Exercises
Chapter 10.1

1. $v_{in}(t) = 10\cos(2t + 40^\circ)$

Input frequency $= 2\ {}^{rad}/_{sec}$, so output frequency must be $\underline{\underline{2\ {}^{rad}/_{sec}}}$

2. $v_{in}(t) = 10\cos(2t + 40^\circ)$

$\left|\frac{V_{out}}{V_{in}}\right| = 0.5$, Input phase - output phase $= 20^\circ$

(a) $2\ {}^{rad}/_{sec}$ (same as input)

(b) $\left|\frac{V_{out}}{V_{in}}\right| = 0.5 \Rightarrow |V_{out}| = 0.5\,|V_{in}| = 0.5\,(10)$

$|V_{out}| = 5\,V$

(c) $\underset{\substack{\nearrow \\ \text{input phase}}}{40^\circ} - \phi = 20^\circ \Rightarrow \phi = 20^\circ$

Note: output wave form is thus:

$V_{out}(t) = 5\cos(2t + 20^\circ)$

Chapter 10.2

Exercises

1.

(a) $\frac{3}{\sqrt{2}}(1+j)$

(b) $5(-1+j)$

(c) $j2.5$

(d) -6

2.

(a) $2\sqrt{2}\, e^{-j45°}$

(b) $3\, e^{-j90°}$

(c) 6

(d) $\sqrt{10}\, e^{j18.4°}$

3.

(a) $$\frac{-j}{2(j+1)} = \frac{1\angle -90°}{2\sqrt{2}\angle 45°} = \frac{1}{2\sqrt{2}}\angle -135° = \underline{\underline{\frac{1}{2\sqrt{2}}\, e^{j(-135°)}}}$$

(b) $$\frac{2-j2}{4+j4} = \frac{2\sqrt{2}\angle -45°}{4\sqrt{2}\angle +45°} = \frac{1}{\underline{\underline{2}}}\angle -90° = \underline{\underline{\frac{1}{2}\, e^{j(-90°)}}} = -\frac{j}{2}$$

(c) $$2e^{j45°}\frac{2}{j+1} = 2\angle 45° \frac{2}{\sqrt{2}\angle 45°} = \frac{4}{\sqrt{2}} = \underline{\underline{2\sqrt{2}}}$$

(d) $$j + \frac{2}{j} = j + \frac{2}{1\angle 90°} = j + 2\angle -90° = j - j2 = -j = \underline{\underline{e^{-j90}}}$$

Chapter 10.2
Exercises

4\. (a) $3 \angle -60^\circ$ or $3e^{-j60^\circ}$

(b) $-2 \angle 45^\circ = 2 \angle 225^\circ$ or $2e^{j225^\circ}$

(c) $\sin(6t) = \cos(6t - 90^\circ) \Rightarrow 1 \angle -90^\circ$ or $1e^{-j90}$

(d) $7 \angle 0^\circ = 7$ or $7e^{j0^\circ}$

5\. (a) $5\sqrt{2} \cos(100t - 45^\circ)$

(b) $3 \cos(3t + \pi)$

(c) $2 \cos(\pi t - 30^\circ) + 4 \cos(4t + 20^\circ)$

Exercises
chapter 10.3

1.

$u(t) = 2\cos(3t)$

let $u(t) = 2e^{j3t}$, $y(t) = \underline{Y} e^{j3t} \Rightarrow \frac{dy}{dt} = j3\underline{Y} e^{j3t}$

$$2\frac{dy}{dt} + 6y(t) = u(t) \Rightarrow 2\left[j3\underline{Y}e^{j3t}\right] + 6\left[\underline{Y}e^{j3t}\right] = 2e^{j3t}$$

