

Real Analog: Chapter 1

Revised April 18, 2017

1 Introduction and Chapter Objectives

In this chapter, we introduce all fundamental concepts associated with circuit analysis. Electrical circuits are constructed in order to direct the flow of electrons to perform a specific task. In other words, in circuit analysis and design, we are concerned with transferring electrical energy in order to accomplish a desired objective. For example, we may wish to use electrical energy to pump water into a reservoir; we can adjust the amount of electrical energy applied to the pump to vary the rate at which water is added to the reservoir. The electrical circuit, then, might be designed to provide the necessary electrical energy to the pump to create the desired water flow rate.

This chapter begins with introduction to the basic parameters which describe the energy in an electrical circuit: *charge*, *voltage*, and *current*. Movement of charge is associated with electrical energy transfer. The energy associated with charge motion is reflected by two parameters: voltage and current. Voltage is indicative of an electrical energy change resulting from moving a charge from one point to another in an electric field. Current indicates the rate at which charge is moving, which is associated with the energy of a magnetic field. We will not be directly concerned with charge, electrical fields, or magnetic fields in this course, we will work almost exclusively with voltages and currents. Since *power* quantifies the rate of energy transfer, we will also introduce power in this chapter.

Electrical circuits are composed of interconnected *components*. In this chapter, we will introduce two basic types of components: *power supplies* and *resistors*. Power supplies are used to provide power to our electrical circuits, and resistors dissipate electrical power by converting it to heat. These two types of components will allow us to introduce and exercise virtually all available circuit analysis techniques. Electrical components are described in terms of the relationships between the voltages and currents at their terminals, these relationships are called the *voltage-current characteristics* of the device. In this chapter, we will introduce voltage-current characteristics for power supplies and resistors. In later chapters, we will introduce additional circuit components, but our circuit analysis approaches will not change – we will simply substitute voltage-current characteristics for these components as appropriate to model future circuits.

Finally, we introduce the two fundamental rules of circuit analysis: *Kirchhoff's Current Law* and *Kirchhoff's Voltage Law*. These rules form the basis of all circuit analysis techniques used throughout this textbook.

Please pay special attention to the *passive sign convention* introduced in this chapter. Voltages and currents have signs – they can be positive or negative – and these signs are crucial to understanding the effect of these parameters on the energy transferred by the circuit. No useful circuit analysis can be performed without following the passive sign convention.

In summary, this chapter introduces virtually all the basic concepts which will be used throughout this textbook. After this chapter, little information specific to electrical circuit analysis remains to be learned – the remainder of

the textbook is devoted to developing analysis methods used to increase the efficiency of our circuit analysis and introducing additional circuit components such as capacitors, inductors, and operational amplifiers. The student should be aware, however, that all of our circuit analysis is based on energy transfer among circuit components; this energy transfer is governed by Kirchhoff's Current Law and Kirchhoff's Voltage Law and the circuit components are modeled by their voltage-current relationships.

After Completing this Chapter, You Should be Able to:

- Define voltage and current in terms of electrical charge
- State common prefixes and the symbols used in scientific notation
- State the passive sign convention from memory
- Determine the power absorbed or generated by a circuit element, based on the current and voltage provided to it
- Write symbols for independent voltage and current sources
- State from memory the function of independent voltage and current sources
- Write symbols for dependent voltage and current sources
- State governing equations for the four types of dependent sources
- State Ohm's Law from memory
- Use Ohm's Law to perform voltage and current calculations for resistive circuit elements
- Identify nodes in an electrical circuit
- Identify loops in an electrical circuit
- State Kirchhoff's current law from memory, both in words and as a mathematical expression
- State Kirchhoff's voltage law from memory, both in words and as a mathematical expression
- Apply Kirchhoff's voltage and current laws to electrical circuits

1.1 Basic Circuit Parameters and Sign Conventions

This section introduces the basic engineering parameters for electric circuits: *voltage*, *current*, and *power*. The international system of units is commonly used to describe the units of these parameters; this system as it relates to electrical circuit analysis is briefly discussed in this section.

This section also introduces the *passive sign convention*. It is extremely important when analyzing electrical circuits to use the correct sign convention between the voltage across a circuit element and the current going through the element. Some of the most common errors of beginning students are associated with applying incorrect sign conventions when analyzing circuits.

1.1.1 Electrical Charge

Electron flow is fundamental to operation of electric circuits; the concept of *charge* can be used to describe the distribution of electrons in the circuit. Charge can be represented as either positive or negative – generally relative to some reference level. Charge is represented by the variable q and is measured in *coulombs*, abbreviated as C . The charge of one electron corresponds to $-1.6022 \times 10^{-19} C$. Charge can only exist in integer multiples of the charge of a single electron. Charge, however, is not widely used in electrical circuit analysis; voltage and current are more convenient ways to represent the electric charge in a system.

1.1.2 Voltage

Voltage is energy per unit charge. Energy is specified relative to some reference level; thus, voltages are more accurately specified as voltage *differences* between two points in a circuit. The voltage difference between two points can be thought of as a difference in potential energy between charges placed at those two points. Units of

voltage are volts, abbreviated V. The voltage difference between two points indicates the energy necessary to move a unit charge from one of the points to the other. Voltage differences can be either positive or negative.

Mathematically, voltage is expressed in differential form as:

$$v = \frac{dw}{dq} \quad \text{Eq. 1.1}$$

Where v is the voltage difference (in volts), w is the energy (in joules), and q is the charge (in coulombs). The differences in equation (1.1) are all defined relative to different spatial positions; thus, the differentials dw and dq are between two different points in space, and the voltage is defined as being between these same two spatial points.

1.1.3 Current

Current is the rate at which charge is passing a given point. Current is specified at a particular point in the circuit, and is not relative to a reference. Since current is caused by charge in motion, it can be thought of as indicating *kinetic* energy.

Mathematically, current is represented as:

$$i = \frac{dq}{dt} \quad \text{Eq. 1.2}$$

Where i is the current in amperes, q is the charge in coulombs, and t is the time in seconds. Thus, current is the time rate of change of charge and units of charge are coulombs per second, or *amperes* (abbreviated as A).

1.1.4 Power

An electrical system is often used to drive a non-electrical system (in an electric stove burner, for example, electric energy is converted to heat). Interactions between electrical and non-electrical systems are generally described in terms of *power*. Electrical power associated with a particular circuit element is the product of the current passing through the element and the voltage difference across the element. This is often written as:

$$p(t) = v(t) \cdot i(t) \quad \text{Eq. 1.3}$$

Where $p(t)$ is the *instantaneous* power at time t , $v(t)$ is the voltage difference at time t , and $i(t)$ is the current at time t . Power can either be absorbed by a circuit element or generated by a circuit element; the determination as to whether the element is absorbing or generating power can be made by the relative signs of the values of voltage and current. These sign conventions are an important issue, and will be addressed separately in the next chapter. Units of power are watts, abbreviated W.

1.1.5 International System of Units and Prefixes

We will use the international system of units (SI). The scales of parameters that are of interest to engineers can vary over many orders of magnitudes. For example, voltages experienced during lightning strikes can be on the order of 10^7V , while voltages measured from an electroencephalograph (EEG) can be on the order of 10^{-4}V . For this reason, numbers represented in SI units are often associated with a prefix, which helps account for the order-of-magnitude variations in numbers. Table 1 below provides a list of common prefixes and the symbols used to represent them.

Multiple	Prefix	Symbol
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

Table 1.1. SI prefixes.

1.1.6 Passive Sign Convention

A general two-terminal electrical circuit element is shown in Fig. 1.1. In general, there will be some current, i , flowing through the element and some voltage difference, v , across its terminals. Note that we are currently representing both voltage and current as constants, but none of the assertions made in this chapter change if they are functions of time.

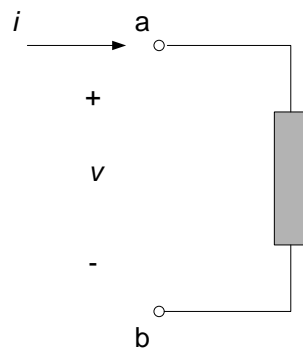
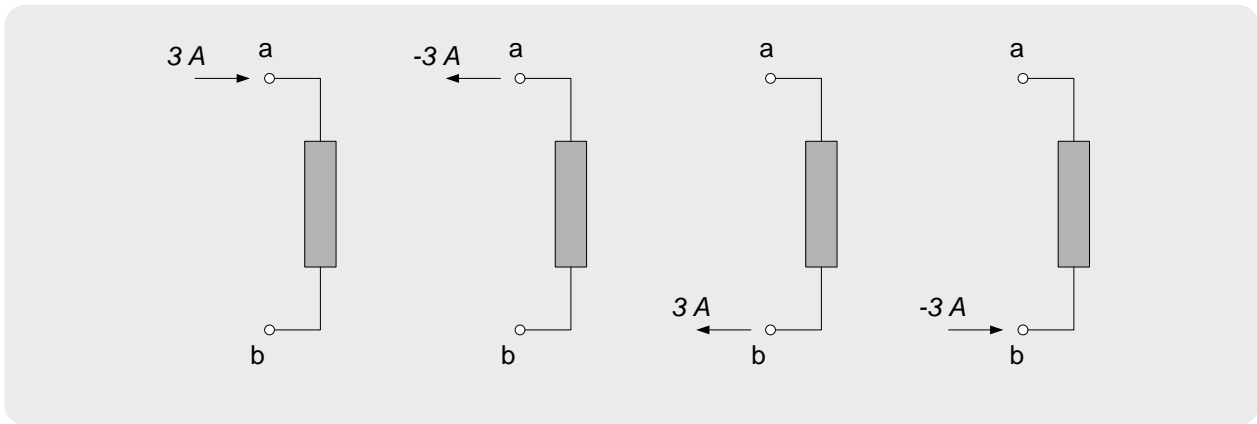


Figure 1.1. General circuit element and passive sign convention.

The assumed direction of the current, i , passing through the element is shown by the arrow on Fig. 1.1. In Fig. 1.1, i is assumed to be positive if it is going into node a. A negative value of i simply indicates a change in direction of the current – if i is negative, the current is going into node b (or, equivalently, out of node a). We will assume that our circuit elements do not accumulate charge, so any current entering node a must leave node b.

Example 1.1:

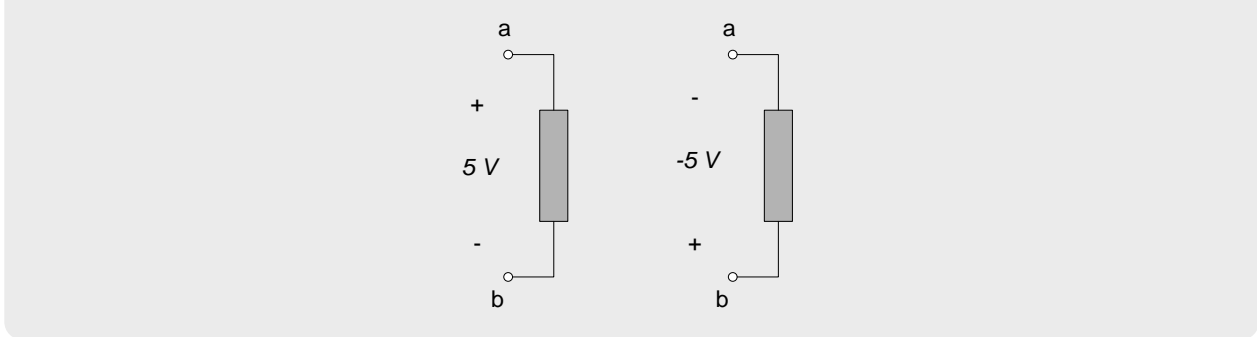
Three amperes (3 A) of current is passing through a circuit element connecting nodes a and b. The current is flowing from node a to node b. The physical situation can be represented schematically by any of the figures shown below – all four figures represent the same current flow and direction.



The assumed polarity of the voltage difference v across the element is shown by the + and - signs on Fig. 1.1. The polarity shown in Fig. 1.1 indicates that a positive value for v indicates that the voltage at the terminal marked with a + sign is higher than the voltage at the terminal marked with a - sign (that is, the voltage at node a is higher than the voltage at node b). A negative value for v simply reverses this polarity (negative voltage means that the voltage at node b is higher than the voltage at node a).

Example 1.2:

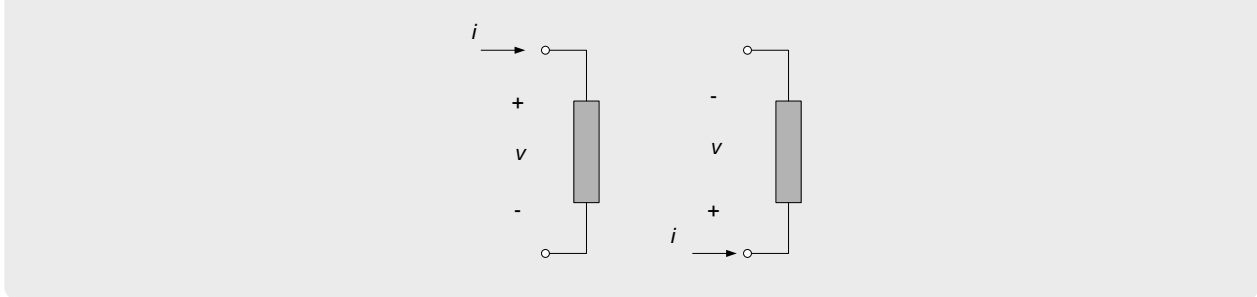
A 5 volt (5 V) voltage potential difference is applied across a circuit element connecting nodes a and b. The voltage at node a is positive relative to the voltage at node b. The physical situation can be represented schematically by either of the figures shown below – both figures represent the same voltage potential difference.



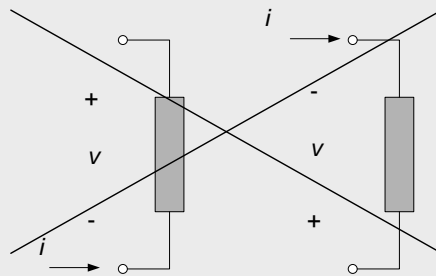
The assumed voltage polarity and current direction are not individually significant - the assumed direction of voltage polarity relative to current direction is important. To satisfy our sign convention, we will assume that positive current enters the node at which the positive voltage polarity is defined. This sign convention is called the *passive sign convention*. In the passive sign convention, the relative assumed sign convention between voltage and current is as shown in Fig. 1.1.

Example 1.3:

The passive sign convention is satisfied for either of the two voltage-current definitions shown below - the current is assumed to enter the positive voltage node.



The passive sign convention is not satisfied for either of the two voltage current definitions shown below – the current is assumed to enter the negative voltage node.



Note: Many students attempt to choose current directions and voltage polarities so that their calculations result in positive values for voltages and currents. In general, this is a waste of time - it is best to arbitrarily assume either a voltage polarity of current direction for each circuit element.

Choice of a positive direction for current dictates the choice of positive voltage polarity, per Fig. 1.1. Choice of a positive voltage polarity dictates the choice of positive current direction, per Fig. 1.1.

Analysis of the circuit is performed using the above assumed signs for voltage and current. The sign of the results indicates whether the assumed choice of voltage polarity and current direction was correct. A positive magnitude of a calculated voltage indicates that the assumed sign convention is correct; a negative magnitude indicates that the actual voltage polarity is opposite to the assumed polarity. Likewise, a positive magnitude of a calculated current indicates that the assumed current direction is correct; a negative magnitude indicates that the current direction is opposite to that assumed.

1.1.7 Voltage Subscript and Sign Conventions

The assumed sign convention for voltage potentials is sometimes expressed by using subscripts. The first subscript denotes the node at which the positive voltage polarity is assumed and the second subscript is the negative voltage polarity. For example, v_{ab} denotes the voltage difference between nodes a and b , with node a assumed as having positive voltage relative to node b . Switching the order of the subscripts changes the assumed polarity of the voltage difference and this the sign of the voltage, so $v_{ab} = -v_{ba}$. Since our passive sign convention dictates the direction of current relative to voltage polarity, a circuit element whose voltage difference is denoted as v_{ab} will have positive current entering node a .

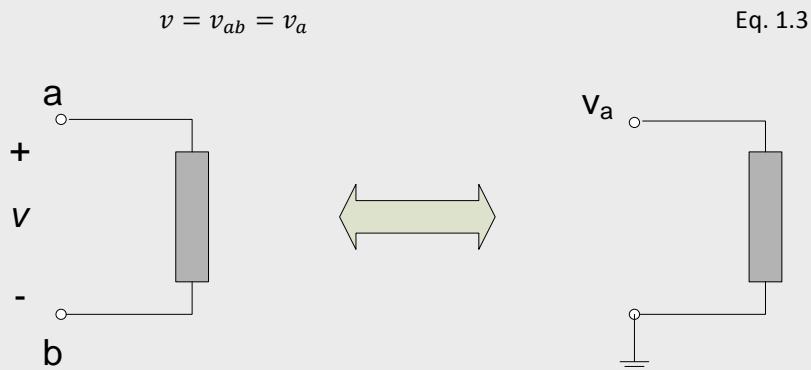
1.1.8 Reference Voltages and Ground

For convenience, voltage differences are often not explicitly stated as being differences between two potential levels - a node will simply be referred to as having some “voltage”. This voltage must still be interpreted as a voltage difference, however. The difference in this case is assumed to be relative to some reference voltage, with the reference generally assumed to be 0V. The reference voltage is often referred to as *ground*. The symbol, displayed below, is used to denote the ground or reference voltage from which all other voltages are measured. When this convention is used, voltages at a node are often identified with a single subscript. For example, v_a would be the voltage at node a , relative to ground. It is assumed that positive voltages are positive relative to ground and negative voltages are negative relative to ground.



Example 1.4:

The two figures below show identical ways of specifying the voltage across a circuit element. In the circuit to the left, the voltage v is the voltage potential between nodes a and b, with the voltage at node a being assumed positive relative to voltage at node b. This can be equivalently specified as v_{ab} . In the figure to the right below, node b has been specified as our ground with the use of the ground symbol. In this figure, the voltage at node a can be specified simply as v_a , with the polarity being assumed positive relative to ground which is implied to be 0V. Thus, for the figures below:



1.1.9 Power and Sign Conventions

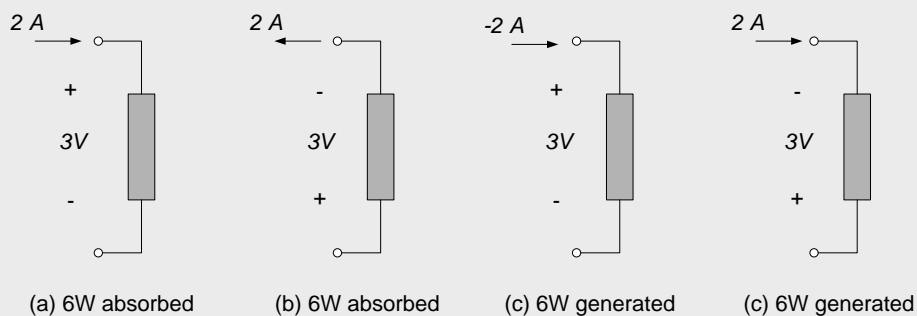
The sign of the voltage across an element relative to the sign of the current through the element governs the sign of the power. Equation (1.3) above defines power as the product of the voltage times current:

$$P = vi \tag{Eq. 1.4}$$

The power is *positive* if the signs of voltage and current agree with the passive sign convention - that is, if a positive current enters the positive voltage polarity node. If the power is positive, the element is said to be absorbing power. The power is *negative* if the signs of voltage and current disagree with the passive sign convention - that is, if positive current enters the negative voltage polarity node. If the power is negative, the element is said to be *generating* power.

Example 1.5:

In Fig. (a) below, the element agrees with the passive sign convention since a positive current is entering the positive voltage node. Thus, the element of Fig. (a) is absorbing energy. In Fig. (b), the element is absorbing power - positive current is leaving the negative voltage node, which implies that positive current enters the positive voltage node. The element of Fig. (c) generates power; negative current enters the positive voltage node, which disagrees with the passive sign convention. Fig. (d) also illustrates an element which is generating power, since positive current is entering a negative voltage node.

