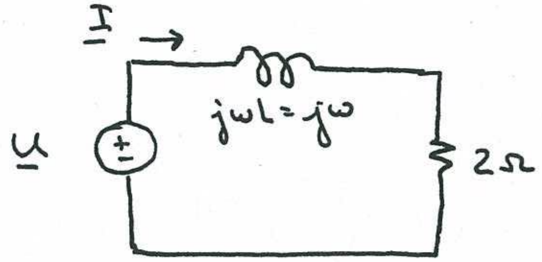
Exercises
chapter 11.1

1. Frequency domain model:



$$\underline{I} = \frac{\underline{u}}{2+j\omega} \Rightarrow \frac{\underline{I}}{\underline{u}} = \frac{1}{2+j\omega}$$

2. $u(t) = 3\cos(2t-60°) + 4\cos(4t+30°) + 7\cos(6t+45°)$

- $\omega = 2$ rad/s $\Rightarrow \underline{u}_1 = 3\angle{-60°}$, $|H(j2)| = 0$, $\angle H(j2) = 0°$

$$\underline{Y}_1 = \underline{u}_1 (0\angle 0°) = 0\angle{-60°}$$

- $\omega = 4$ rad/s. $\Rightarrow \underline{u}_2 = 4\angle 30°$, $|H(j4)| = 1$, $\angle H(j4) = 0°$

$$\underline{Y}_2 = \underline{u}_2 (1\angle 0°) = 4\angle 30°$$

- $\omega = 6$ rad/s $\Rightarrow \underline{u}_3 = 7\angle 45°$, $|H(j6)| = 0$, $\angle H(j6) = 0°$

$$\underline{Y}_3 = \underline{u}_3 (0\angle 0°) = 0\angle 45°$$

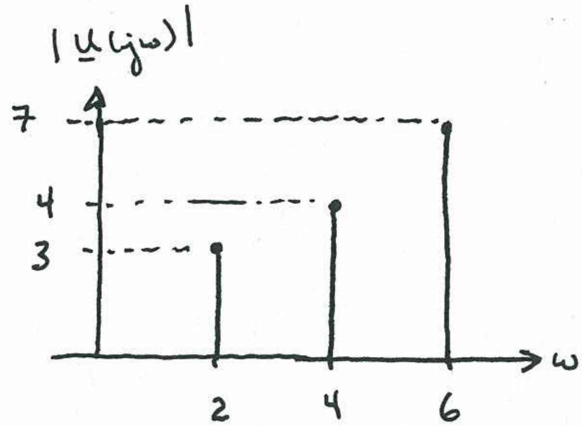
Superimpose $\underline{Y}_1, \underline{Y}_2, \underline{Y}_3$ to get $y(t)$:

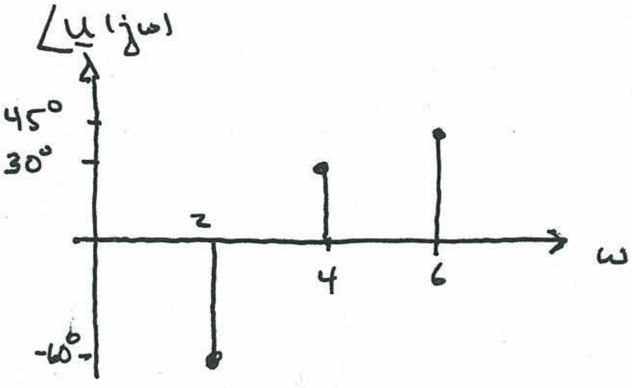
$$y(t) = 0\cos(2t-60°) + 4\cos(4t+30°) + 0\cos(6t+45°)$$

$$y(t) = 4\cos(4t+30°)$$

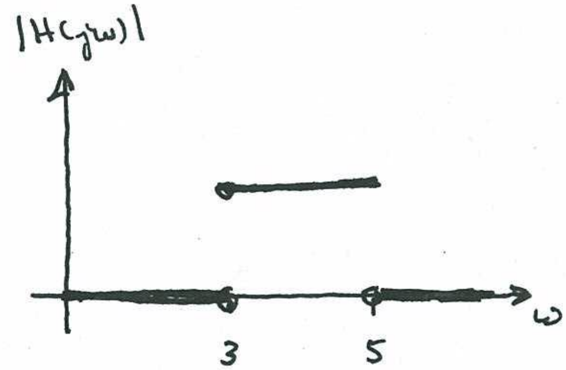
1.