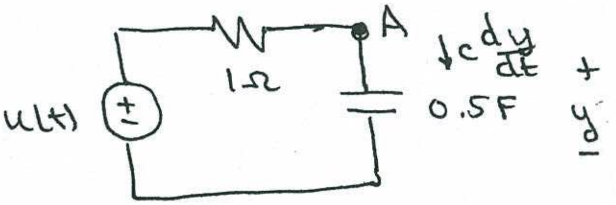
$$j6\underline{Y}\cancel{e^{j3t}} + 6\underline{Y}\cancel{e^{j3t}} = 2\cancel{e^{j3t}}$$

$$\underline{Y} = \frac{2}{6+j6} = \frac{2}{6\sqrt{2}\angle 45^\circ} = \frac{1}{3\sqrt{2}}\angle -45^\circ$$

$$\underline{\underline{y(t) = \frac{1}{3\sqrt{2}}\cos(3t - 45^\circ)}}$$

2.

(a)



KCL at A:

$$\frac{u - y}{1\Omega} = 0.5F\,\frac{dy}{dt}$$

$$\frac{dy}{dt} + 2y = 2u$$

(b) $u(t) = 3\cos(2t) \Rightarrow$ Let $u(t) = 3e^{j2t}$ & $y(t) = \underline{Y}e^{j2t}$

$$(j2)\underline{Y}\cancel{e^{j2t}} + 2\underline{Y}\cancel{e^{j2t}} = 2(3\cancel{e^{j2t}})$$

$$\underline{Y} = \frac{6}{2+j2} = \frac{6}{2\sqrt{2}\angle 45^\circ} = \frac{3}{\sqrt{2}}\angle -45^\circ \Rightarrow \underline{\underline{y(t) = \frac{3}{\sqrt{2}}\cos(2t - 45^\circ)}}$$

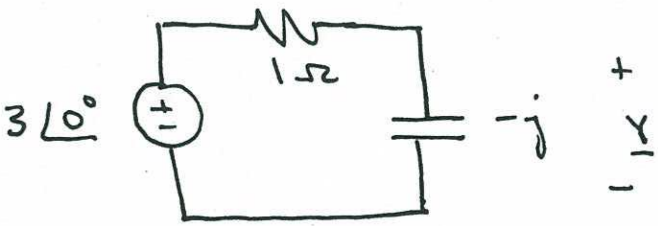
Exercises

Chapter 10.4.

1. We did this problem in exercise 2 of chapter 10.3, but let's do it differently here. (This approach won't be formally introduced until chapter 10.5)

- At a frequency of 2 rad/sec, the capacitor impedance is $\frac{1}{j(2)(0.5)} = -j$.

- Substitute the frequency-domain representation of the input and the capacitor into the circuit:



Now use voltage divider formula to find $\underline{Y}$:

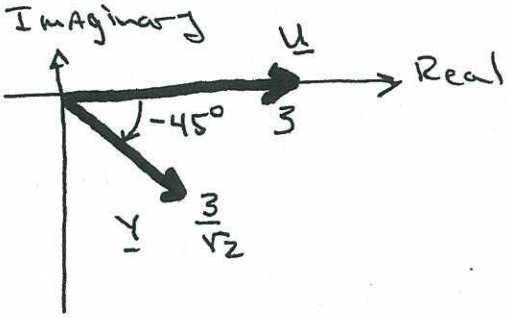
$$\underline{Y} = 3\angle 0^\circ \left(\frac{-j}{1-j} \right) = 3\angle 0^\circ \left(\frac{1\angle -90^\circ}{\sqrt{2}\angle -45^\circ} \right)$$

$$\underline{Y} = \frac{3}{\sqrt{2}} \angle -45^\circ \Rightarrow \underline{y(t) = \frac{3}{\sqrt{2}} \cos(2t - 45^\circ)}$$

Exercises
Chapter 10.4

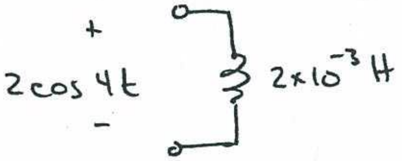
2.