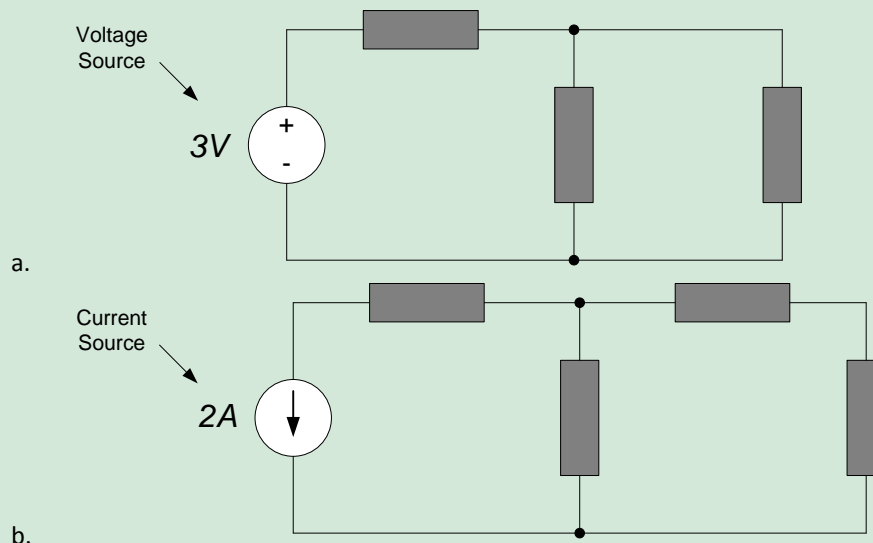


Section Summary

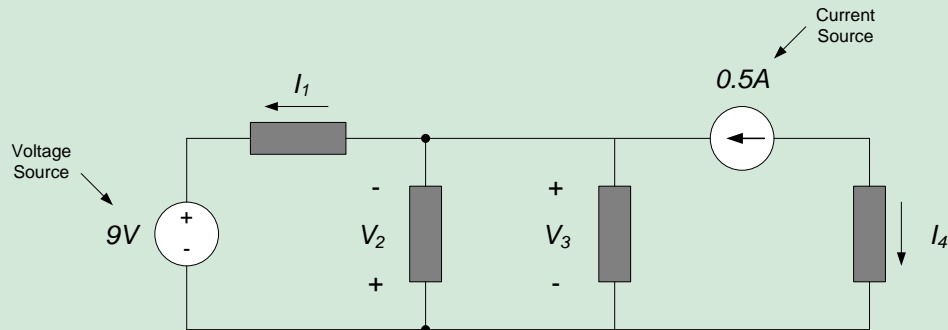
- In this text, we will be primarily concerned with the movement of electrical charge. Electrical charge motion is represented by voltage and current. Voltage indicates the energy change associated with the movement of a charge from one location to another, while current is indicative of the rate of current motion past a particular point.
 - Voltage is an energy difference between two physically separated points. The polarity of a voltage is used to indicate which point is to be assumed to be at the higher energy level. The positive terminal (+) is assumed to be at a higher voltage than the negative terminal (-). A negative voltage value simply indicates that the actual voltage polarity is opposite to the assumed polarity.
 - The sign of the current indicates the assumed direction of charge motion past a point. A change in the sign of the current value indicates that the current direction is opposite to the assumed direction.
- The assumed polarity of the voltage across a passive circuit element must be consistent with the assumed current direction through the element. The assumed positive direction for current must be such that positive current enters the positive voltage terminal of the element. Since this sign convention is applied only to passive elements, it is known as the *passive sign convention*.
 - The assumed current direction or the assumed voltage polarity can be chosen arbitrarily, but once one parameter is chosen, the other must be chosen to agree with the passive sign convention.
- The power absorbed or generated by an electrical circuit component is the product of the voltage difference across the element and the current through the element: $p=iv$. The relative sign of the voltage and current are set according to the passive sign convention. Positive power implies that the voltage and current are consistent with the passive sign convention (the element absorbs or dissipates energy) while negative power indicates that the relative signs between voltage and current are opposite to the passive sign convention (the element generates or supplies energy to the circuit).

1.1 Exercises

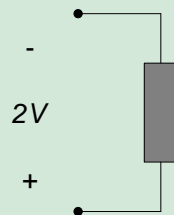
1. Assign reference voltage and current directions to the circuit elements represented by the shaded boxes in the circuits below.



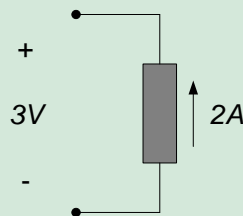
2. Either the reference voltage polarity or the reference current direction is provided for the circuit elements below. Provide the appropriate sign convention for the missing parameters.



3. Determine the magnitude and direction of the current in the circuit element below if the element absorbs 10W.



4. Determine the power absorbed or supplied by the circuit element below. State whether the power is absorbed or supplied.



1.2 Power Sources

Circuit elements are commonly categorized as either *passive* or *active*. A circuit element is passive if the total amount of energy it delivers to the rest of the circuit (over all time) is non-positive (passive elements can temporarily deliver energy to a circuit, but only if the energy was previously stored in the passive element by the circuit). An active circuit element has the ability to create and provide power to a circuit from mechanisms external to the circuit. Examples of active circuit elements are batteries (which create electrical energy from chemical processes) and generators (which create electrical energy from mechanical processes, such as spinning a turbine).

In this section we consider some very important active circuit elements: voltage and current sources. We will discuss two basic types of sources: *independent sources* and *dependent sources*. Independent sources provide a specified voltage or current, regardless of what is happening elsewhere in the circuit to which they are connected - batteries and generators are generally considered to be independent sources. Dependent sources provide a voltage or current based on a voltage or current elsewhere in the circuit (the source voltage or current is dependent upon some other voltage or current). Dependent sources are often used in the mathematical modeling of common devices such as metal-oxide semiconductor field-effect transistors (MOSFETs) and bipolar junction transistors (BJTs).

1.2.1 Independent Voltage Sources

An independent voltage source maintains a specified voltage across its terminals. The symbol used to indicate a voltage source delivering a voltage $v_s(t)$ is shown in Fig. 1.2. As indicated in Fig. 1.2, the voltage supplied by the source can be time-varying or constant (a constant voltage is a special case of a time-varying voltage). An alternate symbol that is often used to denote a constant voltage source is shown in Fig. 1.3; we, however, will generally use the symbol of Fig. 1.2 for both time-varying and constant voltages.

Note that the sign of the voltage being applied by the source is provided on the source symbol - there is no need to assume a voltage polarity for voltage sources. The current direction, however, is unknown and must be determined (if necessary) from an analysis of the overall circuit.

Ideal voltage sources provide a specified voltage regardless of the current flowing through the device. Ideal sources can, obviously, provide infinite power; all real sources will provide only limited power to the circuit. We will discuss approaches for modeling non-ideal sources in later chapters.

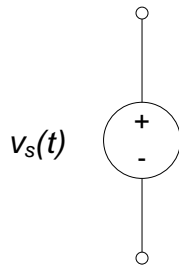


Figure 1.2. Independent voltage source.

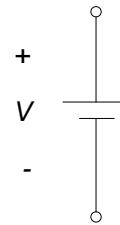


Figure 1.3. Constant voltage source.

1.2.2 Independent Current Sources

An independent current source maintains a specified current. This current is maintained regardless of the voltage differences across the terminals. The symbol used to indicate a current source delivering a current $i_s(t)$ is shown in Fig. 1.4. The current supplied by the source can be time-varying or constant.

Note that the sign of the current being applied by the source is provided on the source symbol - there is no need to assume a current direction. The voltage polarity, however, is unknown and must be determined (if necessary) from an analysis of the overall circuit.

Ideal current sources provide a specified current regardless of the voltage difference across the device. Ideal current sources can, like ideal voltage sources, provide infinite power; all real sources will provide only limited power to the circuit. We will discuss approaches for modeling non-ideal current sources in later chapters.

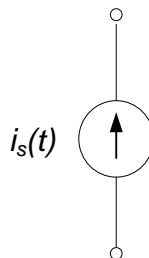


Figure 1.4. Independent current source.

1.2.3 Dependent Sources

Dependent sources can be either voltage or current sources; Fig. 1.5(a) shows the symbol for a dependent voltage source and Fig. 1.5(b) shows the symbol for a dependent current source. Since each type of source can be controlled by either a voltage or current, there are four types of dependent current sources:

- Voltage-controlled voltage source (VCVS)
- Current-controlled voltage source (CCVS)
- Voltage-controlled current source (VCCS)
- Current-controlled current source (CCCS)



Figure 1.5. Symbols for dependent sources.

Figure 1.6 illustrates the voltage-controlled dependent sources, and Fig. 1.7 illustrates the current-controlled dependent sources. In all cases, some electrical circuit exists which has some voltage and current combination at its terminals. Either the voltage or current at these terminals is used to set the voltage or current of the dependent source. The parameters μ and β in Figs. 1.6 and 1.7 are dimensionless constants. μ is the *voltage gain* of a VCVS and β is the *current gain* of a CCCS. The parameter r is the voltage-to-current ratio of a CCVS and has units of volts/ampere, or *ohms*. The parameter g is the current-to-voltage ratio of a VCCS and has units of amperes/volt, or *siemens*. The units of ohms and siemens will be discussed in more depth in section 1.3.

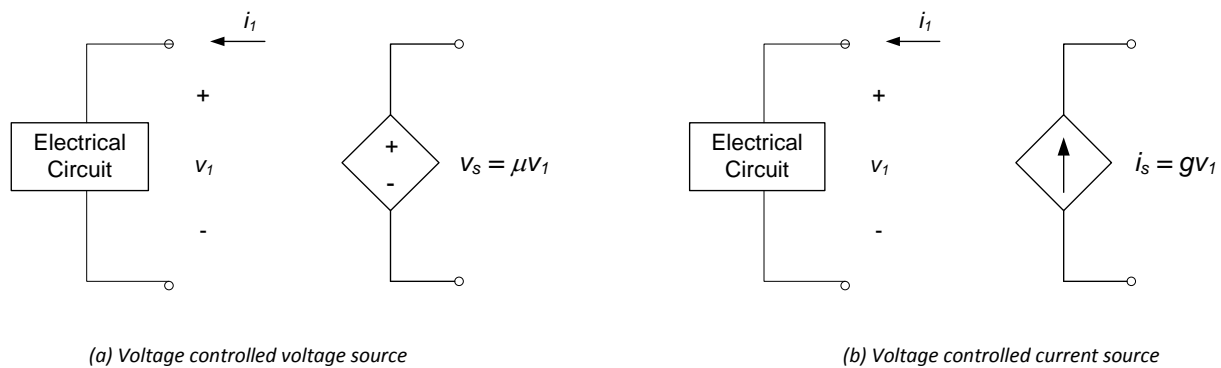


Figure 1.6. Voltage-controlled dependent sources

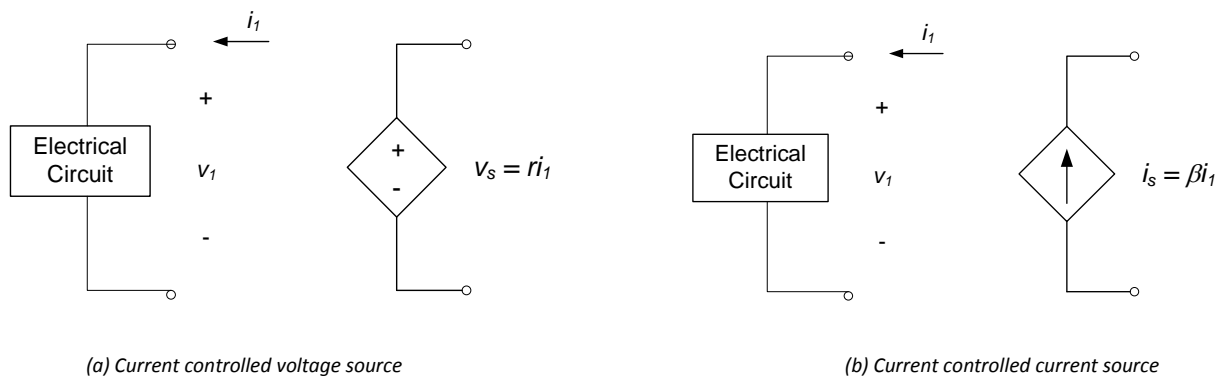


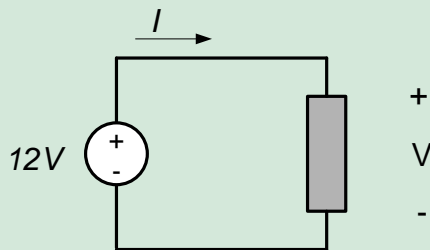
Figure 1.7. Current-controlled dependent sources

Section Summary

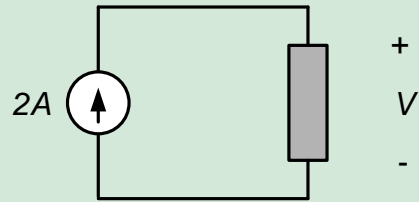
- Circuit elements can be either active or passive. Active elements provide electrical energy from a circuit from sources outside the circuit; active elements can be considered to create energy (from the standpoint of the circuit, anyway). Passive elements will be discussed in section 1.3, when we introduce resistors. Active circuit elements introduced in this section are ideal independent and dependent voltage and current sources.
 - Ideal independent sources presented in this section are voltage and current sources. Independent voltage sources deliver the specified voltage, regardless of the current demanded of them. Independent current sources provide the specified current, regardless of the voltage levels required to provide this current. Devices such as batteries are often modeled as independent sources.
 - Dependent sources provide a voltage or current which is controlled by a voltage or current elsewhere in the circuit. Devices such as operational amplifiers and transistors are often modeled as dependent sources. We will revisit the subject of dependent sources in chapter 5 of this text, when we discuss operational amplifier circuits.

1.2 Exercises:

1. The ideal voltage source shown in the circuit below delivers 12V to the circuit element shown. What is the current I through the circuit element?



2. The ideal current source shown in the circuit below delivers 2A to the circuit element shown. What is the voltage difference V across the circuit element?



1.3 Resistors and Ohm's Law

Resistance is a property of all materials - this property characterizes the loss of energy associated with passing an electrical current through some conductive element. Resistors are circuit elements whose characteristics are dominated by this energy loss. Since energy is always lost when current is passed through an electrical circuit element, all electrical elements exhibit resistive properties which are characteristic of resistors. Resistors are probably the simplest and most commonly used circuit elements.

All materials impede the flow of current through them to some extent. Essentially, this corresponds to a statement that energy is always lost when transferring charge from one point in a circuit to another - this energy loss is generally due to heat generation and dissipation. The amount of energy required to transfer current in a particular element is characterized by the *resistance* of the element. When modeling a circuit, this resistance is represented by *resistors*. The circuit symbol for a resistor is shown in Fig. 1.8. The value of resistance is labeled in Fig. 1.8 as R . i in Fig. 1.8 is the current flowing through the resistor and v is the voltage drop across the resistor, caused by the energy dissipation induced by the resistor. The units of resistance are ohms, abbreviated Ω .

The relationship between voltage and current for a resistor is given by *Ohm's Law*:

$$v(t) = Ri(t) \quad \text{Eq. 1.5}$$

Where voltage and current are explicitly denoted as functions of time. Note that in Fig. 1.8, the current is flowing from a higher voltage potential to a lower potential, as indicated by the polarity (+ and -) of the voltage and the arrow indicating direction of the current flow. The relative polarity between voltage and current for a resistor must be as shown in Fig. 1.8; the current enters the node at which the voltage potential is highest. Values of resistance, R are always positive, and resistors always absorb power.

Note: The voltage-current relationship for resistors always agrees with the passive sign convention. Resistors always absorb power.

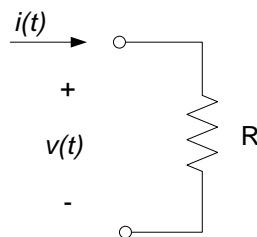


Figure 1.8. Circuit symbol for resistor.

Figure 1.9 shows a graph of v vs. i according to equation (1.5); the resulting plot is a straight line with slope R . Equation (1.5) thus describes the voltage-current relationship for a *linear* resistor. Linear resistors do not exist in reality - all resistors are *nonlinear*, to some extent. That is, the voltage-current relationship is not exactly a straight line for all values of current (for example, all electrical devices will fail if enough current is passed through them). Fig. 1.10 shows a typical nonlinear voltage-current relationship; however, many nonlinear resistors exhibit an

approximately linear voltage-current characteristic over some range of voltages and currents; this is illustrated in Fig. 1.10. We will assume for now that any resistor we use is operating within a range of voltages and currents over which its voltage-current characteristic is linear and can be approximated by equation (1.5).

Note: For the most part, we will consider only linear resistors in this text. These resistors obey the linear voltage-current relationship shown in equation (1.5). All real resistors are nonlinear to some extent, but can often be assumed to operate as linear resistors over some range of voltages and currents.

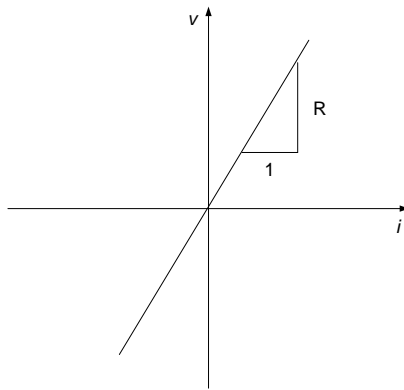


Figure 1.9. Linear resistor voltage vs. current characteristic.

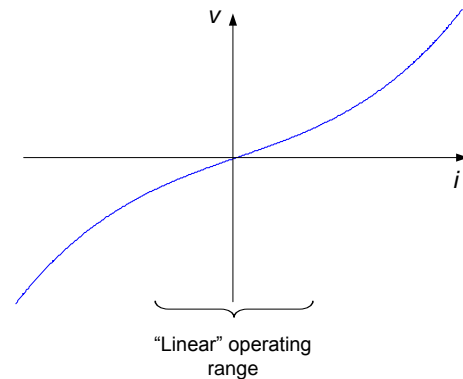


Figure 1.10. Typical nonlinear resistor voltage vs. current characteristic.

1.3.1 Conductance

Conductance is an important quantity in circuit design and analysis. Conductance is simply the reciprocal of resistance, defined as:

$$G = \frac{1}{R} \quad \text{Eq. 1.6}$$

The unit for conductance is siemens, abbreviated *S*. Ohm's law, written in terms of conductance, is:

$$i(t) = Gv(t) \quad \text{Eq. 1.7}$$

Some circuit analyses can be performed more easily and interpreted more readily if the elements' resistance is characterized in terms of conductance.

Note: In section 1.2, we characterized a current-controlled voltage source in terms of a parameter with units of ohms, since it had units of volts/amp. We characterized a voltage-controlled current source in terms of a parameter with units of siemens, since it had units of amps/volts.

1.3.2 Power Dissipation

Instantaneous power was defined by equation (1.3) in section 1.1 as:

$$P(t) = v(t) \cdot i(t)$$

For the special case of a resistor, we can re-write this (by substituting equation (1.5) into the above) as:

$$P(t) = Ri^2(t) = \frac{v^2(t)}{R} \quad \text{Eq. 1.8}$$

Likewise, we can write the power dissipation in terms of the conductance of a resistor as:

$$P(t) = \frac{i^2(t)}{G} = Gv^2(t)$$

Eq. 1.9

Note: We can write the power dissipation from a resistor in terms of the resistance or conductance of the resistor and either the current through the resistor or the voltage drop across the resistor.

1.3.3 Practical Resistors

All materials have some resistance, so all electrical components have non-zero resistance. However, circuit design often relies on implementing a specific, desired resistance at certain locations in a circuit; resistors are often placed in the circuit at these points to provide the necessary resistance. Resistors can be purchased in certain standard values. Resistors are manufactured in a variety of ways, though most commonly available commercial resistors are carbon composition or wire-wound. Resistors can have either a fixed or variable resistance.

Fixed resistors provide a single specified resistance value and have two terminals, as shown in Fig. 1.5 above. *Variable* resistors or *potentiometers* (commonly called “pots”) have three terminals, two are “fixed” and one is “movable”. The symbol for a variable resistor is shown in Fig. 1.11. The resistance between two of the terminals - R_{23} in Fig. 1.11 - of a variable resistor can be set as some fraction of the overall resistance of the device - R_{13} in Fig. 1.11. The ratio of R_{23} to R_{13} is generally set by a dial or screw on the side of the device.