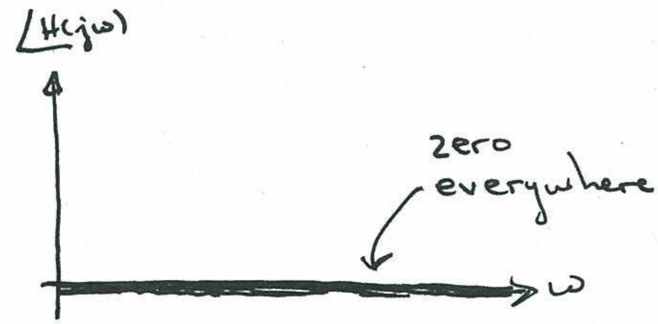
(a)

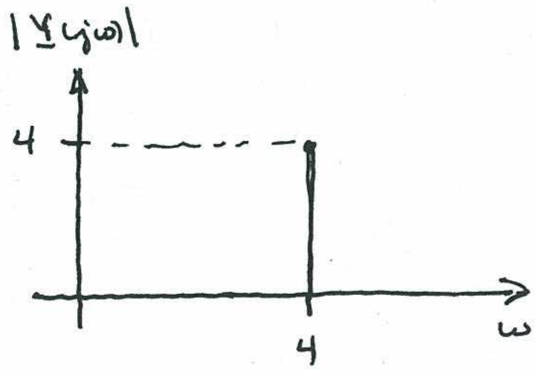

$|\underline{U}(j\omega)|$
7
4
3
2
4
6
ω

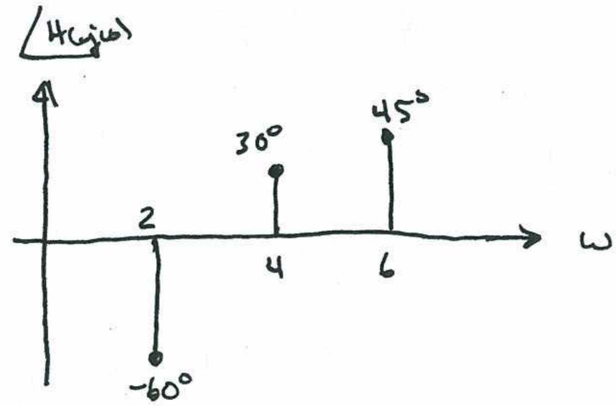

$\angle \underline{U}(j\omega)$
45°
30°
2
4
6
ω
-60°

(b)


