak of the curve occurs when the load resistance is equal to the source resistance, $R_L = R_S$.

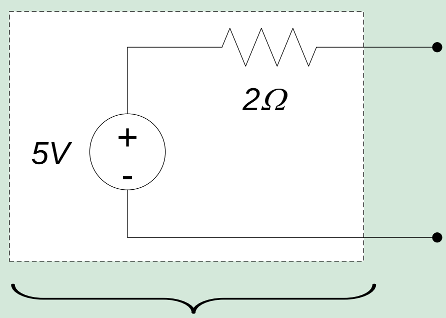


Section Summary

- In many cases, power supplies can be modeled as ideal power supplies, as presented in section 1.2. However, in some cases representation as a power supply as ideal results in unacceptable errors. For example, ideal power supplies can deliver infinite power, which is obviously unrealistic.
- In this section, we present a simple model for a non-ideal power supply.
  - Our non-ideal voltage source consists of an ideal voltage source in series with a resistance which is internal to the power supply.
  - Our non-ideal current supply consists of an ideal current source in parallel with a resistance which is internal to the power supply.
- Voltage and current divider formulas allow us to easily quantify the effects of the internal resistances of the non-ideal power supplies. Our analysis indicates that the non-ideal effects are negligible, as long as the resistance of the load is large relative to the internal resistance of the power supply.

2.4 Exercises

1. A voltage source with an internal resistance of 2Ω as shown below is used to apply power to a 3Ω resistor. What voltage would you measure across the 3Ω resistor?



2. The voltage source of exercise 1 above is used to apply power to a 2kΩ resistor. What voltage would you measure across the 2kΩ resistor?

2.5 Practical Voltage and Current Measurement

The process of measuring a physical parameter will almost invariably change the parameter being measures. This effect is both undesirable and, in general, unavoidable. One goal of any measurement is to affect the parameter being measured as little as possible.

The above statement is true of voltage and current measurements. An ideal voltmeter, connected in parallel with some circuit element, will measure the voltage across the element without affecting the current flowing through the element. Unfortunately, any real or practical voltmeter will draw some current from the circuit it is connected to; this *loading effect* will change the circuit's operating conditions, causing some difference between the measured voltage and the corresponding voltage without the voltmeter present in the circuit. Likewise, an ideal ammeter, connected in series with some circuit element, will measure current without affecting the voltage in the circuit. A practical ammeter, like a practical voltmeter, will introduce loading effects which change the operation of the circuit on which the measurement is being made.

In this section, we introduce some effects of measuring voltages and currents with practical meters.

2.5.1 Voltmeter and Ammeter Models

We will model both voltmeters and ammeters as having some internal resistance and a method for displaying the measured voltage difference or current. Fig. 2.7 shows schematic representations of voltmeters and ammeters.

The ammeter in Fig. 2.7(a) has an internal resistance R_M; the current through the ammeter is i_A and the voltage difference across the ammeter is V_M. The ammeter's voltage difference should be as small as possible - an ammeter, therefore, should have an extremely small internal resistance.

The voltmeter in Fig. 2.7(b) is also represented as having an internal resistance R_M; the current through the meter is i_A and the voltage difference across the meter is R_M. The current through the voltmeter should be as small as possible - the voltmeter should have an extremely high internal resistance.

The effects of non-zero ammeter voltages and non-zero voltmeter currents are explored in more detail in the following subsections.

2.5.2 Voltage Measurement

Consider the circuit shown in Fig. 2.8(a). A current source, i_s, provides current to a circuit element with resistance, R. We want to measure the voltage drop, V, across the circuit element. We do this by attaching a voltmeter across the circuit element as shown in Fig. 2.8(b).

In Fig. 2.8(b) the voltmeter resistance is in parallel to the circuit element we wish to measure the voltage across and the combination of the circuit element and the voltmeter becomes a current divider. The current through the resistor R then becomes:

$$i = i_s \frac{R_M}{R+R_M} \qquad \text{Eq. 2.26}$$

The voltage across the resistor R is then, by Ohm's law:

$$V = i_s \frac{R \cdot R_M}{R+R_M} \qquad \text{Eq. 2.27}$$

If $R_M >> R$, this expression simplifies to:

$$V \approx i_s \frac{R \cdot R_M}{R_M} = R \cdot i_s \qquad \text{Eq. 2.28}$$

And negligible error is introduced into the measurement - the measured voltage is approximately the same as the voltage without the voltmeter. If, however, the voltmeter resistance is comparable to the resistance R, the simplification of equation (2.28) is not appropriate and significant changes are made to the system by the presence of the voltmeter.

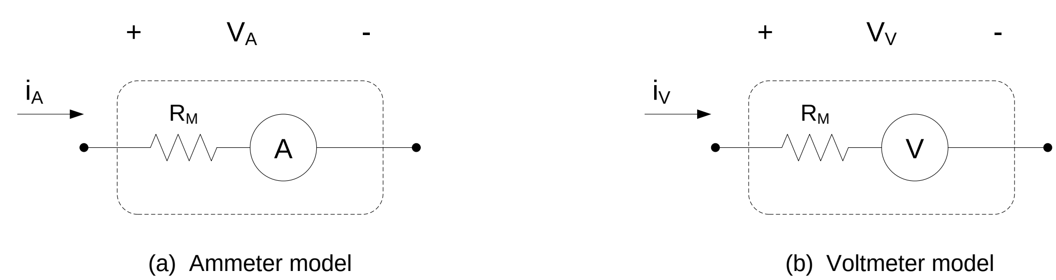


Figure 2.7. Ammeter and voltmeter models.

2.5.3 Current Measurement

Consider the circuit shown in Fig. 2.9(a). A voltage source, V_S, provides power to a circuit element with resistance, R. We want to measure the current, *i*, through the circuit element. We do this by attaching an ammeter in series with the circuit element as shown in Fig. 2.9(b).