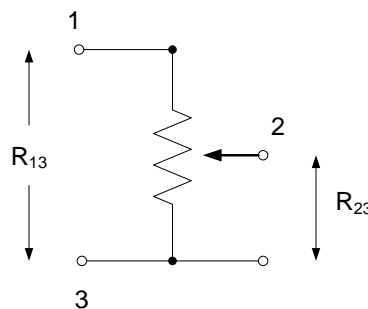
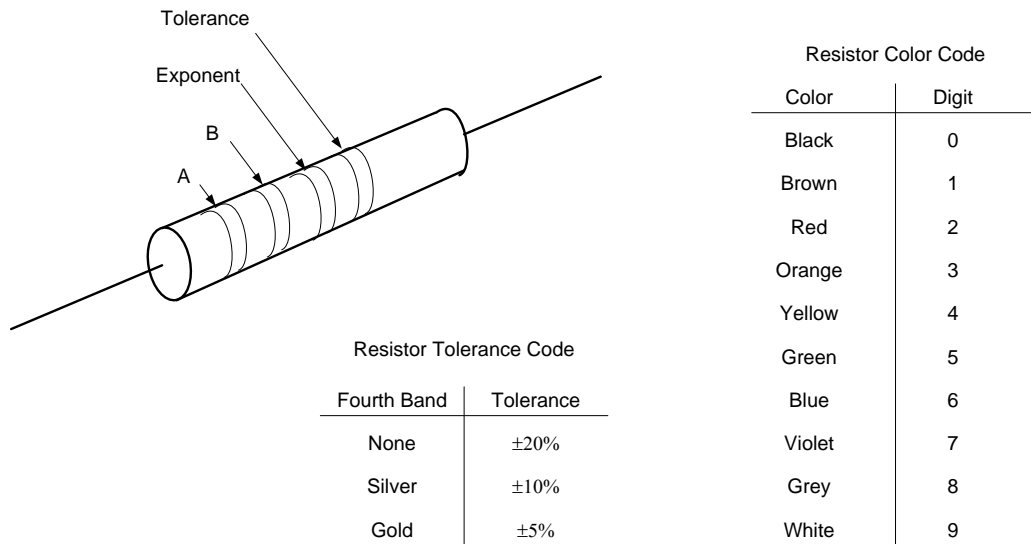


Figure 1.11. Schematic for variable resistor.

Resistors, which are physically large enough, will generally have their resistance value printed directly on them. Smaller resistors generally will use a color code to identify their resistance value. The color coding scheme is provided in Fig. 1.12. The resistance values indicated on the resistor will provide a *nominal* resistance value for the component; the actual resistance value for the component will vary from this by some amount. The expected tolerance between the allowable actual resistance values and the nominal resistance is also provided on the resistor, either printed directly on the resistor or provided as an additional color band. The color-coding scheme for resistor tolerances is also provided in Fig. 1.12.



$$\text{Resistance} = (10 \times A + B) \times 10^{\text{Exponent}} \pm \text{Tolerance}$$

Figure 1.12. Resistor color code.

Example 1.6:

A resistor has the following color bands below. Determine the resistance value and tolerance.

- First band (A): Red
- Second band (B): Black
- Exponent: Orange
- Tolerance: Gold

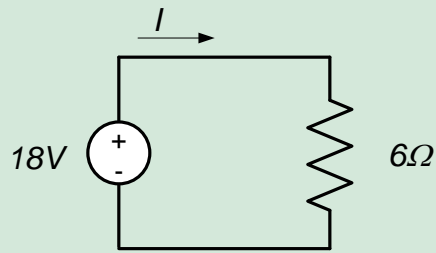
$$\text{Resistance} = (20+0) \times 10^3 \pm 5\% = 20 \text{ k}\Omega \pm 1 \text{ k}\Omega$$

Section Summary

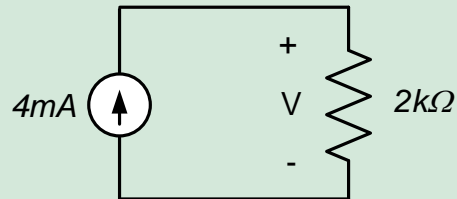
- The relationship between voltage and current for a resistor is Ohm's Law: $v=iR$. Since a resistor only dissipates energy, the voltage and current for a resistor must always agree with the passive sign convention.
- As noted in section 1.2, circuit elements can be either active or passive. Resistors are passive circuit elements. Passive elements can store or dissipate electrical energy provided to them by the circuit; they can subsequently return energy to the circuit which they have previously stored, but they cannot create energy. Resistors cannot store electrical energy; they can only dissipate energy by converting it to heat.

1.3 Exercises

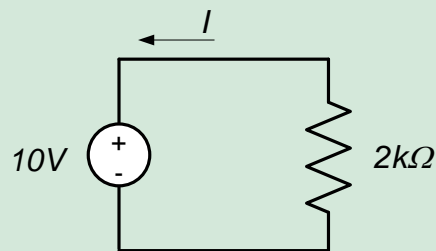
1. The ideal voltage source shown in the circuit below delivers 18V to the resistor shown. What is the current i through the resistor?



2. The ideal current source shown in the circuit below delivers 4mA to the resistor shown. What is the voltage difference V across the resistor?



3. The ideal voltage source shown in the circuit below delivers 10V to the resistor shown. What is the current I in the direction shown?



1.4 Kirchhoff's Laws

This section provides some basic definitions and background information for two important circuit analysis tools: Kirchhoff's Current Law and Kirchhoff's Voltage Law. These laws, together with the voltage-current characteristics of the circuit elements in the system, provide us with the ability to perform a systematic analysis of any electrical network.

We will use a *lumped-parameters* approach to a circuit analysis. This means that the circuit will consist of a number of discrete elements, connected by *perfect conductors*. Perfect conductors have no resistance, thus there is no voltage drop across a perfect conductor regardless of how much current flows through it. There is no energy stored or dissipated by a perfect conductor. All energy dissipation and energy storage is thus assumed to reside (or is *lumped*) in the circuit elements connected by the perfect conductors.

The lumped parameters approach toward modeling circuits is appropriate if the voltage and currents in the circuit change slowly relative to the rate with which information can be transmitted through the circuit. Since information propagates in an electrical circuit at a rate comparable to the speed of light and circuit dimensions are relatively small, this modeling approach is often appropriate.

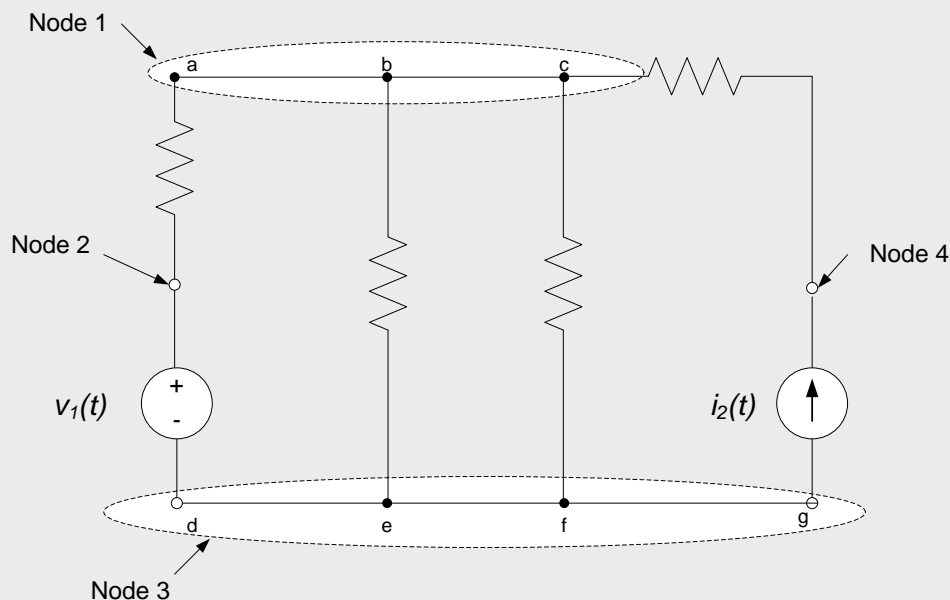
An alternate approach to circuit analysis is a *distributed-parameters* approach. This approach is considerably more mathematically complicated than the lumped parameters approach, but is necessary when dimensions become very large (as in cross-country power transmission) or when signals are varying extremely rapidly (such as the rate of bit transmission in modern computers).

1.4.1 Nodes

Identification of circuit nodes will be extremely important to the application of Kirchhoff's Laws. A *node* is a point of connection of two or more circuit elements. A node has a single, unique voltage. Since there is no voltage drop across a perfect conductor, any points in a circuit which are connected by perfect conductors will be at the same voltage and will thus be part of the same node.

Example 1.7

The circuit below has four nodes, as shown. A common error for beginning students is to identify points a, b, and c as being separate nodes, since they appear as separate points on the circuit diagram. However, these points are connected by perfect connectors (no circuit elements are between points a, b, and c) and thus the points are at the same voltage and are considered electrically to be at the same point. Likewise, points d, e, f, and g are at the same voltage potential and are considered to be the same node. Node 2 interconnects two circuit elements (a resistor and a source) and must be considered as a separate node. Likewise, node 4 interconnects two circuit elements and qualifies as a node.

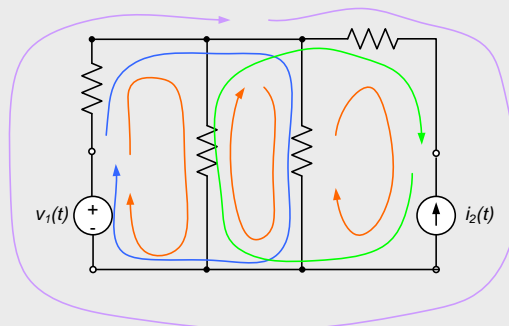


1.4.2 Loops

A *loop* is any closed path through the circuit which encounters no node more than once.

Example 1.8

There are six possible ways to loop through the circuit of the previous example. These loops are shown below.



1.4.3 Kirchhoff's Current Law

Kirchhoff's Current Law is one of the two principle approaches we will use for generating the governing equations for an electrical circuit. Kirchhoff's Current Law is based upon our assumption that charges cannot accumulate at a node.

Kirchhoff's Current Law (commonly abbreviated in these chapters as KCL) states:

The algebraic sum of all currents entering (or leaving) a node is zero.

A common alternate statement for KCL is:

The sum of the currents entering any node equals the sum of the currents leaving the node.

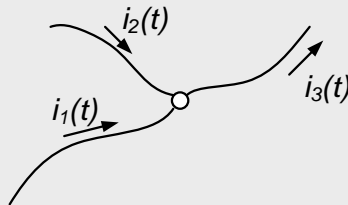
A general mathematical statement for Kirchhoff's Current Law is:

$$\sum_{k=1}^N i_k(t) = 0 \quad \text{Eq. 1.10}$$

Note: Current directions (entering or leaving the node) are based on assumed directions of currents in the circuit. As long as the assumed directions of the currents are consistent from node to node, the final result of the analysis will reflect the actual current directions in the circuit.

Example 1.9:

In the figure below, the assumed directions of $i_1(t)$, $i_2(t)$ and $i_3(t)$ are as shown.



If we (arbitrarily) choose a sign convention such that currents entering the node are positive then currents leaving the node are negative and KCL applied at this node results in:

$$i_1(t) + i_2(t) - i_3(t) = 0$$

If, on the other hand, we choose a sign convention that currents entering the node are negative, then currents leaving the node are positive and KCL applied at this node results in:

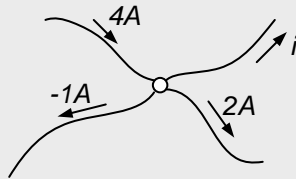
$$-i_1(t) - i_2(t) + i_3(t) = 0$$

These two equations are the same; the second equation is simply the negative of the first equation. Both of the above equations are equivalent to the statement:

$$i_1(t) + i_2(t) = i_3(t)$$

Example 1.10:

Use KCL to determine the value of the current i in the figure below.

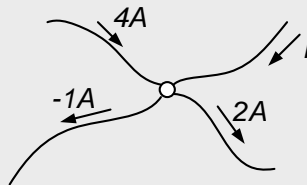


Summing the currents entering the node results in:

$$4A - (-1A) - 2A - i = 0 \Rightarrow i = 4A + 1A - 2A = 3A$$

and $i = 3A$, leaving the node.

In the figure below, we have reversed our assumed direction of i in the above circuit:



Now, if we sum currents entering the node:

$$4A - (-1A) - 2A + i = 0 \Rightarrow i = -4A - 1A + 2A = -3A$$

So now $i = -3A$, entering the node. The negative sign corresponds to a change in direction, so we can interpret this result to a $+3A$ current leaving the node, which is consistent with our previous result. Thus, the assumed current direction has not affected our results.

We can generalize Kirchhoff's Current Law to include any enclosed portion of a circuit. To illustrate this concept, consider the portion of a larger circuit enclosed by a surface as shown in Fig. 1.13 below. Since none of the circuit elements within the surface store charge, the total charge which can be stored within any enclosed surface is zero. Thus, the net charge entering an enclosed surface must be zero. This leads to a generalization of our previous statement of KCL:

The algebraic sum of all currents entering (or leaving) any enclosed surface is zero.

Applying this statement to the circuit of Fig. 1 results in:

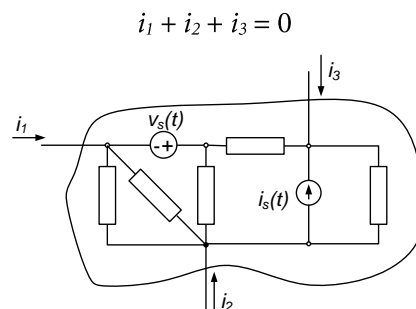


Figure 1.13 KCL applied to closed surface.

1.4.4 Kirchhoff's Voltage Law

Kirchhoff's Voltage Law is the second of two principle approaches we will use for generating the governing equations for an electrical circuit. Kirchhoff's Voltage Law is based upon the observation that the voltage at a node is unique.

Kirchhoff's Voltage Law (commonly abbreviated in these chapters as KVL) states:

The algebraic sum of all voltage differences around any closed loop is zero.

An alternate statement of this law is:

The sum of the voltage rises around a closed loop must equal the sum of the voltage drops around the loop.

A general mathematical statement for Kirchhoff's Voltage Law is:

$$\sum_{k=1}^N v_k(t) = 0 \quad \text{Eq. 1.11}$$

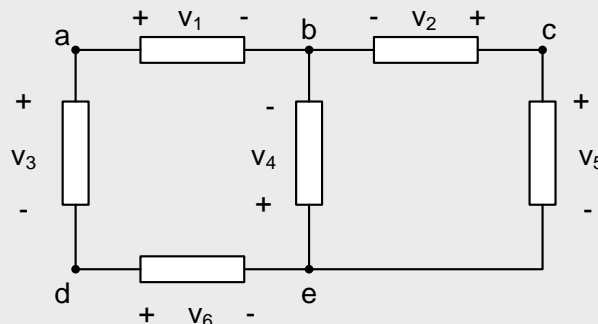
Where V_k is the k^{th} voltage difference in the loop and N is the total number of voltage differences in the loop.

Note: Voltage polarities are based on assumed polarities of the voltage differences in the loop. As long as the assumed directions of the voltages are consistent from loop to loop, the final result of the analysis will reflect the actual voltage polarities in the circuit.

Example 1.11:

In the figure below, the assumed (or previously known) polarities of the voltages v_1 , v_2 , v_3 , v_4 , v_5 , and v_6 are as shown. There are three possible loops in the circuit: a-b-e-d-a, a-b-c-e-d-a, and b-c-e-b. We will apply KVL to each of these loops.

Our sign convention for applying signs to the voltage polarities in our KVL equations will be as follows: when traversing the loop, if the positive terminal of a voltage difference is encountered before the negative terminal, the voltage difference will be interpreted as positive in the KVL equation. If the negative terminal is encountered first, the voltage difference will be interpreted as negative in the KVL equation. We use this sign convention for convenience; it is not required for proper application of KVL, as long as the signs on the voltage differences are treated consistently.



Applying KVL to the loop a-b-e-d-a, and using our sign convention as above results in:

$$v_1 - v_4 - v_6 - v_3 = 0$$

The starting point of the loop and the direction that we loop in is arbitrary; we could equivalently write the same loop equation as loop d-e-b-a-d, in which case our equation would become:

$$v_6 + v_4 - v_1 + v_3 = 0$$

This equation is identical to the previous equation, the only difference is that the signs of all variables has changed and the variables appear in a different order in the equation.

We now apply KVL to the loop b-c-e-b, which results in:

$$-v_2 + v_5 + v_4 = 0$$

Finally, application of KVL to the loop a-b-c-e-d-a provides:

$$v_1 - v_2 + v_5 - v_6 - v_3 = 0$$

Application Examples: Solving for Circuit Element Variables

Typically, when analyzing a circuit, we will need to determine voltages and/or currents in one or more elements in the circuit. In this chapter, we discuss use of the tools presented in previous chapters for circuit analysis.

The complete solution of a circuit consists of determining the voltages and currents for every elements in the circuit. A complete solution of a circuit can be obtained by:

4. Writing a voltage-current relationship for each element in the circuit (e.g. write Ohm's Law for the resistors).
5. Applying KCL at all but one of the nodes in the circuit.
6. Applying KVL for all but one of the loops in the circuit.

This approach will typically result in a set of N equations in N unknowns, the unknowns consisting of the voltages and currents for each element in the circuit. Methods exist for defining a reduced set of equations or a complete analysis of a circuit; these approaches will be presented in later chapters.

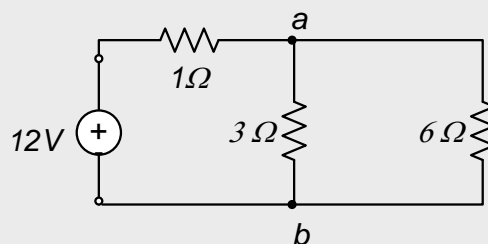
If KCL is written for every node in the circuit and KVL written for every loop in the circuit, the resulting set of equations will typically be over-determined and the resulting equations will, in general, not be independent. That is, there will be more than N equations in N unknowns and some of the equations will carry redundant information.

Generally, we do not need to determine all the variables in a circuit. This often means that we can write fewer equations than those listed above. The equations to be written will, in these cases, be problem dependent and are often at the discretion of the person doing the analysis.

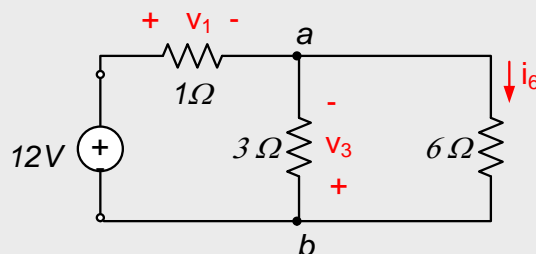
Examples of using Ohm's Law, KVL, and KCL for circuit analysis are provided below.

Example 1.12:

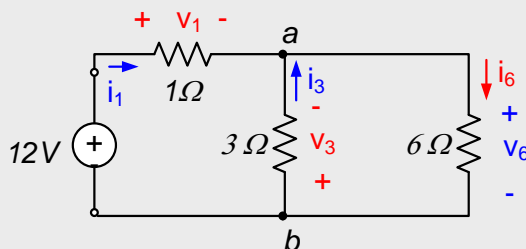
For the circuit below, determine v_{ab} .



We are free to arbitrarily choose either the voltage polarity or the current direction in each element. Our choices are shown below:



Once the above voltage polarities and current directions are chosen, we must choose all other parameters in a way that satisfies the passive sign convention. (Current must enter the positive voltage polarity node.) Our complete definition of all circuit parameters is shown below:



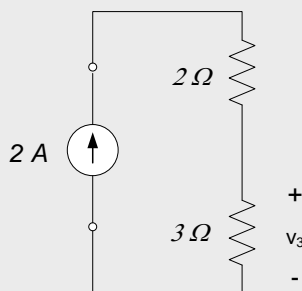
We now apply the steps outlined above for an exhaustive circuit analysis.

- Ohm's law, applied for each resistor, results in:
 $v_1 = (1\Omega)i_1$; $v_3 = (3\Omega)i_3$; $v_6 = (6\Omega)i_6$
- KCL, applied at node a:
 $i_1 + i_3 - i_6 = 0$
- KVL, applied over any two of the three loops in the circuit:
 $-12V + v_1 - v_3 = 0$
 $v_3 + v_6 = 0$

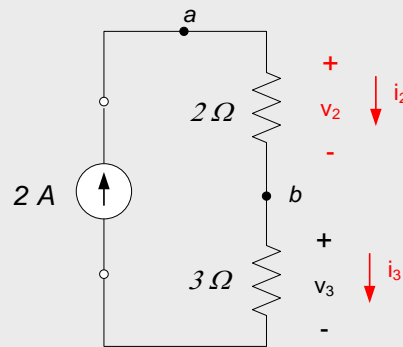
The above provide six equations in six unknowns. Solving these for v_3 results in $v_3 = -8V$. Since $v_3 = -v_{ab}$, $v_{ab} = 8V$

Example 1.13:

Determine v_3 in the circuit shown below.