$|H(j\omega)|$
3
5
ω

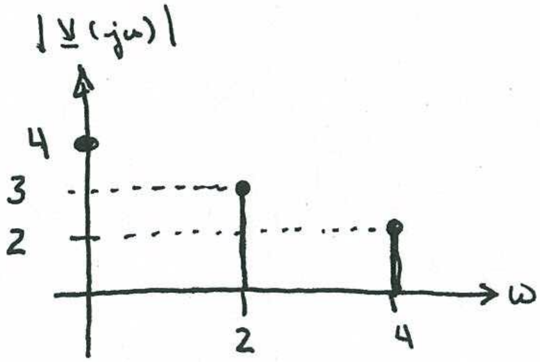

$\angle H(j\omega)$
zero everywhere
ω

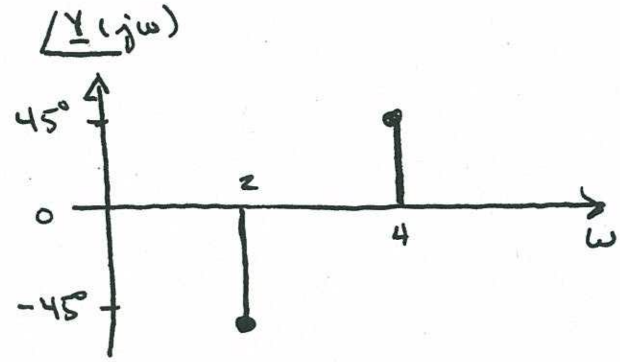

$|\underline{Y}(j\omega)|$
4
4
ω


$\angle H(j\omega)$
30°
45°
2
4
6
ω
-60°

Exercises
Chapter 11.2

2. $v(t) = 4 + 3\cos(2t - 45^\circ) + 2\cos(4t + 45^\circ)$

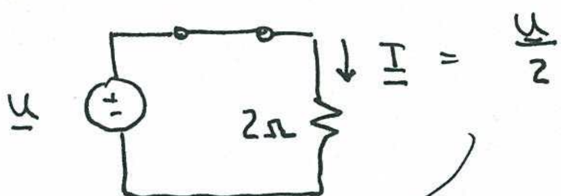


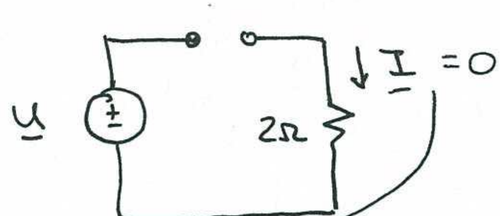


3.

$\omega \to 0$, inductor shorts:

$\omega \to \infty$, inductor opens





$\frac{\underline{I}}{\underline{V}} = \frac{1}{2}$

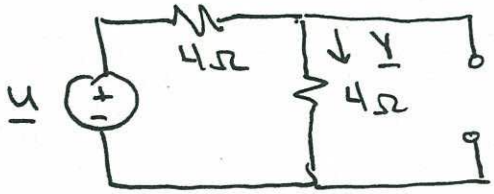
$\frac{\underline{I}}{\underline{V}} = 0$

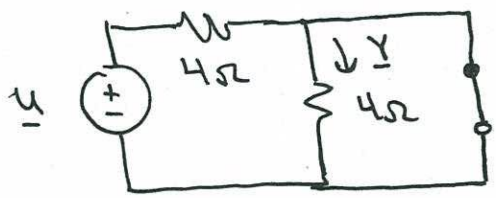
Both check ✓

4.

$\omega \to 0$:

$\omega \to \infty$:



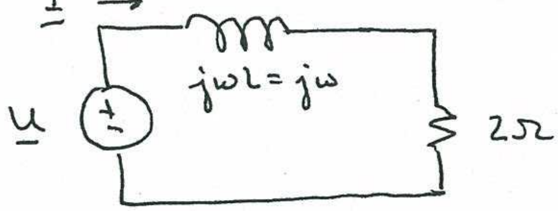


$\underline{Y} = \frac{\underline{V}}{8}$

$\underline{Y} = 0$

Exercises
Chapter 11.3

1. Frequency domain circuit:



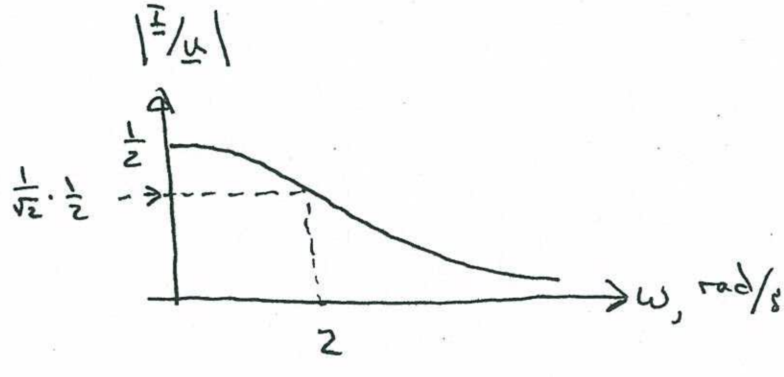
$$\underline{I} = \frac{\underline{U}}{2+j\omega} \Rightarrow \underline{\underline{\frac{\underline{I}}{\underline{U}} = \frac{1}{2+j\omega}}} \Rightarrow \begin{cases} \left|\frac{\underline{I}}{\underline{U}}\right| = \frac{1}{\sqrt{2^2+\omega^2}} \\ \angle \underline{I}/\underline{U} = -\tan^{-1}\left(\frac{\omega}{2}\right) \end{cases}$$

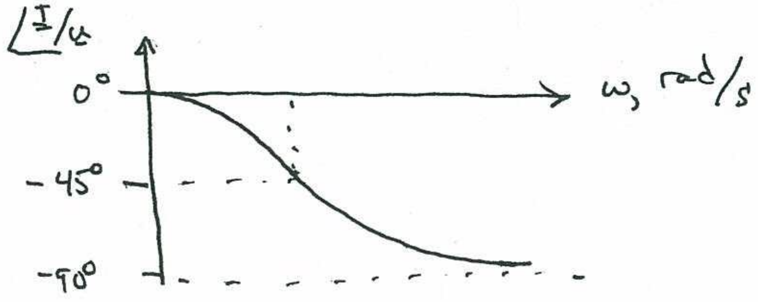
$$\left|\frac{\underline{I}}{\underline{U}}\right|_{max} = \frac{1}{2} \quad (\text{when } \omega = 0)$$

$$\underline{\underline{\omega_c = 2 \ \text{rad/sec}}} \Rightarrow \left|\frac{\underline{I}}{\underline{U}}\right|_{\omega=\omega_c} = \frac{1}{\sqrt{2^2+2^2}} = \frac{1}{\sqrt{8}} = \underbrace{\frac{1}{\sqrt{2}} \cdot \frac{1}{2}}$$

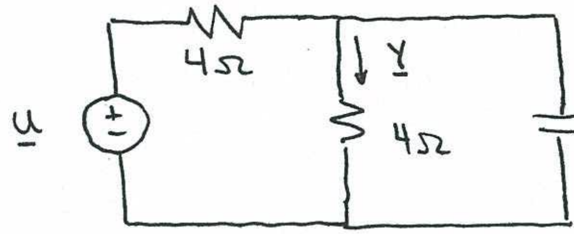
$\frac{1}{\sqrt{2}}$ times maximum value.

2.



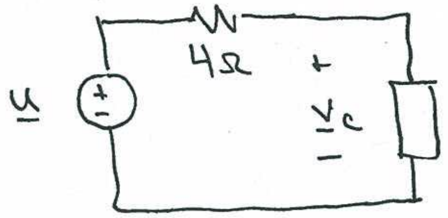


3. Frequency domain circuit:



$$\frac{1}{j\omega(\frac{1}{2})} = \frac{2}{j\omega}$$

Equivalent circuit:



$$Z = \frac{\frac{2}{j\omega}\cdot 4}{\frac{2}{j\omega}+4} = \frac{8}{2+4j\omega} = \frac{4}{2j\omega+1}$$

V_c from voltage divider: $V_c = u\left[\dfrac{\frac{4}{2j\omega+1}}{4+\frac{4}{j2\omega+1}}\right]$

$$V_c = u\,\frac{4}{8+j8\omega} = \frac{1}{2+j2\omega}\,u$$

$$Y = \frac{V_c}{4\,\Omega} \Rightarrow Y = \frac{1}{8+j8\omega}\,u \Rightarrow \underline{\underline{\frac{Y}{u} = \frac{1}{8+j8\omega}}}$$