$\underline{U} = 3\angle 0^\circ$, $\underline{Y} = \frac{3}{\sqrt{2}}\angle -45^\circ$

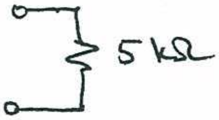


3.

$\Rightarrow \quad Z = \frac{1}{j\omega C} = \frac{1}{j(4)\frac{1}{8}} = \underline{\underline{-j2\Omega}}$



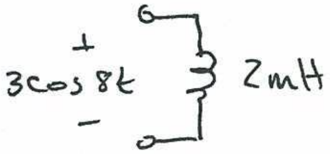
$\Rightarrow \quad Z = j\omega L = j(4)(2\times10^{-3}) = \underline{\underline{j8\,m\Omega}}$



$\Rightarrow \quad \underline{\underline{Z = 5\,k\Omega}}$

4.

$\Rightarrow Z = \frac{1}{j(8)\frac{1}{8}} = -j\underline{\underline{1\Omega}}$

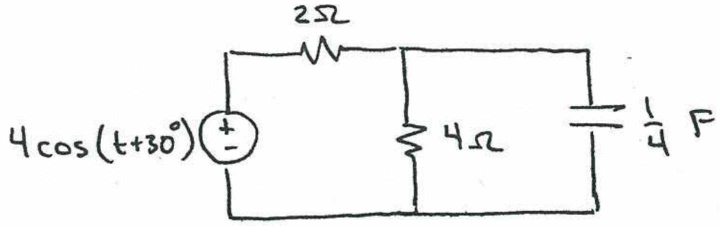


$\Rightarrow Z = j\omega L = j(8)(2\times10^{-3}) = j16\,m\Omega$

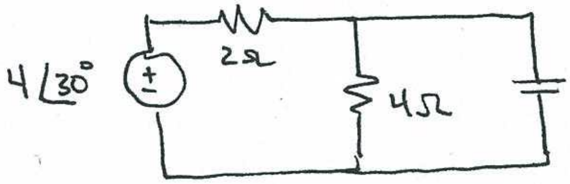
5kΩ resistor's impedance is 5kΩ, independent of frequen

Exercises
Chapter 10.5

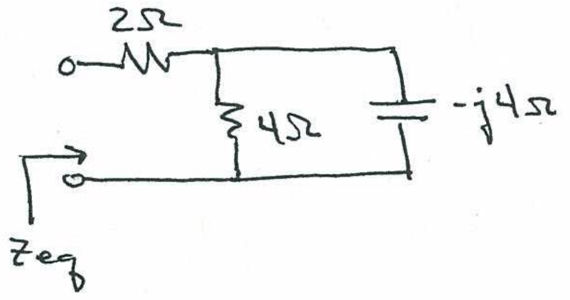
1.



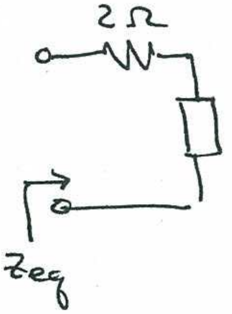
$\downarrow$ $\omega = 1$ rad/sec



$\frac{1}{j\omega C} = \frac{1}{j(1)(\frac{1}{4})} = -j4\Omega$



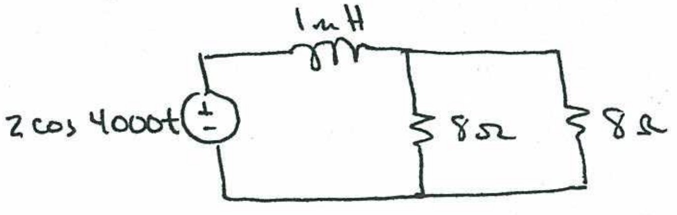
$\Rightarrow$



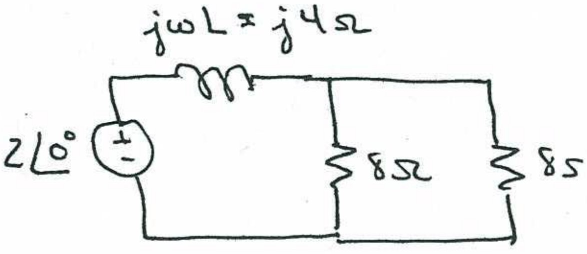
$\frac{(4\Omega)(-j4\Omega)}{(4-j4)\Omega} = 2\sqrt{2}\angle{-45}$