In Fig. 2.9(b) the series combination of the ammeter resistance and the circuit element whose current we wish to measure creates a voltage divider. KVL around the single circuit loop provides:

$$V_s = i(R_M + R) \quad \text{Eq. 2.29}$$

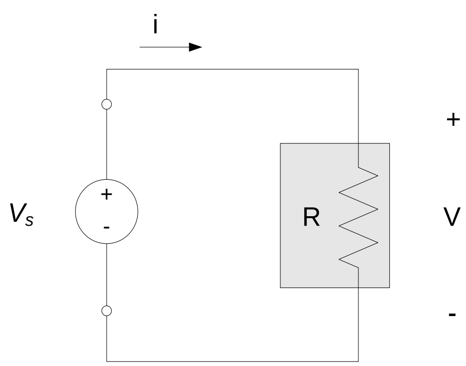
Solving for the current results in:

$$i = \frac{V_S}{R_M + R} \quad \text{Eq. 2.30}$$

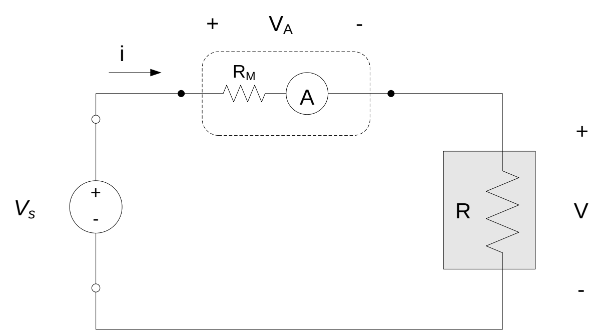
If $R_M << R$, this simplifies to:

$$i \approx \frac{V_S}{R} \quad \text{Eq. 2.31}$$

And the measured current is a good approximation to current in the circuit of Fig. 2.9(a). However, if the ammeter resistance is not small compared to the resistance R, the approximation of equation (2.31) is not appropriate and the measured current is no longer representative of the circuit's operation without the ammeter.



(a) Original circuit

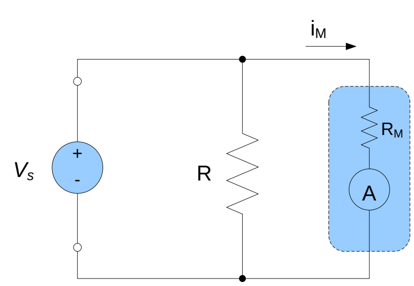


(b) Circuit with ammeter.

Figure 2.9. Current measurement.

Caution

Incorrect connections of ammeters or voltmeters can cause damage to the meter. For example, consider the connection of an ammeter in <u>parallel</u> with a relatively large resistance, as shown below.



In this configuration, the ammeter current, $i_M = \frac{V_S}{R_M}$. Since the ammeter resistance is typically very small, this can result in high currents being provided to the ammeter. This, in turn, may result in excessive power being provided to the ammeter and resulting damage to the device.

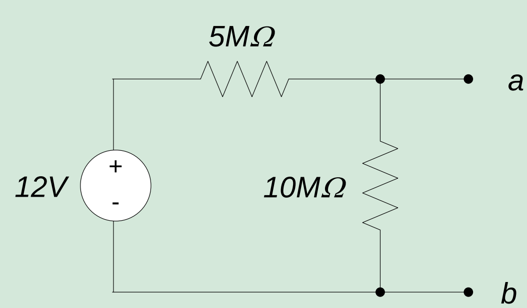
Ammeters are generally intended to be connected in series with the circuit element(s) whose current is being measured. Voltmeters are generally intended to be connected in parallel with the circuit element(s) whose voltage is being measured. Alternate connections can result in damage to the meter.

Section Summary

- Measurement of voltage and/or current in a circuit will always result in some effect on the circuit's behavior - that is, our measurement will always change the parameter being measured. One goal when measuring a voltage or current is to ensure that the measurement effects are negligible.
- In this section, we present simple models for voltmeters and ammeters (voltage and current measurement devices, respectively).
  - Our non-ideal voltmeter consists of an ideal voltmeter (which had infinite resistance, and thus draws no current from the circuit) in parallel with a resistance which is internal to the voltmeter. This model replicates the finite current which is necessarily drawn from the circuit by a real voltmeter.
  - Our non-ideal ammeter consists of an ideal ammeter (which has zero resistance, and thus introduces no voltage drop in the circuit) in series with a resistance which is internal to the ammeter. This resistance allows us to model the finite voltage drop which is introduced into the circuit by a real current measurement.
- Voltage and current divider formulas allow us to easily quantify the effects of the internal resistances of voltage and current meters. Our analysis indicates that the non-ideal effects are negligible, as long as:
  - The resistance of the voltmeter is large relative to the resistance across which the voltage measurement is being made.
  - The resistance of the ammeter is small compared to the overall circuit resistance.

2.5 Exercises

A voltmeter with an internal resistance of 10MΩ is used to measure the voltage v_{ab} in the circuit below. What is the measured voltage? What voltage measurement would you expect from an ideal voltmeter?



Real Analog Chapter 2: Lab Projects

2.1.1: Temperature Measurement System

In this lab assignment, students will design another temperature-measuring circuit. Unlike our previous temperature measuring circuit, the output voltage of this circuit is to be relative to the output voltage at room temperature. The output voltage is to be positive if the temperature is above room temperature, and negative if the temperature is below room temperature. As with our previous temperature measuring circuit, this circuit will use a thermistor to sense temperature changes.

Before beginning this lab, you should be able to:

- State Ohm's law
- Determine the equivalent resistance of series and parallel resistive networks
- State the voltage divider and current divider formulae
- Use a digital mulitmeter to measure resistance, voltage, and current (Labs 1.1 and 1.2.1)
- Use the Analog Discovery's waveform generator to apply constant voltages to a circuit (Lab 1.2.2)
- Use the Analog Discovery voltmeter to measure a constant voltage (Lab 1.2.1)
- Use color codes on resistors to determine the resistor's nominal resistance

After completing this lab, you should be able to:

- Design a thermistor-based circuit to measure temperature
- Use a potentiometer to provide a desired resistance value
- Use multiple power supplies in an electrical circuit.