We choose voltages and currents as shown below. Since v_3 is defined in the problem statement, we define it to be consistent with the problem statement.



KVL around the single loop in the circuit does not help us – the voltage across the current source is unknown, so inclusion of this parameter in a KVL equation simply introduces an additional unknown to go with the equation we write. KVL would, however, be useful if we wished to determine the voltage across the current source.

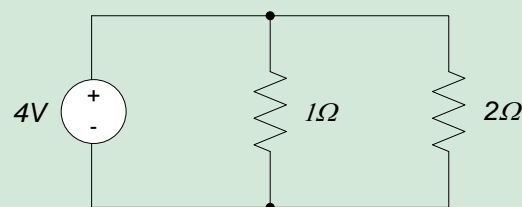
KCL at node a tells us that $i_2 = 2\text{A}$. Likewise, KCL at node b tells us that $i_2 - i_3 = 0$, so $i_3 = i_2 = 2\text{A}$. Ohm's law tells us that $v_3 = (3\Omega)(i_3) = (3\Omega)(2\text{A}) = 6\text{V}$.

Section Summary

- Kirchhoff's Current Law (KCL) and Kirchhoff's Voltage Law (KVL) govern the interactions between circuit elements. Governing equations for a circuit are created by applying KVL and KCL and applying the circuit element governing equations, such as Ohm's Law.
- Kirchhoff's current law states that the sum of the currents entering or leaving a node must be zero. A node in a circuit is a point which has a unique voltage.
 - A node is a point of interconnection between two or more circuit elements. A circuit node has a particular voltage. Nodes can be "spread out" with perfect conductors.
- Kirchhoff's voltage law states that the sum of the voltage differences around any closed loop in a circuit must sum to zero. A loop in a circuit is any path which ends at the same point at which it starts.
 - A loop is a closed path through a circuit. Loops end at the same node at which they start, and typically are chosen so that no node is encountered more than once.

1.4 Exercises:

1. For the circuit below, determine:
 - a) The current through the 2Ω resistor
 - b) The current through the 1Ω resistor
 - c) The power (absorbed or generated) by the 4V power source



Chapter Summary:

- In this text, we will be primarily concerned with the movement of electrical charge. Electrical charge motion is represented by voltage and current. Voltage indicates the energy change associated with the movement of a charge from one location to another, while current is indicative of the rate of current motion past a particular point.
 - Voltage is an energy difference between two physically separated points. The polarity of a voltage is used to indicate which point is to be assumed to be at the higher energy level. The positive terminal (+) is assumed to be at a higher voltage than the negative terminal (-). A negative voltage value simply indicates that the actual voltage polarity is opposite to the assumed polarity.
 - The sign of the current indicates the assumed direction of charge motion past a point. A change in the sign of the current value indicates that the current direction is opposite to the assumed direction.
- Circuit elements can be either active or passive. Active elements provide electrical energy from a circuit, from sources outside the circuit; active elements can be considered to create energy (from the standpoint of the circuit, anyway). Passive elements can store or dissipate electrical energy provided to them by the circuit; they can subsequently return energy to the circuit which they have previously stored, but they cannot create energy.
 - Active circuit elements introduced in this section are independent voltage and current sources, and dependent voltage and current sources.
 - Resistors are the only passive circuit elements introduced in this chapter. Resistors cannot store electrical energy, they can only dissipate energy by converting it to heat.
- The assumed polarity of the voltage across a passive circuit element must be consistent with the assumed current direction through the element. The assumed positive direction for current must be such that positive current enters the positive voltage terminal of the element. Since this sign convention is applied only to passive elements, it is known as the *passive sign convention*.
 - The assumed current direction or the assumed voltage polarity can be chosen arbitrarily, but once one parameter is chosen, the other must be chosen to agree with the passive sign convention.
- The power absorbed or generated by an electrical circuit component is the product of the voltage difference across the element and the current through the element: $p = iv$. The relative sign of the voltage and current are set according to the passive sign convention. Positive power implies that the voltage and current are consistent with the passive sign convention (the element absorbs or dissipates energy) while negative power indicates that the relative signs between voltage and current are opposite to the passive sign convention (the element generates or supplies energy to the circuit).
- The relationship between voltage and current for a resistor is Ohm's Law: $v = iR$. Since a resistor only dissipates energy, the voltage and current for a resistor must always agree with the passive sign convention.
- Kirchoff's voltage law states that the sum of the voltage differences around any close loop in a circuit must sum to zero. A loop in a circuit is any path which ends at the same point at which it starts.
- Kirchoff's current law states that the sum of the currents entering or leaving a node must be zero. A node in a circuit is a point which has a unique voltage.

Real Analog Chapter 1: Lab Projects

1.1: Solderless Breadboards, Open-circuits, and Short-circuits

The purpose of this lab assignment is to provide users with an introduction to some of the equipment which will be used in subsequent lab assignments. The following topics are discussed:

- breadboards
- use of digital multimeters (DMMs) to measure resistance.

The concept of electrical *resistance* is introduced qualitatively in this lab assignment. A more rigorous discussion of resistance will be presented in later modules and explored more thoroughly in later lab assignments.

Before beginning this lab, you should be able to:

- Define the connectivity between holes in a solderless breadboard (see related background material relative to breadboards)
- State how to use a DMM to measure resistance (see related material relative to DMMs)
- State rules governing proper usage of a lab notebook (see related materials relative to lab notebooks)

After completing this lab, you should be able to:

- Use a breadboard to connect electrical components
- Create open circuits and short circuits on a breadboard
- Identify nodes on a breadboard circuit
- Use a digital multimeter to measure resistance

This lab exercise requires:

- Hand-held DMM
- Solderless breadboard

Symbol Key

- DEMO** Demonstrate circuit operation to teaching assistant; teaching assistant should initial lab notebook and grade sheet, indicating that circuit operation is acceptable.
- ANALYSIS** Analysis; include principle results of analysis in laboratory report.
- SIM** Numerical simulation (using PSPICE or MATLAB as indicated); include results of MATLAB numerical analysis and/or simulation in laboratory report.
- DATA** Record data in your lab notebook.

General Discussion:

In this portion of the lab assignment, we will examine the connections between different holes in your breadboard using a DMM as an ohmmeter. Specifically, we will see which connections correspond to open circuits and short circuits. We will use jumper wires to change the connectivity between holes on the breadboard.

Pre-lab: None

Lab Procedures:

In this portion of the lab assignment, we will examine the connections between different holes in your breadboard using a DMM as an ohmmeter. Specifically, we will see which connections correspond to open circuits and short circuits. We will use jumper wires to change the connectivity between holes on the breadboard.

DATA

- Using your DMM as an ohmmeter, connect the leads of the DMM to two holes in the same row on your breadboard, as shown in Figure 1(a) below. The DMM should indicate a very low resistance between the holes – generally fractions of an ohm. Record your results in your lab notebook.

DATA

- Check the resistance between two rows of holes on opposite sides of the central channel of the breadboard, as shown in Figure 1(b) below. Your DMM should indicate a very high – or infinite – resistance between the holes. Record your results in your lab notebook.

Note:

Different ohmmeters have different ways of indicating what they consider to be an infinite resistance. For example, some models may read out an overload (sometimes displayed as OL or a “flashing” number) condition.

DATA

- Check the resistance between two arbitrary holes (not in the same row) of your breadboard (e.g. between nodes a and c in Figure 1(b)). Your DMM should again indicate a very high or infinite resistance. Record your results in your lab notebook.

DATA

- Finally, use a jumper wire to connect two different rows on the breadboard. Add a jumper wire between nodes a and c as shown in Figure 1(c). The DMM should indicate that the resistance between these nodes is now very low. Record your results in your lab notebook.

DATA

- In your lab notebook, comment on whether the connections in lab procedures 1 – 4 correspond to “open circuit” or “short circuit” conditions.

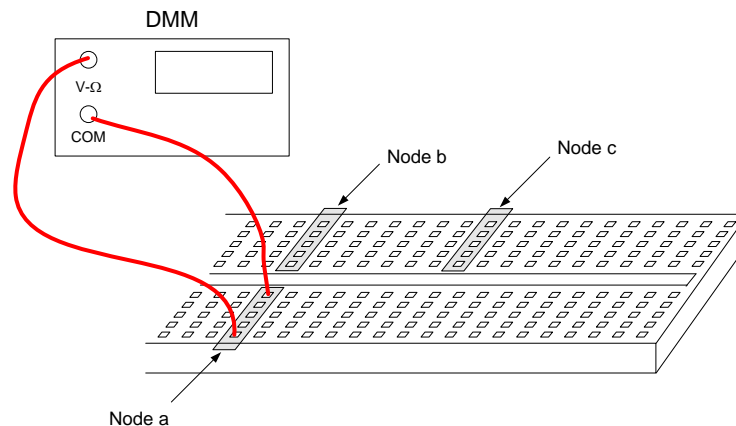


Figure 1(a).

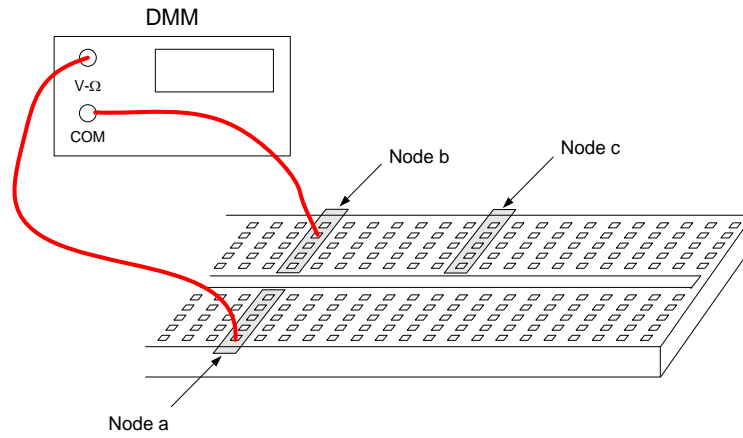


Figure 1(b).

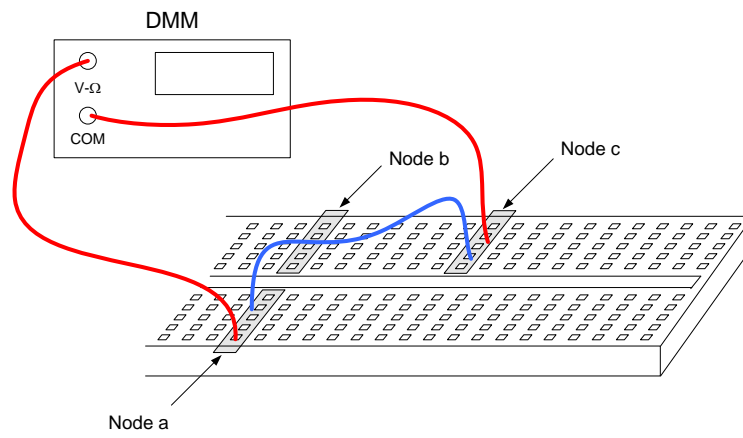


Figure 1(c).

Real Analog Chapter 1: Lab Worksheets

1.1: Breadboards and Ohmmeters (20 points total)

1. Provide the resistance you measured between two holes in same row. State whether these operate as open or closed circuit. (5 pts)
2. Provide the resistance you measured between two holes opposite sides of channel. Characterize as open or closed circuit. (5 pts)
3. Provide the resistance you measured between two arbitrary holes in the breadboard. Characterize as open or closed circuit. (5 pts)
4. Provide the resistance you measured between holes connected by a jumper wire. Characterize as open or closed circuit. (5 pts)

Real Analog Chapter 1: Lab Projects

1.2.1: Independent Power Supplies, Ammeters, and Voltmeters

The purpose of this lab assignment is to provide users with an introduction to some of the equipment which will be used in the next few lab assignments. The following topics are discussed:

- Power supplies
- Use a digital multimeter (DMM) to measure voltage and current

Before beginning this lab, you should be able to:

- State how to use a DMM to measure voltage and current (see related material relative to DMMs)
- Write symbols for independent voltage and current sources
- State the purpose of independent voltage and current sources

After completing this lab, you should be able to:

- Use a digital multimeter to measure voltage and current
- Use the Analog Discovery voltmeter to measure a constant voltage
- Use the Analog Discovery power supplies to provide a constant voltage to a circuit

This lab exercise requires:

- Analog Discovery
- Digilent Analog Parts Kit
- Digital multimeter

Symbol Key:

DEMO

Demonstrate circuit operation to teaching assistant; teaching assistant should initial lab notebook and grade sheet, indicating that circuit operation is acceptable.

ANALYSIS

Analysis; include principle results of analysis in laboratory report.

SIM

Numerical simulation (using PSPICE or MATLAB as indicated); include results of MATLAB numerical analysis and/or simulation in laboratory report.

DATA

Record data in your lab notebook.

General Discussion:

Consider the circuit shown schematically in Fig. 1(a). A DC voltage supply of 5V is connected between nodes a and c to provide power to the circuit. We want to know the current, i , out of the power source, and the voltage across the open circuit, v_{bc} . Since there is an open circuit between nodes b and c, we will expect no current to flow through the circuit ($i = 0$ A) and the voltage difference between nodes b and c should be the same as the voltage applied to the circuit ($v_{bc} = 5$ V).

The schematic of Fig. 1(b) shows the same circuit, with the measurements which will be made explicitly indicated. A DMM is connected between nodes a and b to measure the current i . We will use one oscilloscope channel, connected across nodes b and c to measure the voltage v_{bc} . (Since the voltage v_{bc} will be a constant value, it is not absolutely necessary to measure this value using an oscilloscope. It would be more common to use a DMM to measure the voltage v_{bc} , if one were available.) You may need to use a combination of jumper wires and

connectors with alligator clips to make the connections between the breadboard and the DMMs and power supplies.

Notes:

- A DMM, used as an ammeter, behaves approximately as a short circuit.
- A DMM or oscilloscope, used to measure voltage, behaves approximately as an open circuit.

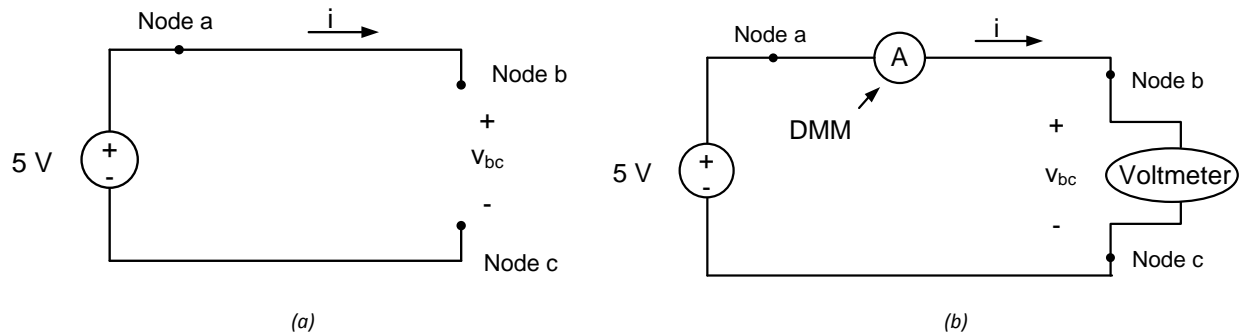


Figure 1. Circuit schematic.

Note:

Constant voltages are commonly measured using DMMs. However, it is easy to use the Analog Discovery Voltmeter instrument to perform this task. The Analog Discovery Voltmeter allows measurement of up to two voltages simultaneously on two different channels. Probes used by the Voltmeter are labeled as **1+** and **1-** (positive and negative terminals for the channel 1 voltage) and **2+** and **2-** (positive and negative terminals for the channel 2 voltage) on the Analog Discovery. Usage of the Voltmeter is briefly discussed below:

- Connect the probes of the desired voltmeter channel to the appropriate nodes of your circuit.
- Click on the “More Instruments” icon on the WaveForms main window and select Voltmeter from the resulting drop-down menu. A Voltmeter window should open.
- Make sure that the Voltmeter is enabled – if the checkmark next to “Enable” is not selected, click on it now¹.
- The measured constant voltages for both channels are displayed in the “DC” row on the Voltmeter window². If the probes for a channel are not connected to a circuit, the Voltmeter reading should be approximately zero for that channel.

The above process can be applied rather automatically, as long as the voltage being measured is constant. We will examine the measurement of voltages, which vary with time in later projects.

Pre-lab: None

¹ The Voltmeter instrument and Scope instruments share the same set of probes. If the Voltmeter instrument is enabled, the Scope instrument is disabled. Disabling the Voltmeter instrument enables the Scope instrument.

² The rows labeled **True RMS** and **AC RMS** are used for measuring voltages which change as a function of time. We’ll discuss these measurements in later lab assignments.

Lab Procedures:**DATA**

1. Connect the circuit shown in Fig. 1(b) except do not connect the power supply to the circuit. Use your DMM to perform the current measurement and channel 1 of your voltmeter to measure the voltage v_{bc} . Note in your lab notebook the voltage and current with no power supplied to the circuit.

DATA

2. Use VP+ on the Analog Discovery to apply the 5V source to the circuit. Turn the power supply on.

3. Measure the voltage v_{bc} and the current i . Note your results in your lab notebook. Do your results agree with your expectations? What is the approximate power generated or absorbed by the power source?

DATA

4. Turn off the power supply, reverse the leads on the voltmeter, turn the power supply back on, and note the resulting voltage and current measurements in your lab notebook. Do the results agree with your expectations? Why or why not? Comment on the agreement (or lack thereof) between your voltage and current measurements in your lab notebook.

DEMO

5. Demonstrate operation of your circuit to the Teaching Assistant. Have the TA initial the appropriate page(s) of your lab notebook and the lab checklist.

5. What are the measured voltages v_{ac} and v_{bc} and the measured current i with the voltmeter leads reversed? Comment on your results vs. expectations, especially relative to the signs of the voltages and current. (5 pts)

6. **DEMO:** Have a teaching assistant initial this sheet, indicating that they have observed your circuits' operation. (4 pts)

TA Initials: _____

Real Analog Chapter 1: Lab Projects

1.2.2: Dependent Sources and MOSFETs

Overview:

In this lab assignment, a qualitative discussion of dependent sources is presented in the context of MOSFETs (Metal Oxide Semiconductor Field Effect Transistors). A simple voltage controlled current source is constructed and tested.

Before beginning this lab, you should be able to:

- Use a DMM to measure voltage and current (Lab 1.2.1)
- Use the Analog Discovery power supplies to apply a fixed 5V voltage (Lab 1.2.1)
- Use the Analog Discovery voltmeter to measure a constant voltage (Lab 1.2.1)
- Write symbols for dependent voltage and current sources
- State governing equations for the four types of dependent sources

After completing this lab, you should be able to:

- Use MOSFETs as dependent sources
- Use the Analog Discovery waveform generators to apply constant voltages

This lab exercise requires:

- Analog Discovery
- Diligent Analog Parts Kit
- Digital multimeter

Symbol Key:

- DEMO** Demonstrate circuit operation to teaching assistant; teaching assistant should initial lab notebook and grade sheet, indicating that circuit operation is acceptable.
- ANALYSIS** Analysis; include principle results of analysis in laboratory report.
- SIM** Numerical simulation (using PSPICE or MATLAB as indicated); include results of MATLAB numerical analysis and/or simulation in laboratory report.
- DATA** Record data in your lab notebook.

General Discussion:

Many common circuit elements are *modeled* as dependent sources – that is, the mathematics describing the operation of the element is conveniently described by the equations governing a dependent source. In this portion of the lab assignment, we will build and test a circuit which acts as a Voltage Controlled Current Source (VCCS).

The primary circuit element used in this assignment is a Metal Oxide Semiconductor Field Effect Transistor (MOSFET). There are two basic types of MOSFETs: *n-channel* and *p-channel*; the discussion presented here is for n-channel MOSFETs, though similar concepts apply to p-channel MOSFETs. A MOSFET is a three-terminal device; the symbol commonly used to represent a MOSFET in circuit diagrams is shown in Fig. 1(a). The three terminals of the device are called the source (S), the drain (D) and the gate (G). Our circuit will employ a ZVN2210A MOSFET; the

physical appearance of this MOSFET is shown in Fig. 1(b), along with the relative locations of the drain, gate and source for that model MOSFET.