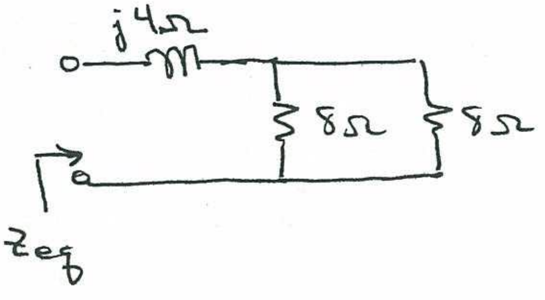
$Z_{eq} = 2\Omega + 2\sqrt{2}\angle{-45°} = 2 + (2 - j2) = \underline{\underline{4 - j2\Omega}} = \underline{\underline{2\sqrt{5}\angle{-26.6°}}}$

2.

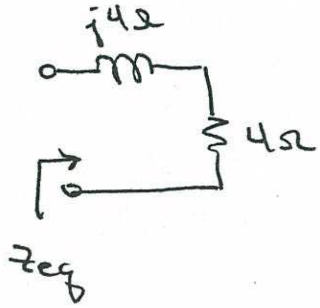


$\xrightarrow{\omega = 4000 \text{ rad/s}}$





$\rightarrow$

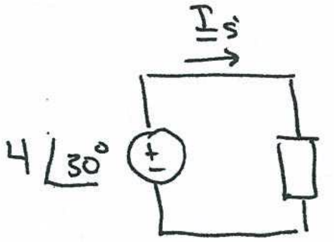


$\Rightarrow$ $\underline{\underline{Z_{eq} = (4 + j4)\Omega}}$

$\underline{\underline{Z_{eq} = 4\sqrt{2}\angle{45°}}}$

Exercises
Chapter 10.5

3. From exercise 1, the equivalent circuit is

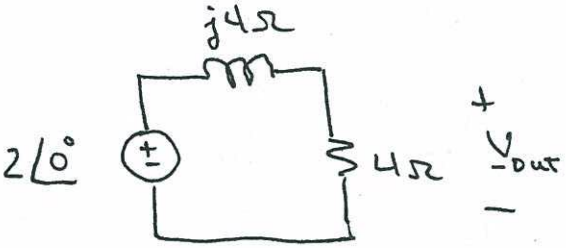


$\underline{Z}_{eq} = 2\sqrt{5}\,\angle{-26.6°}$

$$\underline{I}_s = \frac{4\angle 30°}{2\sqrt{5}\,\angle{-26.6°}} = \frac{2}{\sqrt{5}}\angle 56.6°$$

$$\underline{\underline{i_s(t) = \frac{2}{\sqrt{5}}\cos(t + 56.6°)}}$$

4. From exercise 2, the equivalent circuit is:



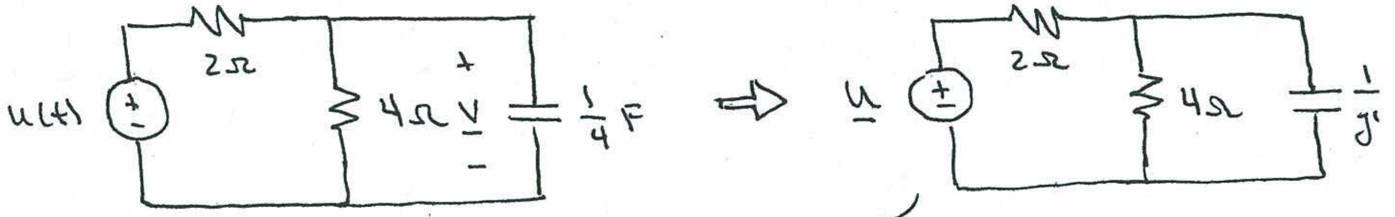
$$\underline{V}_{OUT} = 2\angle 0°\left[\frac{4\Omega}{(4+j4)\,\Omega}\right] = 2\angle 0°\left[\frac{4}{4\sqrt{2}\angle 45°}\right] = \frac{8}{4\sqrt{2}\angle 45°}$$