This lab exercise requires:

- Analog Discovery
- Digilent Analog Parts Kit
- Digital multimeter (optional)

Symbol Key:

DEMO — Demonstrate circuit operation to teaching assistant; teaching assistant should initial lab notebook and grade sheet, indicating that circuit operation is acceptable.

ANALYSIS — Analysis; include principle results of analysis in laboratory report.

SIM — Numerical simulation (using PSPICE or MATLAB as indicated); include results of MATLAB numerical analysis and/or simulation in laboratory report.

DATA — Record data in your lab notebook.

General Discussion:

In this portion of the lab assignment, we will refine the temperature measurement system we designed in Lab 1.4.4. The system will still use a thermistor to detect temperature changes. (Recall that a thermistor is a device whose electrical resistance changes as a function of the temperature of the thermistor. The thermistor we will use has a temperature-resistance curve approximately as shown in Fig. 1. Thermistor operation is discussed in more detail in the companion document to Lab 1.4.4.)

Our design requirements for this assignment are as follows:

1. $\pm$ 5V input voltage to the system
2. Output voltage is 0V $\pm$ 10mV at room temperature (approximately 25°C)
3. Output voltage is positive for temperatures above room temperature, negative for temperatures below room temperature
4. Output voltage increases by a minimum of 1V over a temperature range of 25°C to 37°C. (These temperatures correspond approximately to room temperature and body temperature, respectively.)

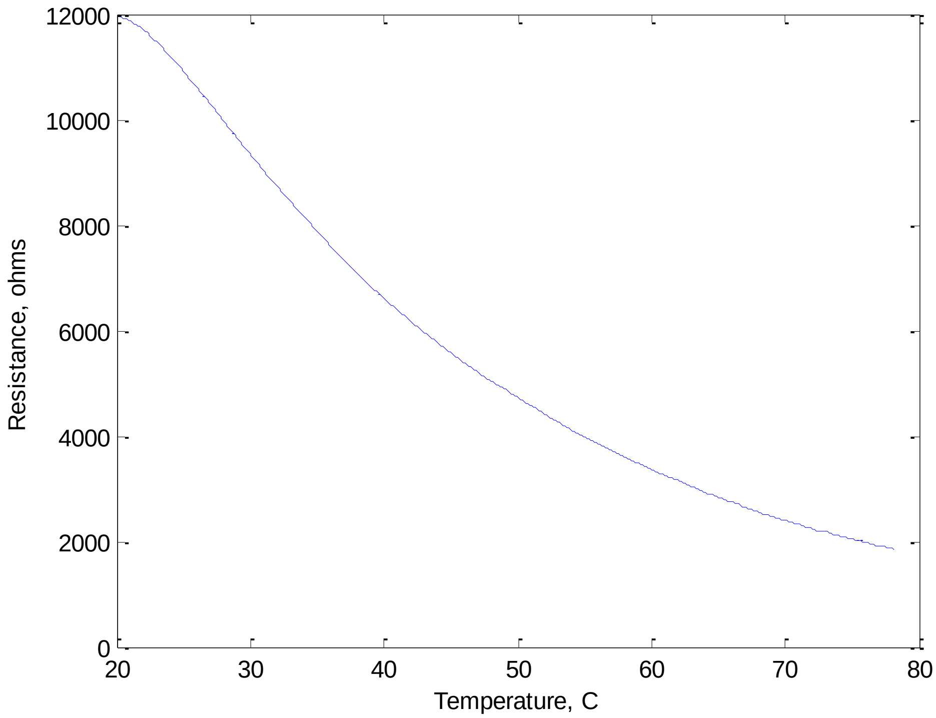


Figure 1. Thermistor temperature-resistance characteristic.

Pre-lab:

ANALYSIS

In the circuit of Figure 2, the resistance R_{TH} is the variable resistance of the thermistor. The voltage v_{out} is the voltage that we will use to indicate temperature. Two 5V voltage supplies are used to apply power to the circuit as shown – note that V_{ba} = +5V and V_{ca} = -5V. V_{out} is measured between nodes d and a with the polarity shown. The design problem is to choose a value for R so that v_{out} satisfies the given design requirements. It is recommended that you choose R based on requirement 2, and then check to see that this resistance satisfies the remaining design requirements.

Be sure to document your analyses (preferably in a lab notebook), including the equation(s) governing the system, your desired value for R, your expected output voltage change over the specified temperature range, and your expected output voltage at room temperature.

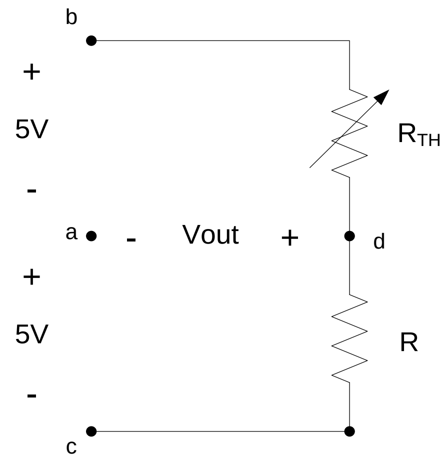


Figure 2. Temperature measurement circuit schematic.

Lab Procedures:

Implement and test the design you created in the pre-lab. It is suggested that you perform at least the following steps when doing this:

DATA

1. Measure the room-temperature resistance of your particular thermistor. Compare this value to the assumed value used in your pre-lab and modify your desired value of R accordingly.

DATA

2. Implement your design. Be sure to record actual resistance values for any fixed resistors used in your design. In order to meet requirement 2, it may be necessary for you to implement a very specific resistance. A potentiometer (variable resistor) can be used to provide an arbitrary resistance value. You can monitor the output voltage while adjusting the potentiometer to ensure that requirement 2 is met. If desired, the potentiometer can be placed in parallel or series with a fixed resistor.