An extremely simplified discussion of a MOSFET's operation is as follows: A "channel" is opened in the MOSFET by application of a voltage at the gate of the MOSFET. This channel allows current to flow from the drain to the source of the MOSFET (i_D in Fig. 1(a)). Thus, if a power supply is connected to the drain of the MOSFET, the MOSFET can be used to control the power supply's current: increasing the gate voltage increases the current out of the power supply. A rough analogy to this process is a valve placed at the base of a water tank – opening the valve allows water to flow out of the tank. Likewise, increasing the gate voltage allows current to "flow" out of the power supply. A MOSFET, therefore, in conjunction with a power supply, can act as a voltage controlled current source in which the drain current is controlled by the gate voltage.

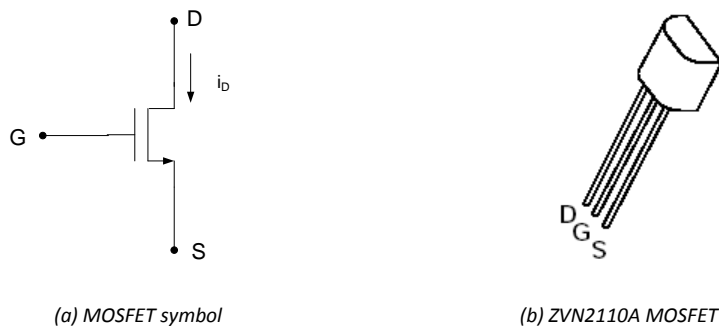


Figure 1. MOSFET symbol and typical physical appearance.

Pre-lab: None

Lab Procedures:

1. Connect the circuit shown in Fig. 2. Two power supplies are used in the circuit. Use channel 1 of your Arbitrary Waveform Generator (W1) to apply the (variable) gate voltage, V_G . Use of the waveform generator to apply constant voltages is presented in Appendix A of this assignment. Use the positive power supply (VP+) to provide a constant 5V to the MOSFET drain; this power supply provides the drain current I_D .

The 100Ω resistor in Fig. 2 is used to limit the amount of current flowing through the MOSFET. If no resistor is used between the power supply and the MOSFET, an excessive amount of current can flow through the MOSFET resulting in damage to the MOSFET and/or the rest of the circuit. The 100Ω resistors in your analog parts kit can be identified by the color bands on the side of the resistor – they will be as shown in Fig. 3. We will discuss resistors in detail in later modules. Use an ohmmeter to measure the resistance of the resistor and record this value in your lab notebook (the actual resistance will most likely be slightly different from 100Ω).

DATA

Connect your DMM as shown in Fig. 2 to measure the current I_D .

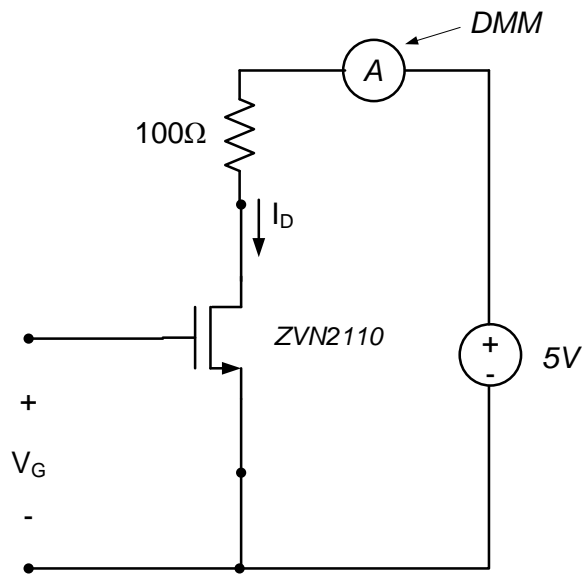


Figure 2. VCCS circuit schematic.

DATA

- MOSFETs have a *threshold voltage*, below which essentially no current passes through the MOSFET. To determine the threshold voltage for our MOSFET, begin with zero voltage applied at the gate by the variable voltage source V_G ($V_G = 0V$). The drain current, with no voltage applied at the gate, should be essentially zero. Gradually increase the MOSFET gate voltage while monitoring the MOSFET drain current I_D . Record in your lab notebook the voltage at which the drain current begins to increase significantly. This is the MOSFET's threshold voltage.

DATA

- Now characterize the MOSFET's relationship between gate voltage and drain current. Starting at the threshold voltage, continue to increase the gate voltage at increments of about 0.3V up to a maximum of about 5V. Record the gate voltages and their corresponding drain currents in your lab notebook. Plot the gate voltage vs. drain current data in your lab notebook. Comment on your observations relative to the data, especially relative to how the circuit behaves like a dependent source.

ANALYSIS

- The parameter g of a VCCS provides a relationship between the rate of change between the applied voltage and the resulting current. This is essentially the slope of the data you plotted in part 3 above. Use the curve of part 3 to estimate the value of g for the circuit you built. Note: Your curve will most likely not be a straight line. Do your best to fit a straight line to the data you acquired in part 3 for your estimate of g .

DEMO

- Demonstrate operation of your circuit to the Teaching Assistant. Have the TA initial the appropriate page(s) of your lab notebook and the lab checklist.

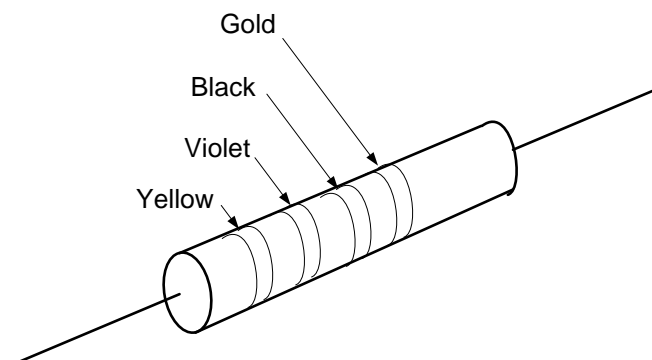
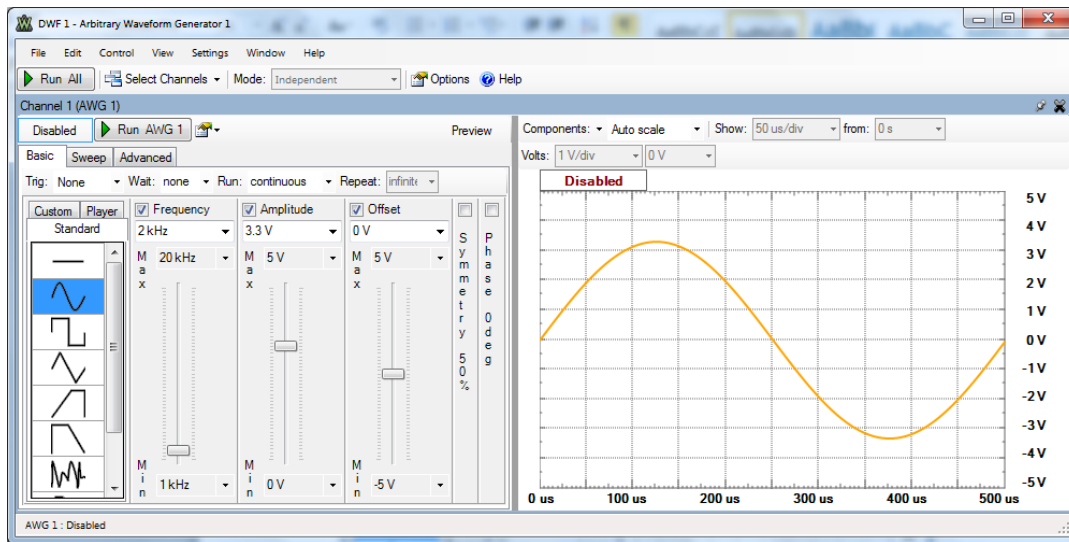



Figure 3. Color bands for 47Ω resistor.

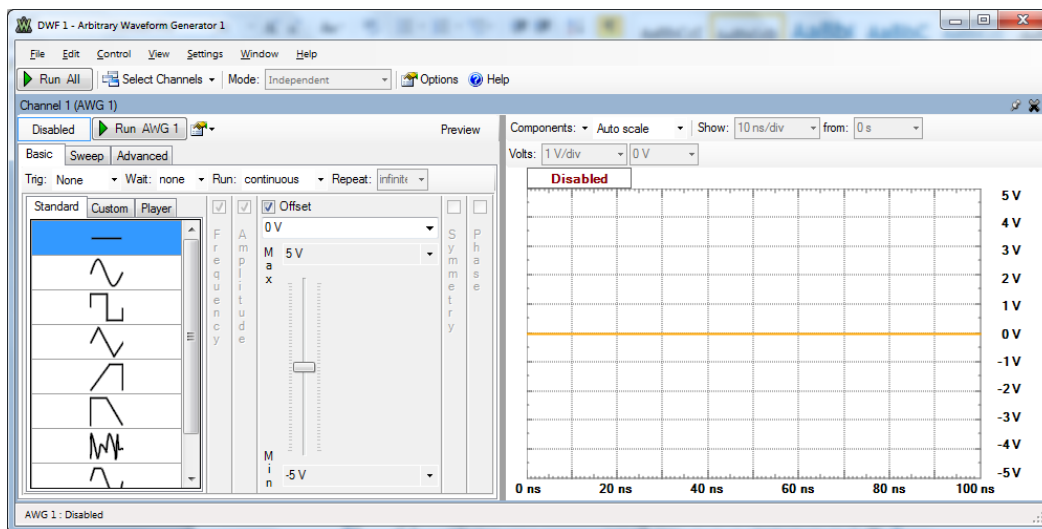
Appendix A: Waveform Generator to Apply Constant Voltages


The Voltage instrument on the Analog Discovery provides the capability to apply fixed $\pm 5\text{V}$ voltages to a circuit. Sometimes, however, it is desirable to apply arbitrary voltages to a circuit. The waveform generator on the Analog Discover provides this capability. To do this, follow the steps below:

1. Click on the **WaveGen** icon in the **WaveForms** main window to open the waveform generator window. This window should appear approximately as shown below:



2. There are a series of icons in a column in the lower-left corner of the screen. These icons allow you to select the “shape” of the waveform which will be applied to your circuit. (If the icons are not visible, click on the **Basic** tab (if it is not already selected) and then click on the **Standard** option. The icons indicating the shape of the available waveforms should appear.) In order to apply a constant voltage, click on the  icon. The waveform generator window should alter its appearance so that it looks as shown below. The button to the immediate left of the “Run AWG 1” button should appear as “**Disabled**”. If it displays **Enabled**, click on it to disable the waveform generator.



3. The value of the constant voltage to be applied to the circuit is specified in the **Offset** column (the value is limited to between $+5\text{V}$ and -5V). The desired value can be set with any of the following approaches:
 - using the slider bar,
 - clicking on the  button in the text box and selecting a value from the resulting drop-down menu, or

- simply typing the desired value in the text box and pressing the **Enter** button on your keyboard.
4. Click on “Run AWG 1” to apply power to channel 1 (W1) of the waveform generator. (Each waveform generator channel has its own interface; you can select the desired channel with the button.) Clicking on the “Run AWG 1” button automatically enables the function generator; the button to the immediate left of the “Run AWG 1” button should now appear as “Enabled”.

Important notes relative to the Enabled/Disabled button:

- The button to the left of the “Run AWG 1” button allows you to “enable” or “disable” the AWG. This button is used to turn off all power provided by the waveform generator channel. Thus, if you are working on your circuit with the waveform generator probes are connected, this button should always be set to “Disabled”.
 - Clicking on “Stop AWG1”, when the waveform generator is running, only removes the time-varying component of the signal, not any constant components. If we are applying only a constant voltage to the circuit (as in this lab assignment), the “Stop AWG 1” button has no effect on the voltages being applied to the circuit.
 - Clicking on “Run AWG 1” automatically enables the waveform generator, so that both time-varying and constant components are applied to the circuit.
 - Closing the waveform generator window also turns off all power applied by the waveform generator.
5. To turn off power to your circuit, click on the “Enabled” button so that it displays “Disabled”.

Real Analog Chapter 1: Lab Worksheets

1.2.2: Dependent Sources and MOSFETs (55 points total)

1. Diagram of circuit, including measured resistance value. (5 pts)
2. What MOSFET threshold voltage. (10 pts)
3. Attach to this worksheet a table providing your measured gate-to-source voltage vs. drain current values and a plot of data. (15 pts)
4. What type of dependent source is the transistor behaving like? Why? (5 pts)
5. Estimated value of g for circuit. Annotate the plot attached to this worksheet, indicating how the value of g was determined. (10 pts)
6. **DEMO:** Have a teaching assistant initial this sheet, indicating that they have observed your circuits' operation. (10 pts)

TA Initials: _____

Real Analog Chapter 1: Lab Projects

1.3.1: Resistors and Ohms Law – Resistance Variations

In this lab, we will experimentally explore the characteristics of resistors. Resistance is measured using an ohmmeter; this measured resistance is compared to a voltage-current characteristic curve measured for the resistor. Some basic statistical data analysis methods are introduced briefly in this lab assignment. MATLAB commands used to perform basic statistical analysis of data are provided in the background material related to this lab assignment.

Before beginning this lab, you should be able to:

- State Ohm's law from memory
- Use a digital multimeter to measure resistance and voltage (Labs 1.1, 1.2.1)
- Use color codes on resistors to determine the resistor's nominal resistance

After completing this lab, you should be able to:

- Calculate the mean, median, and standard deviation values of a set of data

This lab exercise requires:

- Digilent Analog Parts Kit
- Digital multimeter (DMM)

Symbol Key:

- DEMO** Demonstrate circuit operation to teaching assistant; teaching assistant should initial lab notebook and grade sheet, indicating that circuit operation is acceptable.
- ANALYSIS** Analysis; include principle results of analysis in laboratory report.
- SIM** Numerical simulation (using PSPICE or MATLAB as indicated); include results of MATLAB numerical analysis and/or simulation in laboratory report.
- DATA** Record data in your lab notebook.

General Discussion:

Fixed resistors are fabricated with a nominal resistance value. Individual resistors will, in general, not have exactly this resistance – their actual resistance will vary from this nominal value to some extent. In this portion of the lab assignment, we will measure the resistance of several resistors which all have the same nominal resistance, in order to determine the actual resistor-to-resistor resistance variations. These variations will be compared with the manufacturer's specified tolerances.

Important Note:

Since the resistance of individual resistors will vary from the value specified by the manufacturer, it is generally important, when incorporating a resistor in a circuit, to measure the resistor's actual resistance and record that value in your laboratory notebook when you are constructing a circuit. It is possible that the variations in resistance can affect your experimental results.

We will use *statistical analyses* to in this section to examine variations in resistors' resistances. Statistics are a way to quantify variations in results which are due to random effects such as variations in manufacturing, uncertainty in measured quantities, and measurement noise.

Related information:

Some basic information relative to common statistical quantities used to characterize data is provided in Appendix A of this experiment. Basic MATLAB commands which can be used to perform these statistical analyses are also provided in this Appendix

Pre-lab: None

Lab Procedures:

DATA

1. Pick five (5) 2.2KΩ resistors from your analog parts kit. Using an ohmmeter, measure the resistance of each of the resistors and record the values in your lab notebook. Calculate the mean, median, and standard deviation of the resistance values and record these quantities in your lab notebook. Convert these values to percentages of the nominal resistance, and record these results in your lab notebook. Determine the manufacturer's tolerance on the resistance values of the resistance and comment on whether your resistors lie within the specified tolerance.

DATA

2. Repeat procedure 1 above with five (5) 47KΩ resistors.

Appendix A: Basic Statistics

In this appendix, we briefly present some common statistical values used to characterize data which have some random component. We will also present MATLAB commands which can be used to determine these values.

Mean value:

If we have N sample values, $\{y_1, y_2, \dots, y_N\}$, we can determine the mean of the values by the following expression:

$$\bar{y} = \frac{1}{N} (y_1 + y_2 + y_3 + \dots + y_N) = \frac{1}{N} \sum_{i=1}^N y_i \quad \text{Eq. 1}$$

Thus, the mean can be determined by summing all the sample values and dividing by the total number values. It is essentially what we do when we determine an *average* of a series of values.

Median value:

The median value of a set of N sample values $\{y_1, y_2, \dots, y_N\}$ is the value which has the same number of samples above the value as there are below the value. This definition, however, does not necessarily uniquely determine the median value of a set of numbers. We will, therefore, refine our definition as follows:

- If N is an odd number, sort the data so that they are ordered from smallest to largest. The median value is then the middle data value.
- If N is an even number, again sort the data so that they are ordered from smallest to largest. There is no value exactly at the middle of the resulting sequence, so we choose the median value as the mean of the middle two numbers.

Standard deviation:

The standard deviation of a set of values $\{y_1, y_2, \dots, y_N\}$ can be thought of as the amount of "spread" that the numbers have from their mean value. Thus, the standard deviation provides an indication as to how closely grouped the values are around their mean. The definition of standard deviation that we will use is:

$$s = \frac{1}{N-1} \sqrt{\sum_{i=1}^N (y_i - \bar{y})^2} \quad \text{Eq. 2}$$

where s is the standard deviation and \bar{y} is the mean of the values, as defined above. The positive square root is used in the calculation, so that s is always represented as a positive number. Some textbooks use an alternate version of the standard deviation; we will exclusively use equation (2). When using a software package to perform statistical analyses, always check the definition of standard deviation that is used.

MATLAB syntax:

If the variable \mathbf{y} has been defined as a vector of numbers in the workspace, the following MATLAB syntax will calculate the mean, median, and standard deviation of the numbers.

- Mean: **mean (y)**
- Median: **median (y)**
- Standard Deviation: **std (y)**

Real Analog Chapter 1: Lab Worksheets

1.3.1: Resistance Variations (20 points total)

1. 2.2K Ω resistors:
 - a. Attach to this worksheet a table providing your measured resistances (3 pts)

 - b. Provide below the mean and median values, and the standard deviation of your resistance values. (2 pts)

 - c. Provide below the mean, median, and standard deviation as percentages of the nominal resistance. (2 pts)

 - d. Are the estimated variations within the manufacturer's tolerances? Explain. (3 pts)

2. 47K Ω resistors:
 - a. Attach to this worksheet a table providing your measured resistances (3 pts)

 - b. Provide below the mean and median values, and the standard deviation of your resistance values. (2 pts)

 - c. Provide below the mean, median, and standard deviation as percentages of the nominal resistance. (2 pts)

 - d. Are the estimated variations within the manufacturer's tolerances? Explain. (3 pts)

Real Analog Chapter 1: Lab Projects

1.3.2: Resistors and Ohms Law – Voltage-Current Characteristics

In this lab, we will continue to explore the characteristics of resistors. In this lab, we measure several combinations of voltage and current for a resistor and plot the resulting voltage-current characteristic curve measured for the resistor. The resistance of the resistor will be estimated from the slope of the voltage-current characteristic. The slope of the curve will be estimated using linear regression techniques; MATLAB commands used to perform linear regression are provided in the background material associated with this lab assignment.

Before beginning this lab, you should be able to:

- State Ohm's law from memory
- Use a digital multimeter to measure current and voltage (Lab 1.2.1)
- Use color codes on resistors to determine the resistor's nominal resistance
- Use the Analog Discovery's arbitrary waveform generator (AWG) to apply constant voltages to a circuit (Lab 1.2.2)





After completing this lab, you should be able to:

- Determine the least-squares best fit straight line approximating a set of data
- Calculate the correlation coefficient between a set of data and a line approximating the data
- Estimate resistance from measured voltage-current data

This lab exercise requires:

- Analog Discovery
- Digilent Analog Parts Kit
- Digital multimeter

Symbol Key:

- | | |
|---|---|
|  | Demonstrate circuit operation to teaching assistant; teaching assistant should initial lab notebook and grade sheet, indicating that circuit operation is acceptable. |
|  | Analysis; include principle results of analysis in laboratory report. |
|  | Numerical simulation (using PSPICE or MATLAB as indicated); include results of MATLAB numerical analysis and/or simulation in laboratory report. |
|  | Record data in your lab notebook. |

General Discussion:

We have previously noted that the resistance of a component is the slope of the current vs. voltage curve for the component. In this part of the lab assignment, we will measure a current-voltage characteristic curve for a resistor and estimate a resistance from this data. We will compare this resistance from the resistance measured by an ohmmeter.

In order to experimentally determine the current-voltage characteristic for our resistor, we will use the circuit shown schematically in Fig. 1. The arbitrary waveform generator will be used to apply the voltage v_s . We will measure the voltage across the resistor, v_R , and the current through the resistor, i_R , using our DMM. By varying v_s , we can measure a set of values for v_R and i_R and plot v_R vs i_R , as shown in Fig. 2(a). The slope of the line that "best fits" the measured data can then be used to estimate the component's resistance, as shown in Fig. 2(b).

Note:

Do not use the values displayed by the power supply as the resistor voltage and current, v_R and i_R . The values displayed by the power supply may differ from the resistor's voltage and current due to non-ideal power supply effects, such as the power supply internal resistance.