$$\underline{V}_{OUT} = \sqrt{2}\,\angle{-45°}$$

$$\underline{\underline{V_{OUT}(t) = \sqrt{2}\cos(4000t - 45°)}}$$

Exercises
Chapter 10.6

1. $$u(t) = \underset{\substack{\uparrow \\ \omega=1}}{4\cos(t+30^\circ)} + \underset{\substack{\uparrow \\ \omega=2}}{2\cos(2t-45^\circ)}$$



circuit reduction

$$\frac{4\cdot\frac{1}{j\omega c}}{4+\frac{1}{j\omega c}} = \frac{4}{4j\omega c+1} = \frac{4}{j4\omega(\frac{1}{4})+1} = \frac{4}{j\omega+1}$$

Voltage divider:

$$\underline{V} = \underline{u}\,\frac{4/(j\omega+1)}{2+4/(j\omega+1)} = \frac{4}{6+j2\omega}\,\underline{u}$$

$$\omega = 1 \text{ rad/sec} \Rightarrow \underline{V} = \frac{4}{6+j2(1)}\left[4\angle 30^\circ\right] = (0.63\angle -18.4^\circ)(4\angle 30^\circ)$$

$$\underline{V} = 2.52\angle 11.6^\circ$$

$$\omega = 2 \text{ rad/sec} \Rightarrow \underline{V} = \frac{4}{6+j2(2)}\left[2\angle -45^\circ\right] = 1.1\angle -78.7^\circ$$

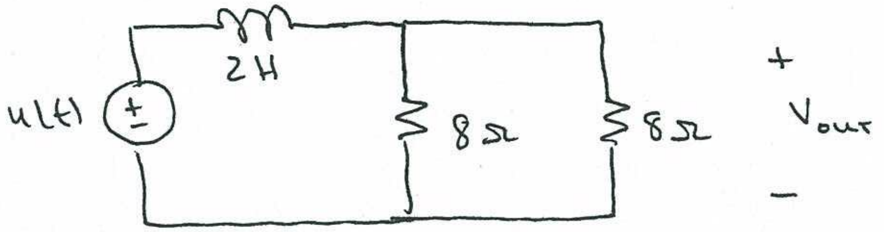
Superimpose in time domain:

$$\underline{\underline{v(t) = 2.52\cos(t+11.6^\circ) + 1.1\cos(2t-78.7^\circ)}}$$

Exercises
Chapter 10.6

2. Total applied voltage:

$$u(t) = u_1(t) + u_2(t) = 4\cos(2t) + \cos(4t)$$



Circuit reduction & conversion to frequency domain

$$\underline{V}_{OUT} = \frac{4}{4+j2\omega} \cdot \underline{u}$$ ← Voltage divider

$$\omega = 2 \text{ rad/sec} \Rightarrow \underline{V}_{out} = \frac{4}{4+j4}(4\angle 0^\circ) = 2\sqrt{2}\angle -45^\circ$$

$$\omega = 4 \text{ rad/sec} \Rightarrow \underline{V}_{OUT} = \frac{4}{4+j8}(1\angle 0^\circ) = 0.45\angle -63.4^\circ$$

Superimpose in time domain to get total response:

$$\underline{\underline{v_{out}(t) = 2\sqrt{2}\cos(2t - 45^\circ) + 0.45\cos(4t - 63.4^\circ)}}$$