DATA

3. Measure and record the voltage response at room temperature. Measure and record the output voltage at the high temperature condition by firmly holding the thermistor between two fingers. Verify that the output voltage becomes negative when the thermistor is below room temperature by holding a cold can (or bottle) of your favorite beverage against the thermistor. Discuss your circuit's performance relative to the design specifications. (e.g. Were requirements met? If not, why?)

DEMO

4. Demonstrate operation of your circuit to the Teaching Assistant Have the TA initial the appropriate page(s) of your lab notebook and the lab checklist.

Real Analog Chapter 2: Lab Worksheets

2.1.1: Temperature Measurement System (50 points total)

1. In the space below, provide your preliminary design from pre-lab. Include your estimate of output voltage at room temperature and output voltage variation resulting from specified temperature variation. Compare the expected results vs. specified performance. (15 pts)

2. Provide the measured thermistor resistance at room temperature; compare this value with data used in pre-lab to design circuit. Design changes resulting from measured thermistor response. (7 pts)

3. Actual resistance values used in implementation of circuit. (3 pts)

4. Measured circuit voltage response. Discuss your results, especially the measured performance vs. the design requirements. (15 pts)

5. **DEMO**: Have a teaching assistant initial this sheet, indicating that they have observed your circuits' operation. (10 pts)

TA Initials: _______

Real Analog Chapter 2: Lab Projects

2.3.1: Series and Parallel Resistances and Circuit Reduction

In this lab, we will examine resistance networks consisting of resistors in series and parallel. We will measure the equivalent resistance of the resistance network and comparing the measured results to analytical expectations. DMMs will be used to measure the voltage and current across individual resistors within series and parallel combinations of resistors; the experimental measurements will be compared to analytical expectations based on the governing equations for voltage and current dividers.

Before beginning this lab, you should be able to:

- State Ohm's law
- Determine the equivalent resistance of series and parallel resistive networks
- State the voltage divider and current divider formulae
- Use a digital mulitmeter to measure resistance, voltage, and current
- Use the Analog Discovery's waveform generator to apply constant voltages
- Use the Analog Discovery voltmeter to measure a constant voltage
- Use color codes on resistors to determine the resistor's nominal resistance

After completing this lab, you should be able to:

- Measure the equivalent resistance of a resistive network
- Measure the voltage and/or current in a resistor in a series or parallel resistance combination

This lab exercise requires:

- Analog Discovery module
- Digilent Analog Parts Kit
- Digital multimeter (optional)

Symbol Key:

DEMO Demonstrate circuit operation to teaching assistant; teaching assistant should initial lab notebook and grade sheet, indicating that circuit operation is acceptable.

ANALYSIS Analysis; include principle results of analysis in laboratory report.

SIM Numerical simulation (using PSPICE or MATLAB as indicated); include results of MATLAB numerical analysis and/or simulation in laboratory report.

DATA Record data in your lab notebook.

General Discussion:

This portion of the lab assignment concerns the circuit shown in Figure 1 below. A power supply is used to apply the 5V voltage difference. We wish to determine the power dissipated by the 1KΩ resistor.

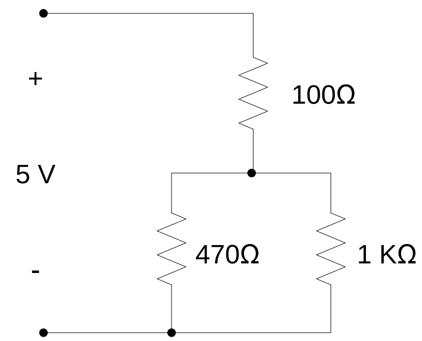


Figure 1. Circuit schematic.

Pre-lab:

ANALYSIS

Analyze the circuit of Figure 1 to estimate the power dissipated by the 1KΩ resistor.

Lab Procedures:

DATA

1. Construct the circuit of Figure 1; measure and record all actual resistance values. Measure the parameters necessary to determine the power dissipated by the 1KΩ resistor. Determine the power dissipated by the 1KΩ resistor. Compare the measured power with your estimate from the pre-lab. Comment on any differences between the estimated and measured values.

DEMO

2. Demonstrate operation of your circuit to the Teaching Assistant. Have the TA initial the appropriate page(s) of your lab notebook and the lab checklist.

Real Analog Chapter 2: Lab Worksheets

2.3.1: Series and Parallel Resistors and Equivalent Resistance (20 points total)

1. Expected power dissipated by 1KΩ resistor (pre-lab analysis). (5 pts)

2. Provide a schematic of the circuit below, including measured resistance values. (3 pts)

3. Measured power dissipated by 1KΩ resistor (provide all measurements taken: actual resistance values, voltages, currents, power calculation). Comment on the agreement between measured and expected power dissipation – calculating a percent difference is always good! (8 pts)

4. **DEMO**: Have a teaching assistant initial this sheet, indicating that they have observed your circuit's operation. (4 pts)

TA Initials: _______

Real Analog Chapter 2: Lab Projects

2.3.2: Series and Parallel Resistances and Circuit Reduction

In this lab assignment, we will perform some simple design-related exercises. Specifically, we will design resistive networks, composed of the available fixed resistors, to provide specified resistances.

Before beginning this lab, you should be able to:

- State Ohm's law
- Determine the equivalent resistance of series and parallel resistive networks
- State the voltage divider and current divider formulae
- Use a digital mulitmeter to measure resistance, voltage, and current
- Use the Analog Discovery's waveform generator to apply constant voltages
- Use the Analog Discovery oscilloscope to measure a constant voltage
- Use color codes on resistors to determine the resistor's nominal resistance

After completing this lab, you should be able to:

- Measure the equivalent resistance of a resistive network
- Measure the voltage and/or current in a resistor in a series or parallel resistance combination

This lab exercise requires:

- Digilent Analog Parts Kit
- Digital multimeter

Symbol Key:

DEMO — Demonstrate circuit operation to teaching assistant; teaching assistant should initial lab notebook and grade sheet, indicating that circuit operation is acceptable.

ANALYSIS — Analysis; include principle results of analysis in laboratory report.