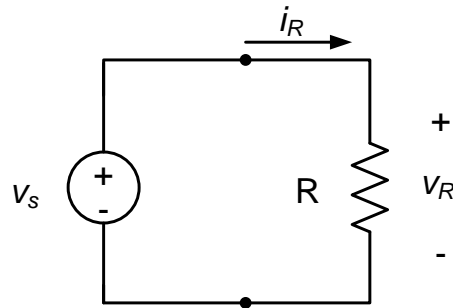


Figure 1. Circuit schematic.

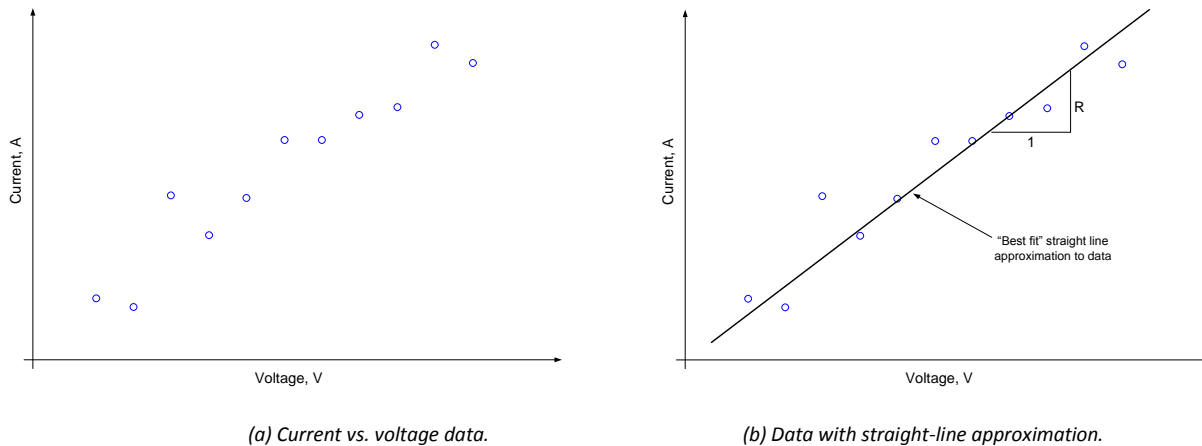


Figure 2. Measured data with “best fit” straight-line approximation.

Pre-lab: None**Lab Procedures:****DATA**

1. Connect the circuit shown in Fig. 1. Use a 100Ω resistor and one of the AWG channels for the variable supply. Use an ohmmeter to measure the actual resistance of your resistor and record the value in your lab notebook.

DATA

2. Vary the supply voltage v_s from 0V to approximately 2V. Measure v_R and i_R for at least 10 different values of v_s over this range of applied voltage (e.g. measure v_R and i_R at approximately 0.2V increments in v_s). Tabulate the measured values of v_R and i_R in your lab notebook. *Note: since we only have one DMM, the voltage and current measurements will have to be performed separately. If you have access to two DMMs, the two measurements can be made simultaneously.*

DEMO

3. Demonstrate operation of your circuit to the Teaching Assistant. Have the TA initial the appropriate page(s) of your lab notebook and the lab checklist.

Related information:

Resistance is estimated in the post-lab exercises using linear regression of these data. Linear regression is discussed in Appendix A of this lab.

Post-lab Exercises:

Determine a least-squares curve fit of the v_R vs. i_R . Plot the resulting line and the measured v_R vs. i_R data on the same graph. Comment on your results. Calculate a correlation coefficient for the data. Comment on your correlation coefficient relative to the qualitative agreement between the line and the data as shown on your plot.

Appendix A: Linear Regression:

Least-squares curve fitting:

Experimental data will always contain some uncertainty, so measured current-voltage data for a resistor will never lie exactly on a straight line – see, for example, Fig. 2. Thus, the notion of a “best” straight-line approximation to the data is rather nebulous. It is simplest to draw by eye a straight line through the plotted data. This approach, while used fairly often, has the drawback that no two engineers are likely to draw the same straight line. Thus, we look for a more objective and readily quantifiable approach toward fitting a line to a set of measured data. One common approach toward determining a line which provides a “best fit” to the available data is *least squares curve fitting*. The basic idea behind the least-squares approach toward fitting a curve to data is as follows:

- We have a set of x, y data, where the x data points are $\{x_1, x_2, \dots, x_N\}$ and the y data points are $\{y_1, y_2, \dots, y_N\}$
- Assume that a straight line will approximate the data. The equation for the straight line is

$$y = mx + b \quad \text{Eq.1}$$

where m and b – the slope of the line and its y -intercept – are unknowns to be determined.

- We define the error between the estimated line and the measured data to be the square of the distance between the line and the data at the x data points. Thus, our error is:

$$E = \sum_{i=1}^N [y_i - (mx_i + b)]^2 \quad \text{Eq. 2}$$

- If we minimize the error of equation (2) with respect to m and b , we obtain the least-squares straight line fit to the data. We will not discuss the mathematical details of this step here – they are rather tedious.
- We can determine how well our straight line agrees with the data by calculating a *correlation coefficient*. The correlation coefficient, r , is calculated by:

$$r = \frac{1}{N-1} \sum_{i=1}^N \left[\left(\frac{x_i - \bar{x}}{s_x} \right) \cdot \left(\frac{y_i - \bar{y}}{s_y} \right) \right] \quad \text{Eq. 3}$$

where \bar{x} and \bar{y} are the means of the x and y data, respectively, and s_x and s_y are the standard deviations of the x and y data. The correlation coefficient is a number between -1 and 1 ($-1 \leq r \leq 1$); it essentially tells us how well our data agrees with the straight line curve fit. If all the data lie exactly on a straight line with positive slope, the correlation coefficient will be identically one ($r = 1$). If the data have noise or

follow a nonlinear relationship, the correlation coefficient will be reduced. Data which are entirely uncorrelated have a correlation coefficient of zero ($r = 0$). A correlation coefficient of -1 simply means that there is a perfect negative relation between x and y – the data will lie on a straight line with negative slope. Fig. 3 provides several examples of data with various degrees of correlation.

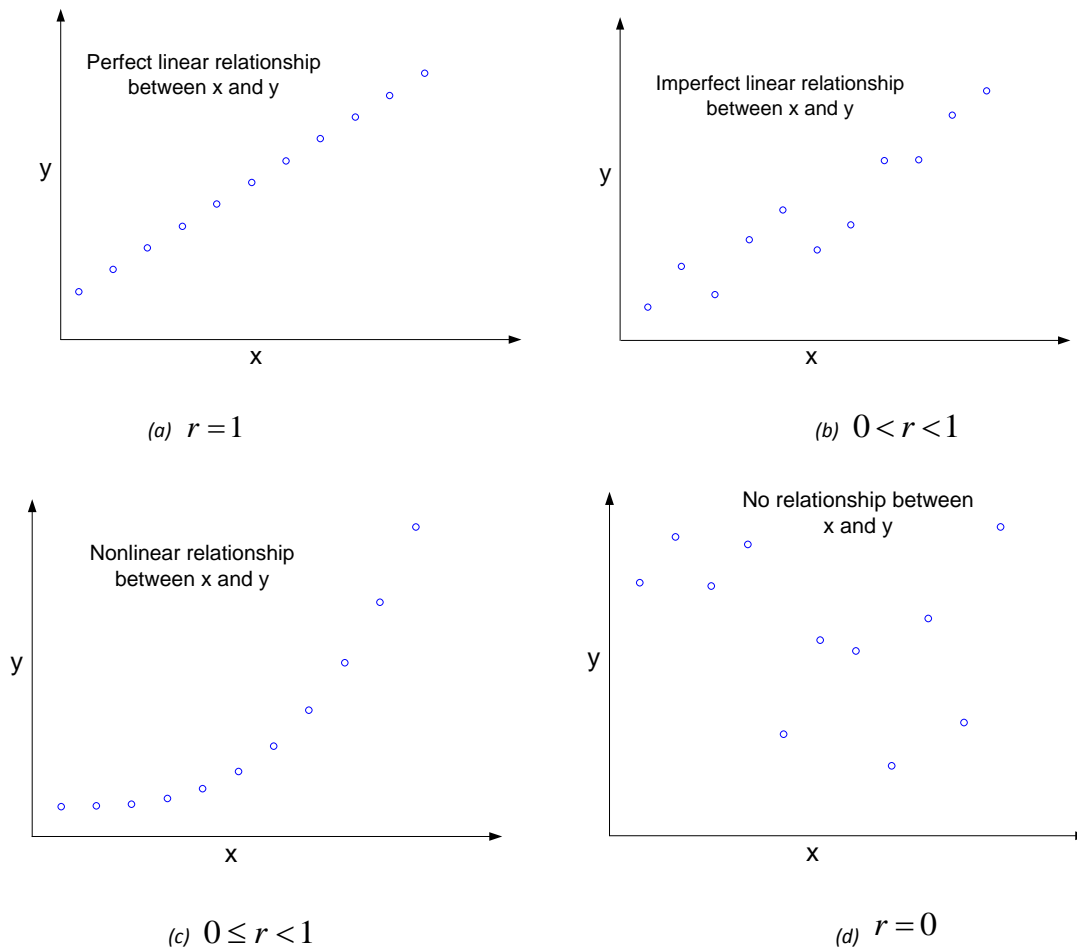


Figure 3. Example data and representative correlation coefficients.

Using MATLAB for least squares curve fitting:

MATLAB's `polyfit` function performs least-squares curve fitting. Polyfit will fit an arbitrary-order polynomial to a set of data. Syntax for the function is

$$\mathbf{p} = \text{polyfit}(\mathbf{x}, \mathbf{y}, \mathbf{n})$$

where \mathbf{x} and \mathbf{y} are vectors containing the data to be fit, \mathbf{n} is the order of polynomial to be fit to the data (a straight line is a first order polynomial, so we will always set $n = 1$). The function returns a vector containing the coefficients of the polynomial which provides a least-squares fit to the data. For $n = 1$ a two-element vector will be returned; the first element of the vector will be the slope of the line (m , in equation (1)) and the second element will be the y-intercept of the line (b , in equation (1)).

MATLAB's `corrcoef` function provides the correlation coefficient of two data sets. Possible syntax for using this function is:

$$\mathbf{r} = \text{corrcoef}(\mathbf{x}, \mathbf{y})$$

where \mathbf{x} and \mathbf{y} are vectors containing the data. This use of the function will return a 2×2 matrix; it will have the following form:

$$r = \begin{bmatrix} r_{xx} & r_{xy} \\ r_{yx} & r_{yy} \end{bmatrix} \quad \text{Eq. 4}$$

This matrix provides correlations between all possible combinations of the data provided to the function. r_{xx} is the correlation between the x data and itself. Likewise, r_{yy} is the correlation between the y data and itself. Since data is always perfectly correlated with itself, $r_{xx} = r_{yy} = 1$ always. r_{xy} is the correlation between the x data and the y data, and r_{yx} is the correlation between the y data and the x data. For us, $r_{xy} = r_{yx}$. Thus, either the r_{xy} or r_{yx} terms will give us the correlation coefficient as defined in equation (3).

Real Analog Chapter 1: Lab Worksheets

1.3.2: Resistance from Measured Current-voltage Data (25 points total)

1. Circuit diagram; measured resistance value of resistor. (5 pts)
2. Attach a table to this worksheet, providing your measured v_R and i_R over range of v_s from 0V – 2V (minimum 10 data points) (8 pts)
3. **DEMO:** Have a teaching assistant initial this sheet, indicating that they have observed your circuits' operation. (5 pts)

TA Initials: _____

4. Provide below an equation for least-squares best fit line to the data and the correlation coefficient between the data and the curve fit. Attach to this worksheet a plot of data vs. best fit line. (7 pts)

Real Analog Chapter 1: Lab Projects

1.4.1: Dusk-to-Dawn Light

In this lab, we will create our first circuit which appears to “do” something which is readily perceivable without instrumentation. The circuit operates as a “dusk-to-dawn” light; the circuit turns a light on when the ambient light level goes below a certain level.

Before beginning this lab, you should be able to:

- State Ohm’s law from memory
- Use Ohm’s law to perform voltage and current calculations for resistive circuit elements
- Use a digital multimeter to measure resistance, current, and voltage (Labs 1.1, 1.2.1)
- Use transistors as dependent sources (Lab 1.2.2)
- Use color codes on resistors to determine the resistor’s nominal resistance
- State KVL and KCL from memory





After completing this lab, you should be able to:

- Use a bipolar junction transistor (BJT) as a dependent source
- Use a photoresistor to measure ambient light levels
- Use an LED as a light source
- Integrate a number of components into an overall system

This lab exercise requires:

- Analog Discovery module
- Digilent Analog Parts Kit
- Digital multimeter (optional)

Symbol Key:

- | | |
|---|---|
|  | Demonstrate circuit operation to teaching assistant; teaching assistant should initial lab notebook and grade sheet, indicating that circuit operation is acceptable. |
|  | Analysis; include principle results of analysis in laboratory report. |
|  | Numerical simulation (using PSPICE or MATLAB as indicated); include results of MATLAB numerical analysis and/or simulation in laboratory report. |
|  | Record data in your lab notebook. |

General Discussion:

In this part of the lab assignment, we will create a light-sensitive lighting system. A photocell – a light-sensitive resistor – will be used to sense the ambient light level. A Bipolar Junction Transistor (BJT) will be used as a switch to turn on a light-emitting diode when the ambient light level becomes low. The circuit we will use is shown in Fig. 1.

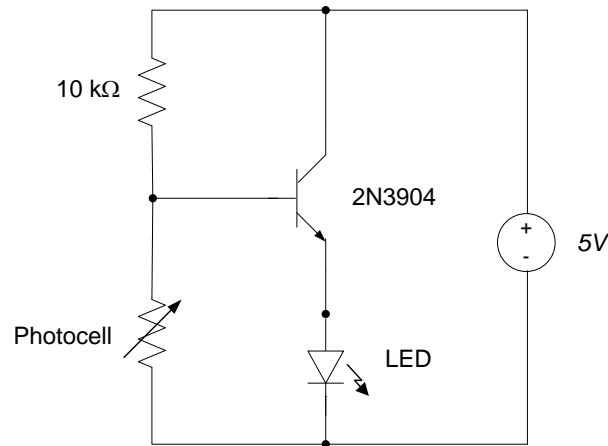


Figure 1. Dusk-to-dawn lighting circuit.

There are several unfamiliar components in the circuit of Fig. 1: a photocell, an LED, and a BJT. A detailed understanding of the operation of these components is beyond the scope of this lab, but a brief overview of their operating characteristics is provided below. Later courses in a typical electrical engineering curriculum will provide background information necessary to fully understand these components.

Diodes and LEDs:

Diodes are two-terminal semiconductor devices that conduct current in only one direction. The terminals of a diode are called the *anode* and the *cathode*; diodes are intended to conduct current from the anode to the cathode. Diodes have a minimum *threshold voltage* (or V_{th} , usually around 0.7V) that must be present between the anode and cathode in order for current to flow. If the anode voltage is not at least V_{th} greater than the cathode voltage, no current will flow. Likewise, if the cathode voltage is greater than the anode voltage, the diode is said to be *reverse-biased* and no current will flow. In an ideal diode, if the diode voltage equals the threshold voltage (plus a small amount), then unlimited current can flow without causing the voltage across the diode to increase. And, if the diode is reverse-biased, no current will flow regardless of reverse-voltage magnitude.

As with diodes, LED's are two-terminal semiconductor devices that conduct current in only one direction (from the anode to the cathode). The small LED chips are secured inside a plastic housing, and they emit light at a given frequency when a small electric current (typically 10mA to 25mA) flows through them. When the voltage difference across the LED exceeds the threshold voltage of the LED, current flows through the LED and light is emitted. If the LED voltage is less than the threshold voltage, no current flows and no light is emitted. LEDs are available in a number of colors; the Digilent analog parts kit contains red, yellow, and green LEDs.

Since LEDs are *polarized* devices, they must be placed in the circuit with the correct orientation; the anode must be at higher voltage potential than the cathode in order for the diode to emit light. An LED schematic symbol is shown in Fig. 2 below, together with a sketch of a physical LED. The anode and cathode on a physical LED can be identified because the anode pin is longer than the cathode pin and the cathode side of the plastic diffusion lens is typically slightly flattened.

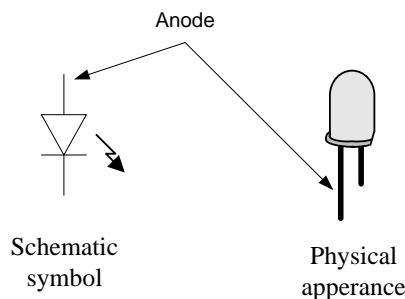


Figure 2. LED schematic symbol and physical appearance.

Bipolar Junction Transistors (BJTs):

In Lab 1.2.1, we used a MOSFET as a voltage controlled current source. Bipolar Junction Transistors, BJTs, are also conveniently modeled as dependent sources. Like MOSFETs, BJTs are three-terminal devices; the terminals of a BJT are called the base (B), the collector (C), and the emitter (E). The symbol commonly used to represent the type of BJT we will be using is shown in Fig. 3(a). Our circuit employs a 2N3904 BJT; the physical appearance of this BJT is shown in Fig. 3(b), along with the relative locations of the base, collector, and emitter for that BJT.



Figure 3. BJT symbol and physical appearance of 2N3904 BJT.

An extremely simplified discussion of a BJT's operation is as follows: application of voltage to the base of the BJT allows current to flow from the collector to the emitter of the BJT. Typically the current flowing into the base of the BJT is much smaller than the collector and emitter currents. Thus, the BJT can be conceptualized as a current controlled current source. Thus, if a power supply is connected to the collector of the BJT, the base voltage of the BJT can be used to control BJT's emitter current: increasing the base voltage typically increases the emitter current. In the case of two relatively discrete values of base voltage, the BJT can act as a switch; low base voltages turn off the switch (the emitter current is zero) while high base voltages turn the switch on (the emitter current is non-zero).

Photocell:

Photocells (sometimes called *photoresistors* or *photoconductors*) are devices whose resistance changes according to the light intensity applied to the sensor. The photocells in the Digilent analog parts kits have resistances which vary from about $5\text{K}\Omega$ at relatively high light levels to about $20\text{K}\Omega$ at relatively low light levels. A variable resistor is commonly indicated on a circuit schematic by a resistor symbol with an arrow through it, as shown in Fig. 4.

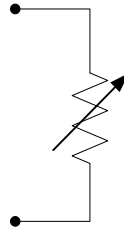


Figure 4. Variable resistor circuit symbol.

Pre-lab:

ANALYSIS

Apply KVL around the outer loop of the circuit of Fig. 1 (as shown in Fig. 5) to determine the voltage V_B for photocell resistances of $5\text{k}\Omega$ and $20\text{k}\Omega$. You may assume that the current into the base of the BJT is negligible.

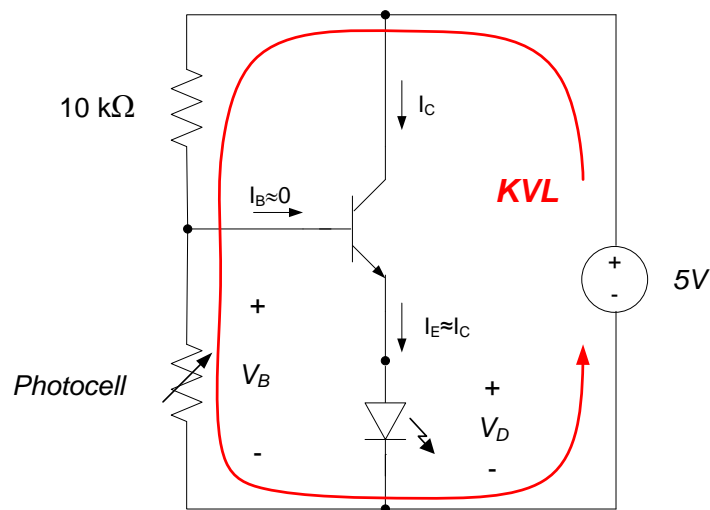


Figure 5. Circuit analysis to determine BJT base voltage.

Lab Procedures:

DATA

1. Construct the circuit shown in Fig. 1. The LED should not light under normal light levels. Using the scope instrument on your Analog Discovery (or a DMM), measure the base voltage of the BJT (V_B in Fig. 5) and the voltage difference across the diode (V_D in Fig. 5). Record these voltages in your lab notebook and compare them with your calculated values from the prelab.