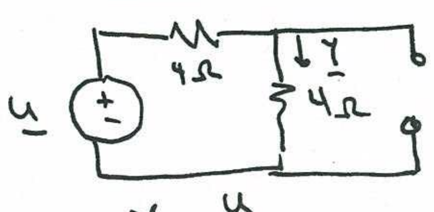
$\underline{\underline{\omega_c = 1\ \text{rad/s}}}$

$\boxed{\omega_c = \frac{1}{\tau}}$

Time constant:

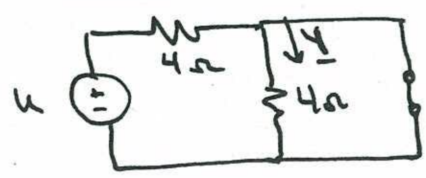
$R_{eq} = 2\,\Omega$, $\tau = R_{eq}C = (2\,\Omega)(\tfrac{1}{2}\,F) = \underline{\underline{1\,\text{sec}}}$

$\underline{\underline{\omega = 0:}}$



$Y = \frac{u}{\ }$

$\underline{\underline{\omega \to \infty:}}$



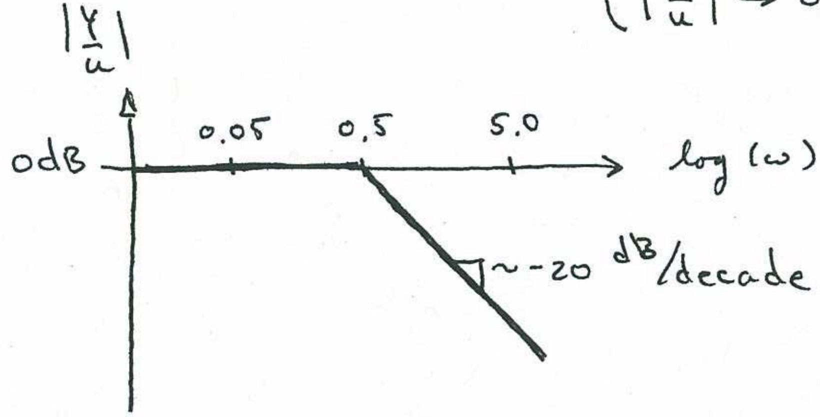
$Y = 0$

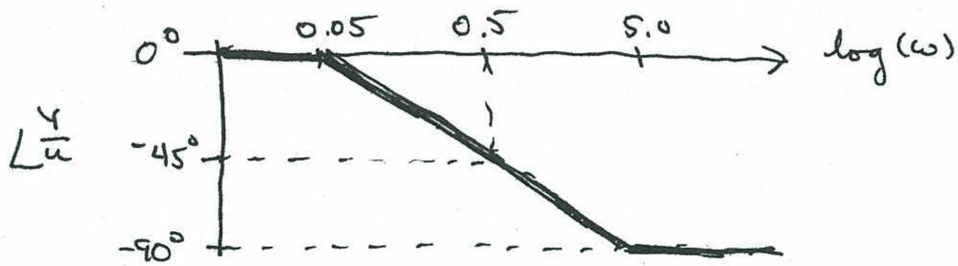
Low pass Filter.

Exercises
Chapter 11.4

1.

$$\frac{Y}{u} = \frac{1}{1+j2\omega} \Rightarrow \begin{cases} \text{DC gain} = 1 = 0\text{ dB} \\ \omega_c = \frac{1}{2}\text{ rad/s} \\ \left|\frac{Y}{u}\right| \to 0 \text{ as } \omega \to \infty \end{cases}$$





Exercises
Chapter 11.4

2.

$$\frac{Y}{U} = \frac{1}{8 + j8\omega} \Rightarrow \begin{cases} \text{DC gain} = \frac{1}{8} = -18 \text{ dB} \\ \omega_c = 1 \text{ rad/sec} \\ \left|\frac{Y}{U}\right| \to 0 \text{ as } \omega \to \infty \end{cases}$$