SIM — Numerical simulation (using PSPICE or MATLAB as indicated); include results of MATLAB numerical analysis and/or simulation in laboratory report.

DATA — Record data in your lab notebook.

General Discussion:

We need resistors with the following resistance values and tolerances:

1. 9KΩ $\pm$ 5%
2. 800Ω $\pm$ 5%
3. 35KΩ $\pm$ 5%

Resistors with these resistances are not included in the analog parts kit; we will use available fixed resistors to construct circuits with the required equivalent resistance.

Pre-lab:

ANALYSIS

Using only fixed-value resistors available in your analog parts kit, design circuits which have the equivalent resistances listed above.

Lab Procedures:

DATA

3. Construct the three circuits you designed in the pre-lab. Use an ohmmeter to measure the equivalent resistance of each of the circuits. Comment on your results – specifically, whether the design requirements were met.

DEMO

4. Demonstrate operation of your circuit to the Teaching Assistant. Have the TA initial the appropriate page(s) of your lab notebook and the lab checklist.

Note:

As always, measure and record the resistance of the individual resistors used in your circuits.

Real Analog Chapter 2: Lab Worksheets

2.3.2: Series and Parallel Resistors and Equivalent Resistance (25 points total)

1. 9KΩ ± 5% resistance. Circuit as designed, actual resistance values used in implementation, measured circuit resistance, discussion of results. (6 pts)

2. 800Ω ± 5%. Circuit as designed, actual resistance values used in implementation, measured circuit resistance, discussion of results. (6 pts)

3. 35KΩ ± 5% Circuit as designed, actual resistance values used in implementation, measured circuit resistance, discussion of results. (6 pts)

4. **DEMO**: Have a teaching assistant initial this sheet, indicating that they have observed your circuit's operation for at least one of the above cases. (7 pts)

TA Initials: _______

Real Analog Chapter 2: Lab Projects

2.4: Non-ideal Power Sources

Though many theoretical models of electrical circuits assume that power supplies are ideal, actual circuit implementations can depend upon non-ideal limitations of the power supplies. In this lab assignment, we will experimentally explore the behavior of non-ideal power sources. The experiments in this assignment illustrate some of the effects of non-ideal power supplies.

Before beginning this lab, you should be able to:

- State Ohm's law
- Determine the equivalent resistance of series and parallel resistive networks
- State the voltage divider and current divider formulae
- Use a digital mulitmeter to measure resistance, voltage, and current
- Use the Analog Discovery's waveform generator to apply constant voltages
- Use the Analog Discovery voltmeter to measure a constant voltage
- Model non-ideal sources

After completing this lab, you should be able to:

- Estimate the internal resistance of a non-ideal power supply
- Describe qualitatively, the effects of power supply internal resistances

This lab exercise requires:

- Analog Discovery module
- Digilent Analog Parts Kit
- Digital multimeter

Symbol Key:

DEMO — Demonstrate circuit operation to teaching assistant; teaching assistant should initial lab notebook and grade sheet, indicating that circuit operation is acceptable.

ANALYSIS — Analysis; include principle results of analysis in laboratory report.

SIM — Numerical simulation (using PSPICE or MATLAB as indicated); include results of MATLAB numerical analysis and/or simulation in laboratory report.

DATA — Record data in your lab notebook.

General Discussion:

Consider the circuit of Figure 1. Our goal is to reduce the circuit resistance to the point where the internal resistance of the power source begins to have an effect on the circuit's behavior.

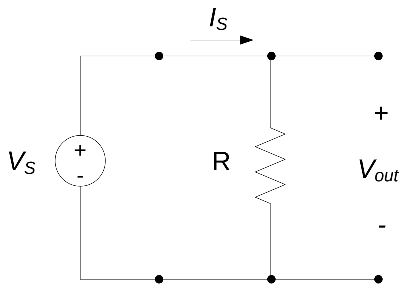


Figure 1. Circuit schematic.

Pre-lab:

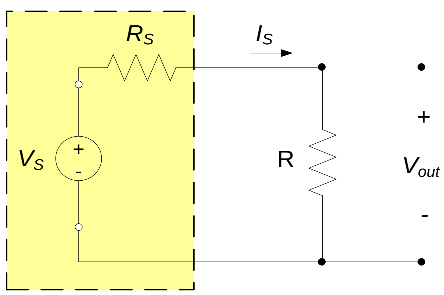
ANALYSIS

If R = 25Ω and V_S = 1V, analyze the circuit of Figure 1 to determine expected values for the measured voltage V_{out}, source current I_S, and power dissipated by the resistor, for the cases in which

a) The voltage source is ideal , and
b) The voltage source is non-ideal, and has an internal resistance R_S. (Note: your answer here will be a function of R_S.

Hint – non-ideal voltage sources:

Per module 1.5.1, a non-ideal voltage source can be modeled as an ideal voltage source in series with an internal resistance as shown in the figure below.



Applying KVL around the loop of the above circuit results in:

$$V_S - R_S I_S = V_{out}$$

Note that we do not have the ability to measure V_S or R_S (they are inside the power supply), but we can potentially measure V_{out} and I_S.

Lab Procedures:

a) Construct the circuit of Figure 1, using $R \approx 25\Omega$ and $V_S \approx 1V$, and estimate the source resistance of your voltage source. The following steps are suggested in order to do this:

DATA

1. Measure and record the actual value of R for your circuit.

DATA

2. Use the waveform generator to apply the voltage V_S = 1V. Measure and record the actual value of V_S by open-circuiting the waveform generator terminals and using a voltmeter or your Analog Discovery module to measure the voltage across the its terminals. Fine tune the waveform generator voltage until you measure exactly 1V across the terminals. (Note that

under these conditions, there is essentially no current supplied by the voltage source so that the internal source resistance is not affecting your voltage measurement.)