DATA

2. Cover the photocell; the LED should light up. Using the scope instrument on your Analog Discovery (or a DMM), measure the base voltage of the BJT (V_B in Fig. 5) and the voltage difference across the diode (V_D in Fig. 5). Record these voltages in your lab notebook and compare them with your calculated values from the prelab.

DEMO

3. Demonstrate operation of your circuit to the Teaching Assistant. Have the TA initial the appropriate page(s) of your lab notebook and the lab checklist.

Real Analog Chapter 1: Lab Projects

1.4.2: Resistive Network Power Dissipation

In this lab assignment, we will use KVL and KCL to analyze a simple circuit. The circuit will be constructed and tested and the actual circuit response compared to the expected response from the analysis.

Before beginning this lab, you should be able to:

- State Ohm's law from memory
- Use a digital multimeter to measure resistance, voltage, and current (Lab 1.1, 1.2.1)
- Use the Analog Discovery's arbitrary waveform generator (AWG) to apply constant voltages to a circuit (Lab 1.2.2)
- Use the Analog Discovery voltmeter to measure a constant voltage (Lab 1.2.1)
- Use color codes on resistors to determine the resistor's nominal resistance
- Use KVL and KCL for circuit analysis





After completing this lab, you should be able to:

- Measure power dissipation of a resistor in a resistive network

This lab exercise requires:

- Analog Discovery
- Digilent Analog Parts Kit
- Digital multimeter (optional)

Symbol Key:

-  Demonstrate circuit operation to teaching assistant; teaching assistant should initial lab notebook and grade sheet, indicating that circuit operation is acceptable.
-  Analysis; include principle results of analysis in laboratory report.
-  Numerical simulation (using PSPICE or MATLAB as indicated); include results of MATLAB numerical analysis and/or simulation in laboratory report.
-  Record data in your lab notebook.

General Discussion:

This portion of the lab assignment concerns the circuit shown in Fig. 1 below. A power supply is used to apply the 5V voltage difference. We wish to determine the power dissipated by the 4.7K Ω resistor.

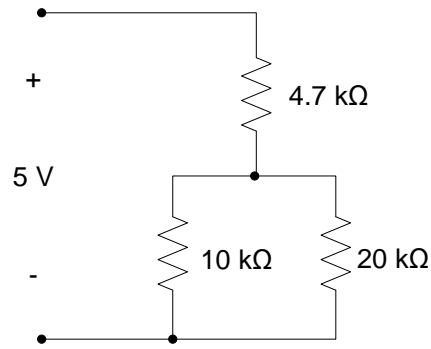


Figure 1. Circuit schematic.

Pre-lab:

ANALYSIS

Analyze the circuit of Fig. 1 to estimate the power dissipated by the 4.7KΩ resistor.

Lab Procedures:

DATA

1. Construct the circuit of Fig. 1. Measure and record the actual resistance values used in your circuit. Measure the parameters (voltages and/or currents) necessary to determine the power dissipated by the 4.7KΩ resistor. Determine the power dissipated by the 4.7KΩ resistor.

ANALYSIS

Compare the measured power with your estimate from the pre-lab. Comment on any differences between the estimated and measured values.

DEMO

2. *Demonstrate operation of your circuit to the Teaching Assistant. Have the TA initial the appropriate page(s) of your lab notebook and the lab checklist.*

Real Analog Chapter 1: Lab Projects

1.4.3: Input Resistance

An extremely important circuit characteristic – the circuit’s input resistance – will be determined using Kirchoff’s laws. The input resistance of a circuit can be a significant design parameter; if the circuit is later incorporated in part of a larger overall system, the circuit’s input resistance can have a significant effect on the behavior of the overall system.

Before beginning this lab, you should be able to:

- State Ohm’s law from memory
- Use a digital multimeter to measure resistance, voltage, and current (Lab 1.1, 1.2.1)
- Use the Analog Discovery’s arbitrary waveform generator (AWG) to apply constant voltages to a circuit (Lab 1.2.2)
- Use the Analog Discovery voltmeter to measure a constant voltage (Lab 1.2.1)
- Use color codes on resistors to determine the resistor’s nominal resistance
- Use KVL and KCL for circuit analysis

After completing this lab, you should be able to:

- Estimate the input resistance of a resistor network from measured voltage-current characteristics

This lab exercise requires:

- Analog Discovery
- Digilent Analog Parts Kit
- Digital multimeter

Symbol Key:

- DEMO** Demonstrate circuit operation to teaching assistant; teaching assistant should initial lab notebook and grade sheet, indicating that circuit operation is acceptable.
- ANALYSIS** Analysis; include principle results of analysis in laboratory report.
- SIM** Numerical simulation (using PSPICE or MATLAB as indicated); include results of MATLAB numerical analysis and/or simulation in laboratory report.
- DATA** Record data in your lab notebook.

General Discussion:

The *input resistance* of a circuit is the effective resistance that a source (or another stage of an overall system) will see when it is connected to the circuit. This value can be an extremely important design parameter for a circuit. In this part of the lab assignment, we will determine the input resistance of the circuit shown in Fig. 1.

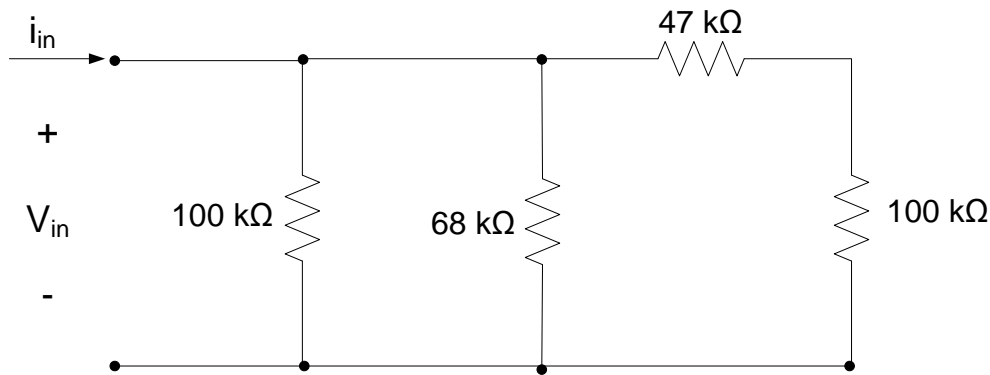


Figure 1. Circuit schematic.

Pre-lab:**ANALYSIS**

Determine the relationship between V_{in} and i_{in} for the circuit shown in Fig. 1. Defining the input resistance according to Ohm's law as $R_{in} = \frac{V_{in}}{i_{in}}$, determine the input resistance of the circuit of Fig. 1.

Lab Procedures:**DATA**

1. Construct the circuit of Fig. 1. Measure and record the actual resistance of all resistors used in your circuit. Use a variable voltage supply (the arbitrary waveform generator of the Analog Discovery will work) to apply V_{in} and i_{in} to the circuit. For at least five different values of V_{in} between 0V and 5V, measure V_{in} and i_{in} .

ANALYSIS

2. Plot the voltage-current characteristic for the five points determined in part 1. Estimate the input resistance of the circuit from the slope of the voltage-current characteristic. (Feel free to draw your own best-fit line to the data points, without performing a least-squares best fit.)

ANALYSIS

3. Calculate the input resistance for each combination of V_{in} and i_{in} measured in part 1. Determine the average input resistance over the five measurements and the standard deviation of the measurements. Compare this value of input resistance to the one determined in part 2.

DEMO

4. Demonstrate operation of your circuits to the Teaching Assistant. Have the TA initial the appropriate page(s) of your lab notebook and the lab checklist.

Real Analog Chapter 1: Lab Worksheets

1.4.3: Circuit Input Resistance (25 points total)

1. Estimated relationship between V_{in} and i_{in} from prelab. Estimated input resistance for circuit (4 pts)
2. Circuit schematic with measured resistance values implemented in circuit (2 pts)
3. Attach to this worksheet a plot of the measured voltage-current data. Your plot should include the five measured points and a straight line approximating the data. In the space below, provide your estimate of the input resistance of the circuit, based on these data. (8 pts)
4. Attach to this worksheet a table including (at least) four combinations of V_{in} and i_{in} applied to circuit, calculated input resistances for each combination, and the average input resistance over all measured values. In the space below, comment on the agreement between the input resistance determined in parts 3 and 4. (6 pts)
5. **DEMO:** Have a teaching assistant initial this sheet, indicating that they have observed your circuits' operation. (5 pts)

TA Initials: _____

Real Analog Chapter 1: Lab Projects

1.4.4: Temperature Measurement System

This lab assignment also includes our first design-related task: we will design a circuit whose output voltage provides a crude temperature measurement. A thermistor – a device whose resistance changes with temperature – is used to sense the temperature. We will create an electrical circuit which uses this resistance change to output a voltage which indicates the temperature of the thermistor.

Before beginning this lab, you should be able to:

- State Ohm’s law from memory
- Use a digital multimeter to measure resistance, voltage, and current (Lab 1.1, 1.2.1)
- Use the Analog Discovery’s arbitrary waveform generator (AWG) to apply constant voltages to a circuit (Lab 1.2.2)
- Use the Analog Discovery voltmeter to measure a constant voltage (Lab 1.2.1)
- Use color codes on resistors to determine the resistor’s nominal resistance
- Use KVL and KCL for circuit analysis

After completing this lab, you should be able to:

- Design a thermistor-based circuit to measure temperature

This lab exercise requires:

- Analog Discovery module
- Digilent Analog Parts Kit
- Digital multimeter (optional)

Symbol Key:

- DEMO** Demonstrate circuit operation to teaching assistant; teaching assistant should initial lab notebook and grade sheet, indicating that circuit operation is acceptable.
- ANALYSIS** Analysis; include principle results of analysis in laboratory report.
- SIM** Numerical simulation (using PSPICE or MATLAB as indicated); include results of MATLAB numerical analysis and/or simulation in laboratory report.
- DATA** Record data in your lab notebook.

General Discussion:

In this portion of the lab assignment, we will design and construct a temperature measurement system. The system will use a *thermistor* to detect temperature changes. A thermistor is a device whose electrical resistance changes as a function of the temperature of the thermistor. The thermistor we will use, for example, has a temperature-resistance curve approximately as shown in Fig. 1. It is worthwhile noting that the relationship between temperature and resistance is not linear. However, during the design process it is common to approximate the data as a straight line – at least over some range of temperatures.

Related Material:

Thermistors are classified as NTC (Negative Temperature Coefficient) or PTC (Positive Temperature Coefficient) depending on whether their resistance decreases or increases with temperature. Thermistor specifications also include their nominal resistance at some temperature. The thermocouple with the temperature-resistance curve shown in Figure 2 is specified as an “NTC 10K Ω @ 25°C” thermistor. Additional information about thermistors can be found in the backup information provided as a companion document on to this lab.

It can be seen from Fig. 1 that the temperature can be inferred from the resistance of the thermistor. However, resistance is not a common quantity to use to represent a physical parameter – it is much more usual to use voltage to represent the parameter. (Voltages are generally easier to use than resistance to represent information. For example, digital logic circuits generally operate based on voltages applied to them.) We will design a circuit which outputs a voltage, from which the temperature of the thermistor can be inferred.

Our design criteria are as follows: Design a circuit like that shown in Fig. 2, containing a thermistor as one of the resistances, which satisfies the following specifications:

1. +5V input voltage to the system
2. Output voltage varies by a minimum of 0.5V over a temperature range of 25°C to 37°C.
3. Output voltage must increase as temperature increases

The selected temperature range provided in the design requirements corresponds (approximately) to the change between room temperature and human body temperature. We can thus check our temperature measurement system by measuring the output voltage when the thermistor is at room temperature and then changing the temperature by firmly holding the thermistor between two fingers. The voltage should increase by at least 0.5V as a result of this temperature change.

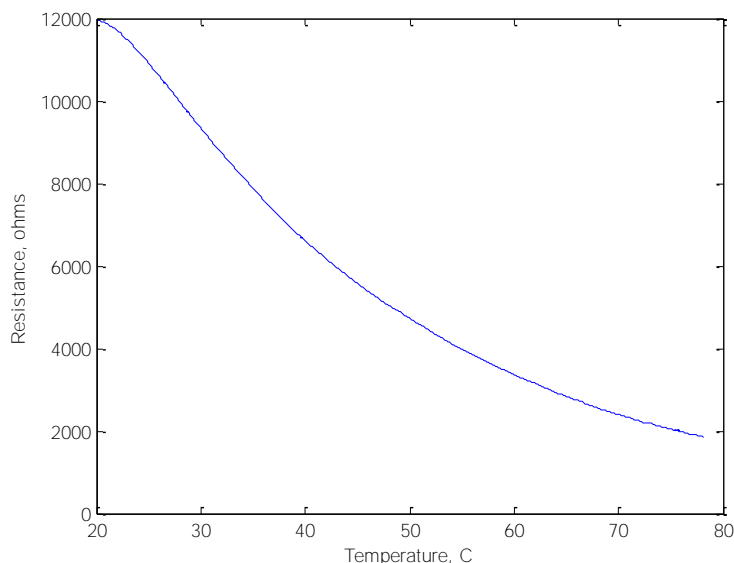


Figure 1. Thermistor temperature-resistance characteristic.

Pre-lab:

In the circuit of Fig. 2, the resistance R_{TH} is the variable resistance of the thermistor. (The arrow through the resistor symbol typically means that the resistance is not necessarily constant.) The voltage v_{out} is the voltage that we will use to indicate temperature. The 5V input voltage is applied across the two resistors as shown. The design problem is to choose a value for R so that v_{out} increases by a minimum of 0.5V over a temperature range of 25°C to 37°C. To do this,

ANALYSIS

1. Analyze the circuit of Fig. 2 to determine v_{out} as a function of R_{TH} and R .
2. Verify that v_{out} increases as temperature increases (or, equivalently, as R_{TH} decreases)
3. Using the temperature-resistance curve provided in Fig. 1, choose a value for R such that v_{out} changes by at least 0.5V over the specified temperature difference.

In your lab notebook, be sure to include the results of your analyses, including the equation(s) governing the system, your desired value for R , your expected output voltage change over the specified temperature range, and your expected output voltage at room temperature.

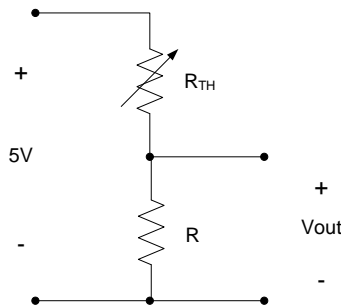


Figure 2. Temperature measurement circuit schematic.

Note on the design process:

In general, design is an *iterative* process. As new information becomes available, the design is revised to ensure that the design requirements are met or the design requirements are revised to ensure that they are realistic. Since this is our first experience with design, we will not iterate on the preliminary design from the pre-lab. Our pre-lab analysis, however, still provides valuable information which can be used to discuss our final design performance relative to system requirements. For example, we may claim that the design requirements were not met because our thermistor did not have the sensitivity provided in Fig. 1 or that our fingertips did not apply the desired 37°C temperature. Either way, we have a reason why we did not meet the design requirements – this can be important when discussing your system performance relative to the design requirements with your customer!

Lab Procedures:

Implement and test your design from the pre-lab. It is suggested that you perform at least the following steps when implementing your design:

DATA

1. Check the resistance variation of your particular thermistor, over the specified temperature variation. To do this, measure and record the resistance of the thermistor at room temperature. Then hold the thermistor firmly between two fingers and record the resulting resistance value. This provides the resistance variation over the desired temperature range.

2. Check your preliminary design generated in the pre-lab with the measured resistance variation determined in step 1 above. It is entirely possible that this step may indicate that your design does not meet the design requirements, however, do not modify your design to meet the design requirements.
- DATA**
3. Implement your design. Be sure to record actual resistance values for any fixed resistors used in your design.
- DATA**
4. Measure the voltage response to the specified temperature change. Record the output voltage at the high and low temperature conditions. Discuss your circuit's performance relative to the design specifications. (e.g. Were requirements met? If not, why?)
- ANALYSIS**
5. Calculate the percent error between the expected performance (based on your pre-lab analysis) and the measured performance.
- DEMO**
6. Demonstrate operation of your circuit to the Teaching Assistant. Have the TA initial the appropriate page(s) of your lab notebook and the lab checklist.

Post-Lab Exercises:

Using the thermistor provided in this lab assignment, design a temperature measurement system which meets the following requirements:

1. The output voltage must increase as temperature increases.
2. The output sensitivity of the device must be at least 0.1 V/°C (or 100 mV per degree Centigrade of temperature change).

Analyze the design to show that the requirements are met. You do not need to implement your design.

1.4.4 Background Information: Resistive Temperature Sensors

Most people have an intuitive concept of temperature, heat or cold is sensed directly via the sense of touch. Temperature is also an extremely important quantity from the standpoint of engineering and physics, in these disciplines we require a more technical definition of temperature. Temperature, engineering terms, provides a measure of the amount of internal kinetic energy that the molecules in an object have. This property governs a number of important physical phenomena, including:

- *Heat transfer.* Heat tends to flow from regions of high temperature to low temperature. The field of heat transfer, therefore, relies heavily upon determining the temperature distribution in a material.
- *Thermodynamics.* Temperature is a primary parameter which determines the thermodynamic state of liquids and gasses. For example, the amount of propane which can safely be stored in a given tank is a strong function of the temperatures that the gas will be subjected to; if the temperature exceeds specified levels, the tank may rupture – with potentially disastrous consequences.

Due to the extreme importance of temperature in many engineering analysis, temperature is one of the most commonly measured physical parameters. Many approaches have been developed to measure temperature. In this module, we will discuss temperature measurement processes which rely upon temperature-induced changes in electrical resistance. Measuring electrical resistance under these circumstances, allows us to infer temperature. Electrical resistance approaches to temperature measurement are extremely popular, since the signals resulting from these types of measurements are readily recorded, transmitted and analyzed.

Two relatively common approaches toward electrical resistance-based temperature measurements are resistive temperature detectors (RTDs) and thermistors. Thermistors are somewhat more widely used than RTDs, primarily because thermistors are considerably less expensive than RTDs. This module is by no means intended to be a comprehensive discussion of how to use either RTDs or thermocouples. We restrict ourselves to a presentation of the principle concepts relative to the operation of these types of temperature measurement device, along with some general information about how these instruments are used in practice.

Before beginning this module, you should be able to:

- Perform basic algebra
- Analyze resistive electrical circuits

After completing this module, you should be able to:

- Write an approximate linear relationship governing the temperature-resistance characteristic of RTDs
- Sketch an electrical circuit to convert resistance change to voltage change
- Write a nonlinear temperature-resistance relationship for thermistors
- List two sources of error commonly arising in resistive temperature measurements

References:

1. Holman, J.P., *Experimental Methods for Engineers*, 3rd edition, McGraw-Hill Book Company, 1978.

Resistive Temperature Detectors:

Many materials, especially metals, have resistivities which increase as temperature increases. Resistive Temperature Detectors (RTDs) utilize this property to provide a temperature measurement. RTDs can be made by winding a thin wire into a coil or depositing a thin film of metal on a plastic or ceramic substrate. This wire or film is then exposed to the temperature being measured; as the temperature increases, the resistivity of the wire or film increases, and the overall electrical resistance of the device increases. Platinum is commonly used in RTDs, since its resistivity varies linearly with temperature over a wide temperature range. RTDs have a number of advantages over other types of temperature measurements: their response is extremely linear over a wide temperature range and their readings are extremely stable. Among their disadvantages is their high cost: the high cost of platinum makes RTDs relatively expensive compared to other temperature measurement devices.

As noted above, the resistance vs. temperature relation is approximately linear. This relationship is commonly defined in terms of a linear temperature coefficient of resistance, α , as follows:

$$R = R_{nom} + \alpha R_{nom} (T - T_{nom}) \quad \text{Eq. 1}$$

where R is the resistance at the temperature T and R_{nom} is the resistance at some nominal (reference) temperature T_{nom} (often taken to be some convenient temperature such as 0°C or 0°F). This can be rearranged to determine the temperature from the measured resistance:

$$T = T_{nom} + \left(\frac{R - R_{nom}}{\alpha \cdot R_{nom}} \right) \quad \text{Eq. 2}$$

The temperature coefficient of resistance for platinum is approximately 0.00392 (reference [1]), though this value depends upon the grade of platinum used. It is always important to determine α accurately for the particular RTD being used, especially if large ranges in temperature are to be measured.

Over very large temperature ranges, the linear relationship of equation (1) between resistance and temperature may not be appropriate. In these cases, a quadratic relationship is generally employed:

$$R = R_{nom} [1 + a(T - T_{nom}) + b(T - T_{nom})^2] \quad \text{Eq. 3}$$

where R , R_{nom} , T , and T_{nom} are as defined above. a and b are experimentally determined constants.

RTD Signal Conditioning:

The resistance changes of RTDs are usually converted to a voltage change by electrical circuitry. Voltage levels have the advantage over resistances in that voltages can be readily transmitted, recorded, and used for control purposes. The resistance-to-voltage conversion is normally performed by a Wheatstone bridge circuit; a very simple Wheatstone bridge circuit is shown in Fig. 1. In the bridge circuit of Fig. 1, the RTD resistance is represented by a resistor with resistance $R+\Delta R$. The resistance R is the nominal resistance of the RTD, the resistance ΔR represents the change in resistance of the strain gage due to some temperature change. There are three fixed resistors in the bridge circuit of Fig. 1; these resistors all have the same resistance as the nominal resistance of the RTD. A fixed voltage, V_s , is applied to the circuit as shown. The output voltage, v_{ab} , is the voltage which is used to represent the change in resistance of the RTD. Without proof, we claim that the output voltage is:

$$v_{ab} = \frac{\Delta R}{2(2R + \Delta R)} \cdot V_s \quad \text{Eq. 4}$$

if $\Delta R \ll R$, which is the usual case for RTDs, this simplifies to:

$$v_{ab} \approx \frac{V_s}{4R} \Delta R \quad \text{Eq. 5}$$

and the output voltage is linearly related to the change in resistance of the RTD. Since the RTD resistance is linearly related to the temperature, the output voltage is thus a linear function of temperature.

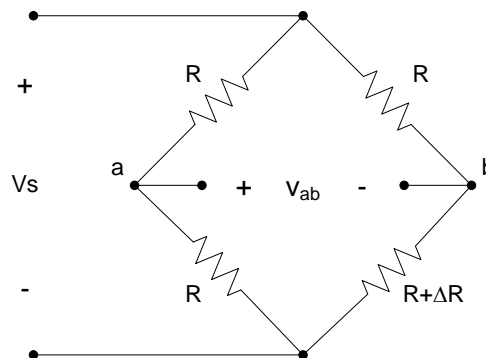


Figure 1. Wheatstone bridge circuit.

Thermistors:

Thermistors are semiconductor devices whose resistance, like RTDs, varies with temperature. In fact, thermistors can be considered to be a special case of an RTD. Thermistors, however, have a number of properties which distinguish them from RTDs as presented in the previous section. These differences are outlined below:

- The resistance of thermistors commonly decreases as temperature increases. Thermistors with this type of resistance-temperature characteristic are called Negative Temperature Coefficient (NTC) thermistors. Thermistors whose resistance increases with temperature are available; these are called Positive Temperature Coefficient (PTC) thermistors.
- Thermistors generally do not exhibit a linear relationship between temperature and resistance. The relationship between temperature and resistance for a thermistor is commonly modeled by an exponential relation. Reference [1], for example, provides the following resistance-temperature relationship:

$$R = R_{nom} e^{\left[\beta \left(\frac{1}{T} - \frac{1}{T_{nom}} \right) \right]}$$

Eq. 6

where R is the resistance at the temperature T and R_{nom} is the resistance at some reference temperature, T_{nom} . This nonlinear resistance-temperature relation is a significant drawback to the use of thermistors; however, the thermistors' advantages often outweigh this disadvantage, making thermistors one of the most common choices for temperature measurement devices.

- Because of the thermistors' nonlinear resistance-temperature relation, thermistors are useful over smaller temperature ranges than RTDs.
- Thermistors generally have a much higher sensitivity than RTDs – the resistance change resulting from a given temperature variation is considerably larger for a thermistor than for an RTD.
- Thermistors generally have a faster temperature response than RTDs. This makes them a better choice for time-varying (transient) temperature changes than RTDs.
- Thermistors are less expensive than RTDs.

Thermistor Signal Conditioning:

A bridge circuit like that of Fig. 1, with the thermistor used for the $R+\Delta R$ resistance, is often used to convert the thermistor's change in resistance to a voltage change. Unlike the RTD, however, the output voltage vs. temperature relation is generally nonlinear. Also, if an NTC thermistor is used, the output voltage of the circuit of Fig. 1 will decrease as temperature increases. Modifications to the bridge circuit of Fig. 1 can alleviate both of these drawbacks – bridge circuits using a second thermistor can help provide a more linear response, while switching the polarity of the output voltage will result in an output voltage which increases with increasing temperature.

Issues associated with resistive temperature measurement:

Several issues must be considered when using resistive temperature measurement devices. When using a bridge circuit like that shown in Fig. 1, the externally supplied voltage V_s will result in some current flowing through the resistive temperature measurement device. This current will induce heating of the resistive temperature measurement device, due to the resistive power dissipation in the device. Care must be taken that this so-called *self heating* of the device does not introduce excessive heating in the device itself, which can invalidate the temperature measurement.

Another common source of error in electrical resistance temperature measurements is the resistance of the leads connecting the device to the bridge circuit used to convert the resistance change to a voltage change. Alternate bridge circuits can compensate for this effect.

4. In the space below, provide your measured circuit voltage response over desired temperature range. Also discuss your results relative to design requirements, including the percent difference between expected and measured values. (8 pts)

5. **DEMO:** Have a teaching assistant initial this sheet, indicating that they have observed your circuits' operation. (5 pts)

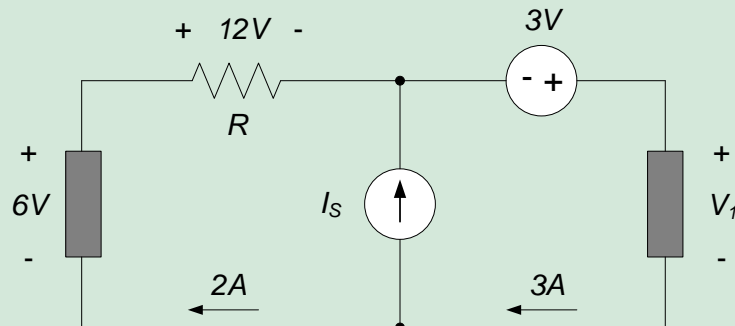
TA Initials: _____

6. Post-lab Exercise: Temperature Measurement System Re-Design (10 pts total)

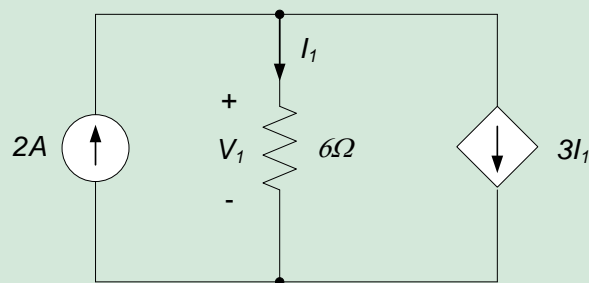
Provide below a schematic of your proposed design, including any voltage supply requirements and component values. Provide a brief analysis of your design's performance, along with a comparison of the performance vs. the design specification (e.g. does your design meet the design requirements?)

Real Analog Chapter 1: Homework

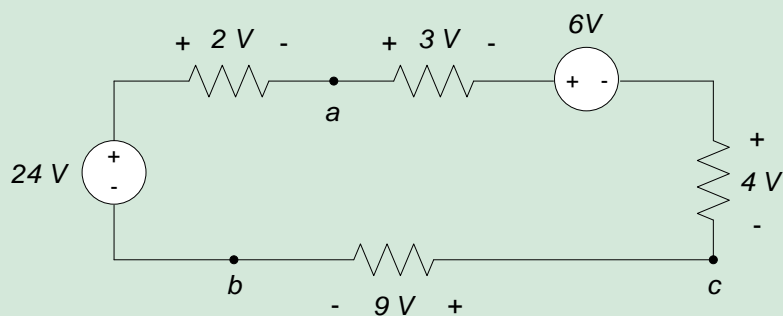
- 1.1 For the circuit below, determine
- the voltage, V_1
 - the resistance, R
 - the current, I_s
 - the power (absorbed or supplied) by the current source
 - the power (absorbed or supplied) by the 3V voltage source



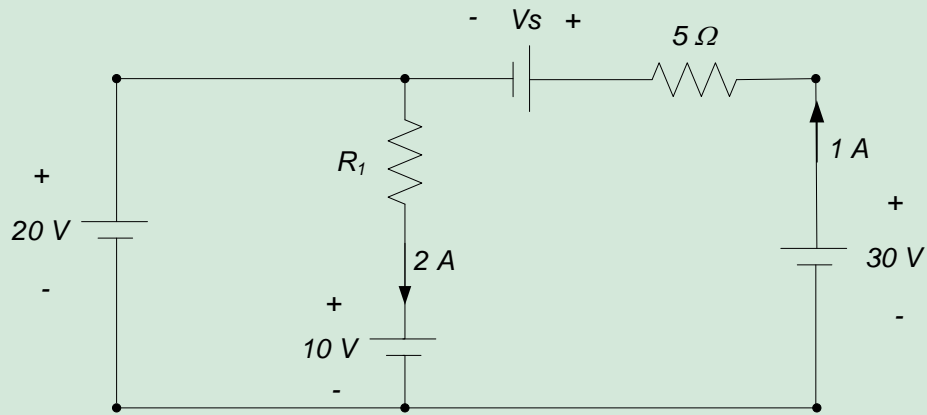
- 1.2 Find I_1 and V_1 in the circuit below.



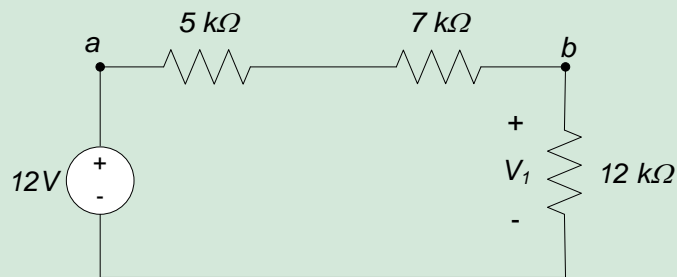
- 1.3 In the circuit below, determine v_{ab} , v_{ac} , and v_{cb} .



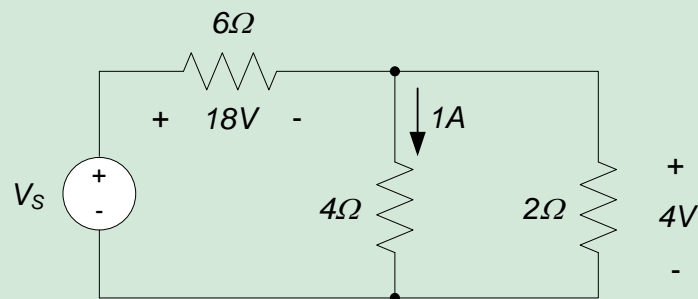
- 1.4 For the circuit shown, find:
- V_s
 - R_1
 - Power absorbed or generated by the 10V source. State whether the power is absorbed or generated.



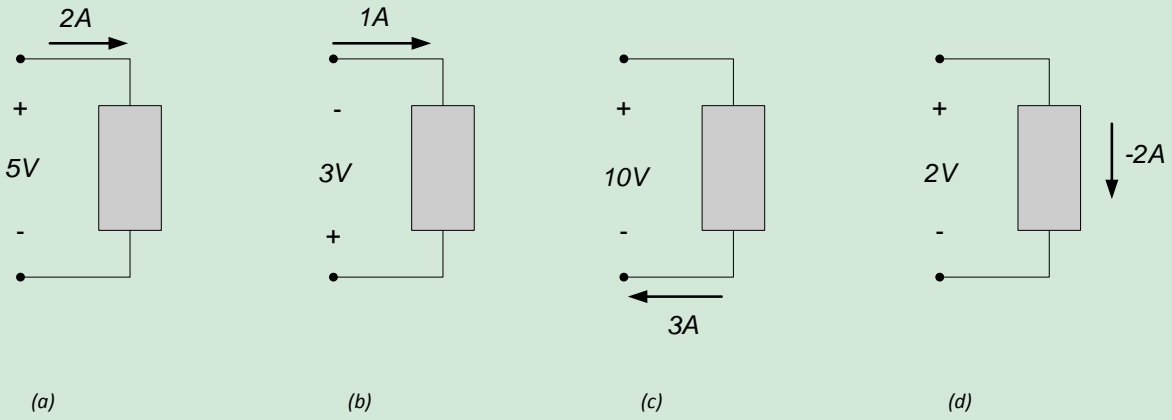
1.5 Find V_{ab} and V_1 in the circuit below.



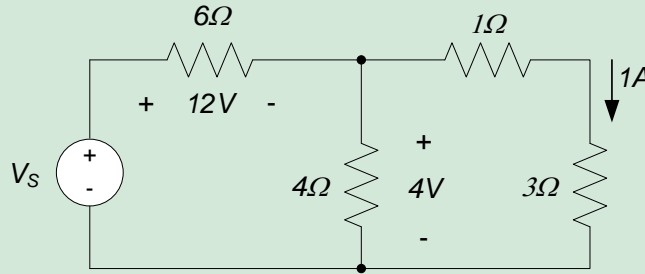
- 1.6 For the circuit below, determine
- the power dissipated by the 6Ω resistor
 - the voltage across the 4Ω resistor
 - the current through the 2Ω resistor



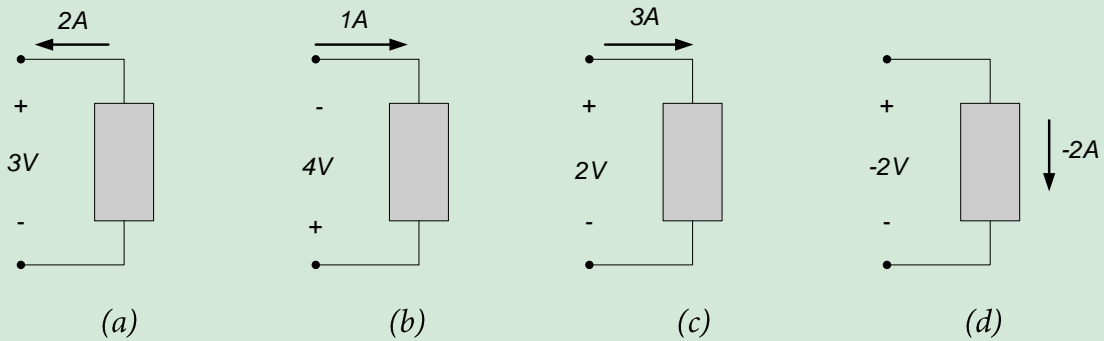
1.7 For the circuit elements below, determine the power generated or absorbed by the element. State whether the power is generated or absorbed.



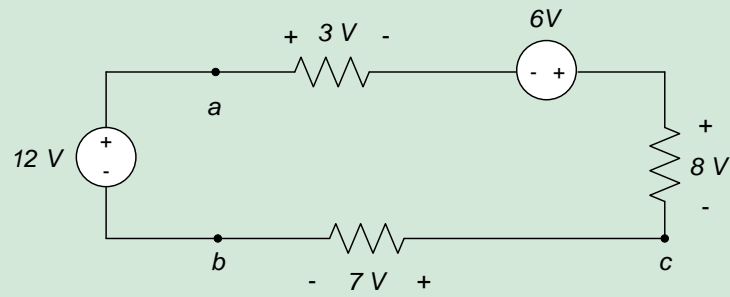
- 1.8 For the circuit below, determine
- the power dissipated by the 6Ω resistor
 - the voltage across the 3Ω resistor
 - the current through the 4Ω resistor



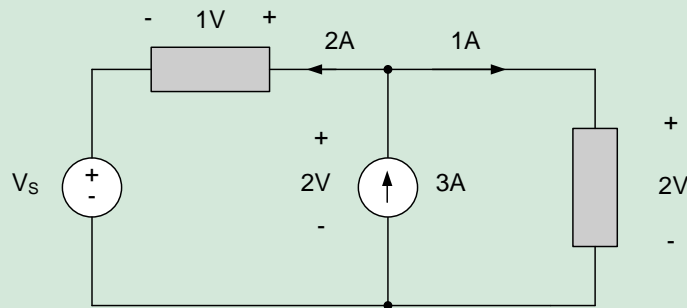
- 1.9 For the circuit elements below, determine the power generated or absorbed by the element. State whether the power is generated or absorbed.



- 1.10 In the circuit below, determine v_{ab} , v_{ac} , and v_{cb} .

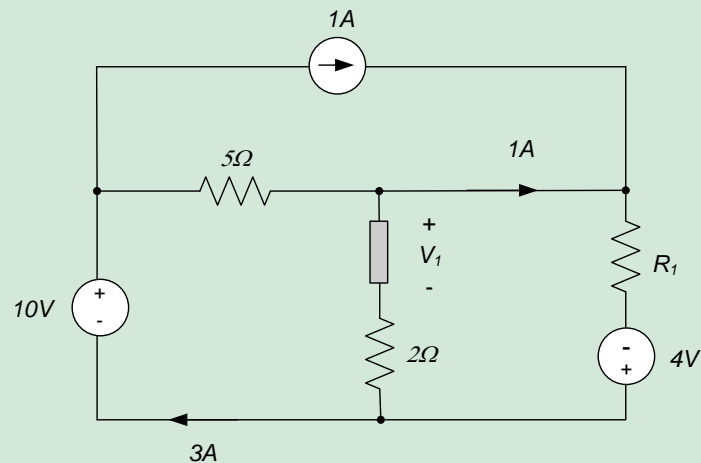


1.11 In the circuit below, determine the power (absorbed or generated) by the voltage source V_s .



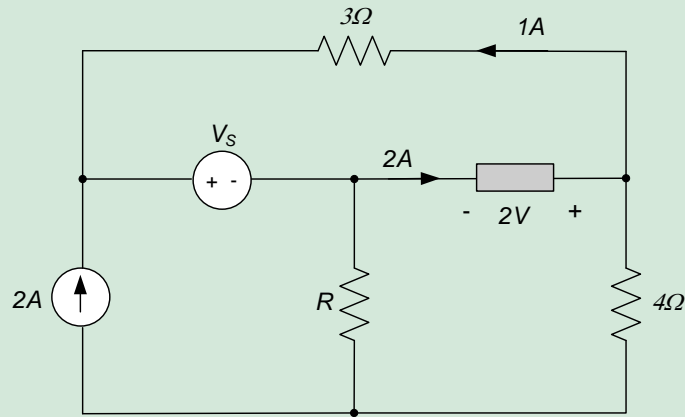
1.12 For the circuit shown, find:

- a) V_1
- b) R_1
- c) The power (generated or absorbed) by the 1A source

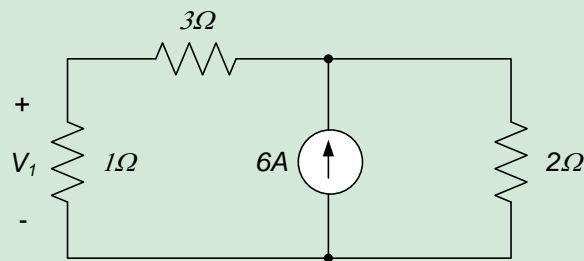


1.13 For the circuit shown, find:

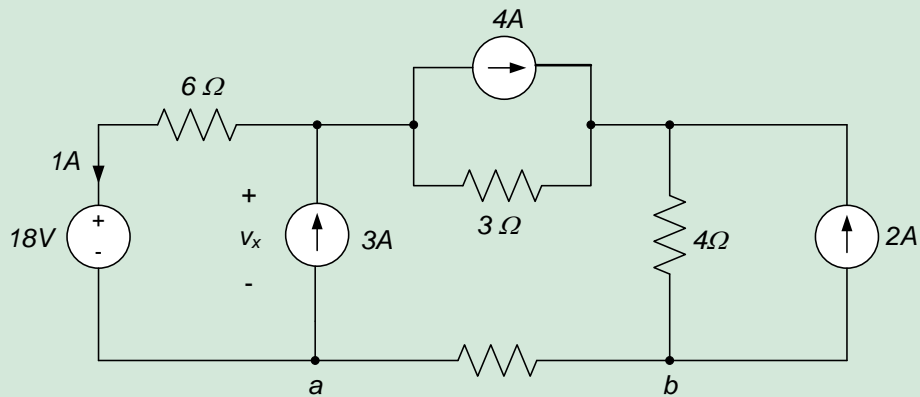
- a) V_s
- b) R
- c) The power (generated or absorbed) by the 2A source



1.14 Write a set of equations from which you can determine the current V_1 in the circuit below. You do not need to solve the equations.



1.15 For the circuit below, determine the voltage v_x , the voltage v_{ab} , and the power (generated or absorbed) by the 3A source.



1.16 For the circuit shown, find the voltage v_x and the current i_4 .