DATA

3. Connect the resistor to your waveform generator. Measure the voltage V_{out} using your Analog Discovery module and the source current I_S using your digital multimeter. Compare the measured voltage with your estimate from part (a) of the pre-lab. Use your measurement of V_{out} and the V_S from part (2) above to estimate the internal resistance of the voltage source. (Note that we are assuming that the ideal voltage source portion of the non-ideal voltage source is not affected by the application of the load resistance R. This is a fairly large assumption!)

DEMO

4. Demonstrate operation of your circuit to the Teaching Assistant. Have the TA initial the appropriate page(s) of your lab notebook and the lab checklist.

b) Repeat the source resistance estimate of part (a) above for two other values of R: $R \approx 30\Omega$ and $R \approx 20\Omega$. Comment on the consistency between the results for the three different values of R.

Note:

This is not a good way to get an accurate estimate of the internal resistance of the power source, but it should give you an idea of the overall concepts involved.

Real Analog Chapter 2: Lab Worksheets

2.4: Non-ideal Power Sources (20 points total)

1. Estimated V_{out}, for both ideal and non-ideal voltage sources, from your pre-lab analysis. (3 pts)

2. Circuit schematic, including measured resistance values. (2 pts)

3. Estimated power source internal resistance, for $R \approx 25\Omega$. (5 pts)

4. **DEMO**: Have a teaching assistant initial this sheet, indicating that they have observed your circuit's operation. (5 pts)

   TA Initials: _______

5. Estimated power source internal resistance for $R \approx 30\Omega$ and $R \approx 20\Omega$. Discussion of consistency of results for various values of R. (5 pts)

Real Analog Chapter 2: Lab Projects

2.5: Practical Voltage and Current Measurement

Theoretical models of electrical circuits often assume that we can determine voltages and currents within the circuit without affecting the circuit's operation. In reality, any time we measure a voltage or current, we alter the circuit's behavior to some extent – sometimes the effects of the measurement process can be very significant. In this lab assignment, we will experimentally explore the behavior of non-ideal meters. The experiments in this assignment illustrate the effects of non-ideal voltage measurements.

Before beginning this lab, you should be able to:

- State Ohm's law
- Determine the equivalent resistance of series and parallel resistive networks
- State the voltage divider and current divider formulae
- Use a digital mulitmeter to measure resistance, voltage, and current
- Use the Analog Discovery's waveform generator to apply constant voltages
- Use the Analog Discovery voltmeter to measure a constant voltage
- Model non-ideal sources

After completing this lab, you should be able to:

- Estimate a voltmeter's internal resistance
- Describe qualitatively the effects of voltmeter internal resistances on voltage measurements

This lab exercise requires:

- Analog Discovery
- Digilent Analog Parts Kit
- Digital multimeter

Symbol Key:

DEMO Demonstrate circuit operation to teaching assistant; teaching assistant should initial lab notebook and grade sheet, indicating that circuit operation is acceptable.

ANALYSIS Analysis; include principle results of analysis in laboratory report.

SIM Numerical simulation (using PSPICE or MATLAB as indicated); include results of MATLAB numerical analysis and/or simulation in laboratory report.

DATA Record data in your lab notebook.

General Discussion:

Very large resistors are used in the voltage divider circuit of Figure 1. Due to these large resistances, measurement of the voltage V_{out} will likely result in measurement errors due to non-ideal voltmeter effects.

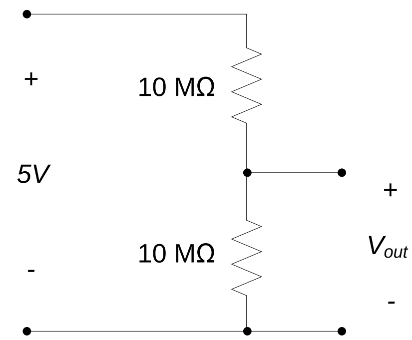


Figure 1. Circuit schematic.

Pre-lab:

ANALYSIS

Analyze the circuit of Figure 1 to determine an expected value for the measured voltage V_{out} for the cases in which

c) The measurement of V_{out} is determined using an ideal voltmeter (a voltmeter with infinite internal resistance), and
d) The measurement of V_{out} is determined using a voltmeter with internal resistance R_M. (In this case, your result will be a formula which depends upon R_M.)

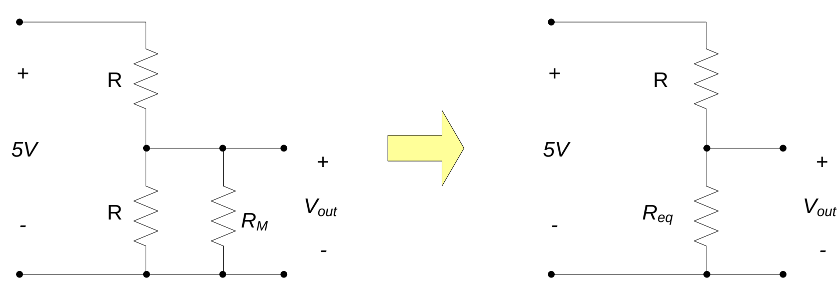
Hint – non-ideal voltmeters:

Per section 2.5 of the text, a voltmeter can be modeled as an equivalent resistance R_M in parallel with the voltage being measured. Thus, the circuit of Figure 1, with the voltmeter resistance included, becomes as shown in the figure to the left below. The parallel combination of the voltmeter and the 10MΩ resistor can then be represented as a single equivalent resistance R_{eq} as shown in the figure to the right below, where

$$R_{\mathrm{eq}} = \frac{(\mathrm{R})(R_M)}{\mathrm{R}+R_M}$$

Therefore,

$$V_{\mathrm{OUT}} = 5\mathrm{V}\left(\frac{\mathrm{R_{eq}}}{\mathrm{R}+R_{eq}}\right)$$



From the above, it can be seen that if *R>>R*$R \gg R_M$, $R_{eq} \approx R$ and the measured V_{out} will be essentially the same as the V_{out} indicated in Figure 1. If, however, this condition is not true, the voltmeter's internal resistiance can have a significant (and generally undesirable) effect on the voltage being measured.

Lab Procedures:

DATA

a) Construct the circuit of Figure 1. Measure the voltage V_{out} using your DMM. Using your pre-lab results, estimate the internal resistance of the voltmeter.

DEMO

b) Demonstrate operation of your circuit to the Teaching Assistant. Have the TA initial the appropriate page(s) of your lab notebook and the lab checklist.

DATA

c) Repeat the test of part (a), except use the voltmeter on your Analog Discovery module to measure V_{out}. Using your pre-lab results, estimate the internal resistance of the scope instrument.

Note:

- This is not a good way to get an accurate estimate of the internal resistance of the voltmeter, but it should give you an idea of the overall concepts involved.
- It is likely that the Analog Discovery internal resistance will be significantly lower than the internal resistance of most commercially available DMMs. This is at least partly due to the fact that the Analog Discovery is primarily intended for making time-varying measurements, while DMMs are intended to measure constant values.

Real Analog Chapter 2: Lab Worksheets

2.5: Practical Voltage and Current Measurement (25 points total)

1. Provide the expected output voltage for both ideal and non-ideal voltmeters, based on your pre-lab analysis. (5 pts)

2. Provide a circuit schematic below; include measured resistance values on your schematic. (2 pts)

3. In the space below, provide your measured output voltage and your estimated value of the internal resistance of the voltmeter of your DMM. Comment on your results, especially relative to the effects of the voltmeter on the quantity being measured. (7 pts)

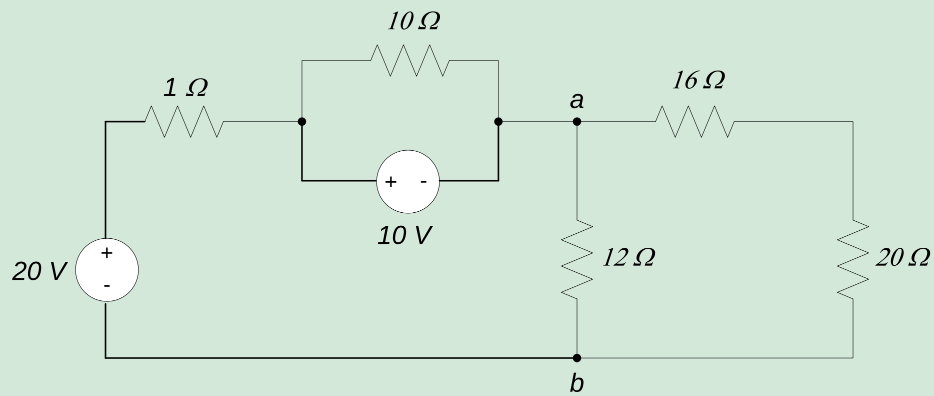
4. In the space below, provide your measured output voltage and your estimated value of the internal resistance of the voltmeter of your Analog Discovery. Comment on your results, especially relative to the effects of the voltmeter on the quantity being measured. (6 pts)

5. **DEMO**: Have a teaching assistant initial this sheet, indicating that they have observed your circuit's operation. (5 pts)

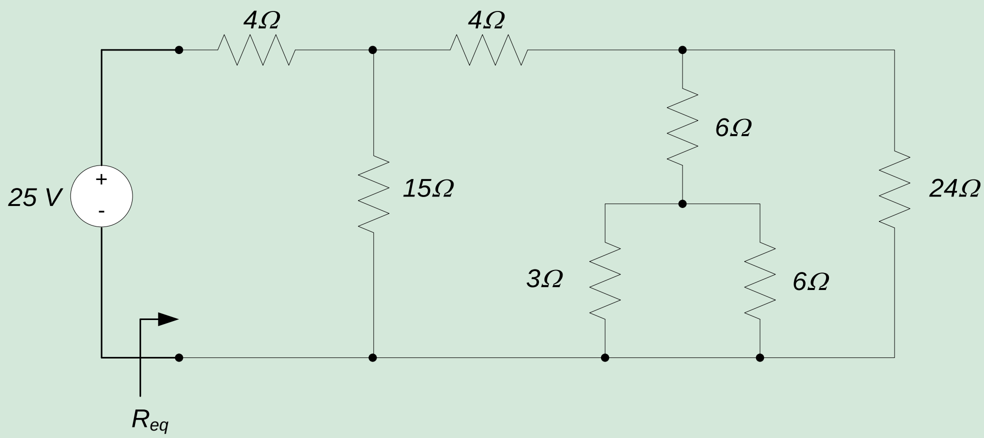
   TA Initials: _______

Real Analog Chapter 2: Homework

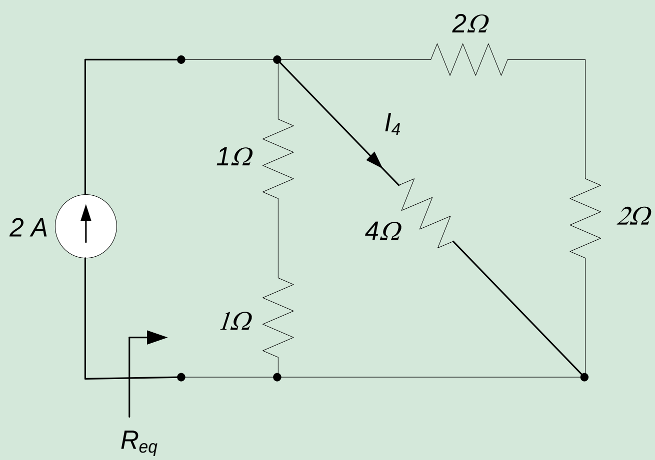
2.1 For the circuit shown, find the voltage v_{ab} and the power (generated or absorbed) by the 20V source.



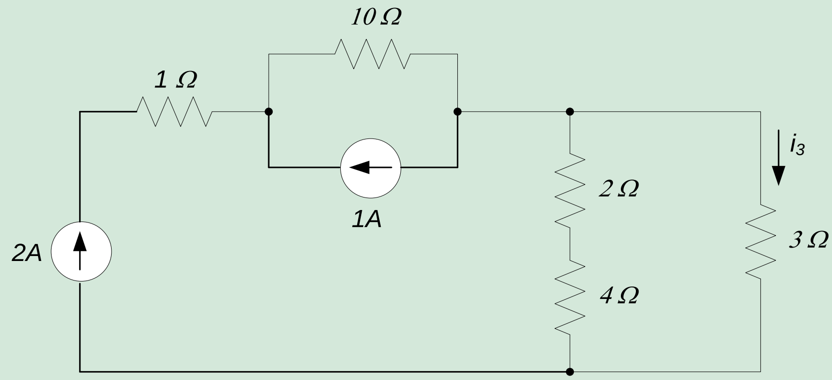
2.2 For the circuit shown, find R_{eq} (the equivalent resistance "seen" by the source) and the current out of the source.



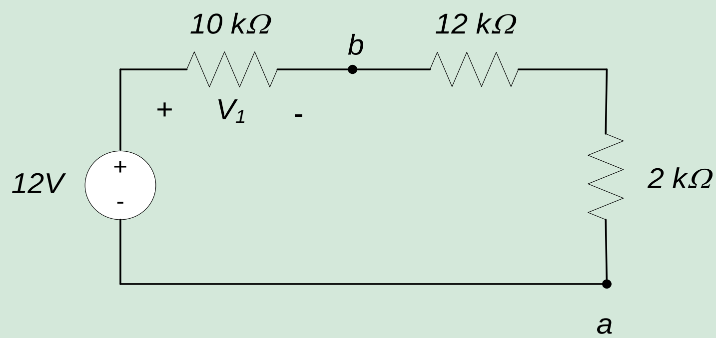
2.3 For the circuit shown, find R_{eq} (the equivalent resistance "seen" by the source) and the current I_4.



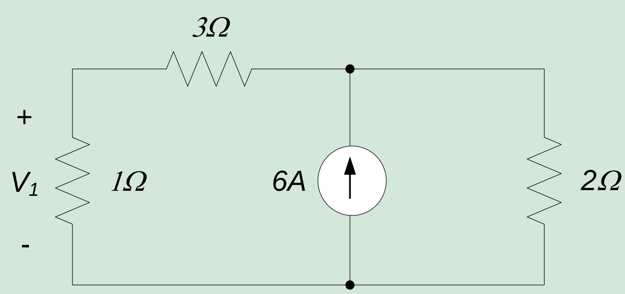
2.4 For the circuit below, determine the current i_3.



2.5 Find V_{ab} and V_1 in the circuit below.

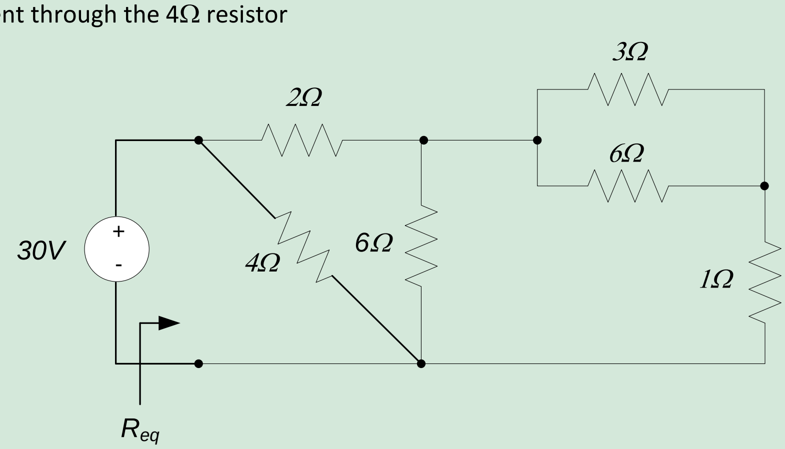


2.6 Determine the voltage V_1 in the circuit below.

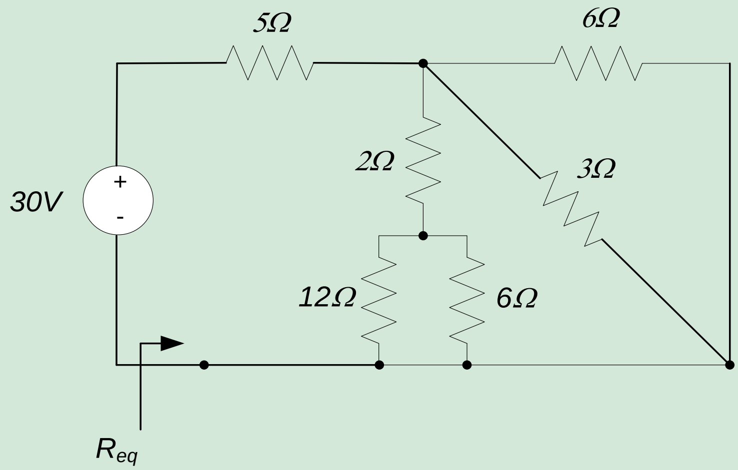


2.7 For the circuit shown, find:

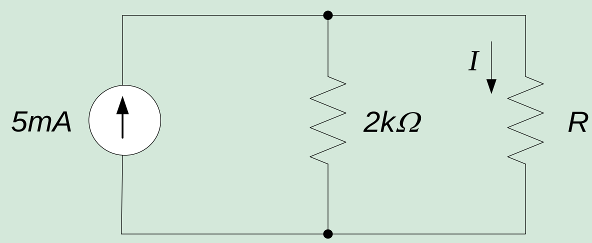
a) R_{eq} (the equivalent resistance "seen" by the source)
b) the power generated by the source
c) The current through the 4Ω resistor



2.8 For the circuit shown, find:
  a) R_{eq} (the equivalent resistance "seen" by the source)
  b) the current delivered by the source
  c) The voltage difference across the 3Ω resistor.
  d) The voltage difference across the 2Ω resistor.



2.9 Determine the value of the resistance R which makes the current I = 2mA.



2.10 Determine the value of the source voltage V_S which makes the voltage V_1 = 2V.